Antonine Physics A2

Topic 15 Supplementary Tutorials

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is a very long Topic which is a catch-all for topics that are not part of the AQA, Edexcel, or OCR syllabuses. They are, however, an essential part of other syllabuses such as the SQA Advanced Higher Syllabus.

Tutorials 15.01 to 15.08 are for the SQA Advanced Higher syllabus.

Tutorials 15.09 to 15.14 are for the Welsh Board and Eduqas Syllabus.

Tutorials 15.15 to 15.17 are for the Cambridge Pre-U and IB syllabuses.

Tutorials 15.18 to 15.20 are for the IB syllabus.

You should NOT attempt to read everything that is in Topic 15. Just do the bits that are relevant to your syllabus.

You may find them helpful in your first year of university study. However, the university level material will be in much greater depth than I have covered here.

Topic 15

1. Cosmology

Tutorial 15.01 General Relativity

SQA Syllabus

Contents

Before you attempt this tutorial, look at Topic 14D *Turning Points in Physics* Tutorials 5 and 6.

15.011 Equivalence Principle

You feel gravity in the form of your weight. As you know, weight is the force you feel as gravity attracts you to the floor. It is the force that will cause you to accelerate at 9.81 m s^{-2}, if a hole appears in the floor under you. If you are in a sealed lift (i.e. with no windows) going upwards at a constant speed, you will feel your normal weight (*Figure 1*).

Figure 1 Feeling your normal weight when going up at constant speed.

This would be confirmed if you stand on a bathroom scales.

Now suppose the cable to the lift breaks, and the lift plunges down the mineshaft. Your weight will feel as if it were zero. This is because you are accelerating at the same rate (9.81 m s^{-2} downwards) as the lift. You would float. A football with you would also float (*Figure 2*).

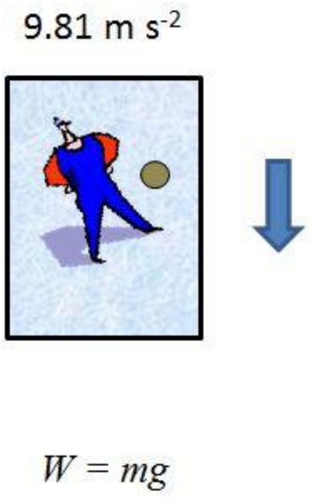

Figure 2 Weightlessness in a falling lift

This feeling of weightlessness is because you, the lift, and the football are all accelerating down the mineshaft at 9.81 m s^{-2}. **They all still have a weight.** However, relative to each other, there is zero acceleration, hence zero force (weight). In your sealed lift cage would not be able to tell whether this was because gravity had been turned off, or that you were accelerating (towards your death). The only way you would see that you were accelerating is if you could see the sides of the mineshaft rushing past you.

Now suppose (belatedly) the emergency braking system on the lift carriage works, you will see the football hit the floor at the same time as you do (*Figure 3*).

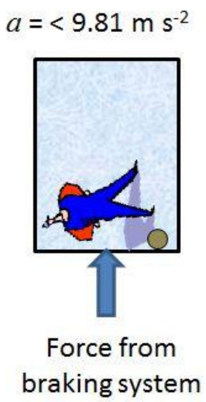

Figure 3 Lift cage slowing down very quickly

In your sealed system, you don't know if gravity has been turned back on, or that there is an upward force that is slowing the lift carriage down. Of course, you are a physicist, and you know that you can't turn off gravity. But the effects are the same.

So, this thought experiment led Albert Einstein in 1907 to come up with the Principle of Equivalence. (He was sitting in his office at the Patents Office in Bern at the time. He was definitely off task.) The principle states:

In a sealed system, it is impossible to distinguish the physical effects due to gravity and acceleration.

We know that the gravitational field strength, g, is a force per unit mass (N kg^{-1}) and it is an acceleration (m s^{-2}).

15.012 Effect of Gravity on Light

We know that light travels in straight lines. Now suppose we have another lift carriage. This one works properly and can accelerate upwards. In this lift, there is a small hole through which a beam of light can pass. At exactly the same place on the other side, there is a second hole (*Figure 4*).

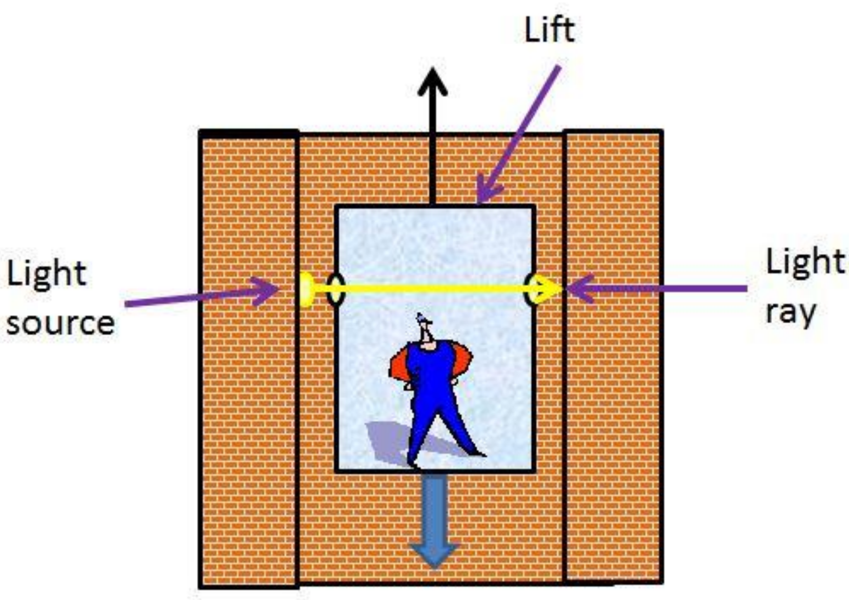

Figure 4 Effect of gravity on light

Therefore, the ray of light will pass through the second hole as shown. However, suppose we make the lift accelerate upwards, and we tracked a particular photon. From outside the lift, we know that the photon will continue to travel in a straight line. However, if we were sealed in the lift, we would see that the photon would appear to move downwards in a parabolic path (*Figure 5*).

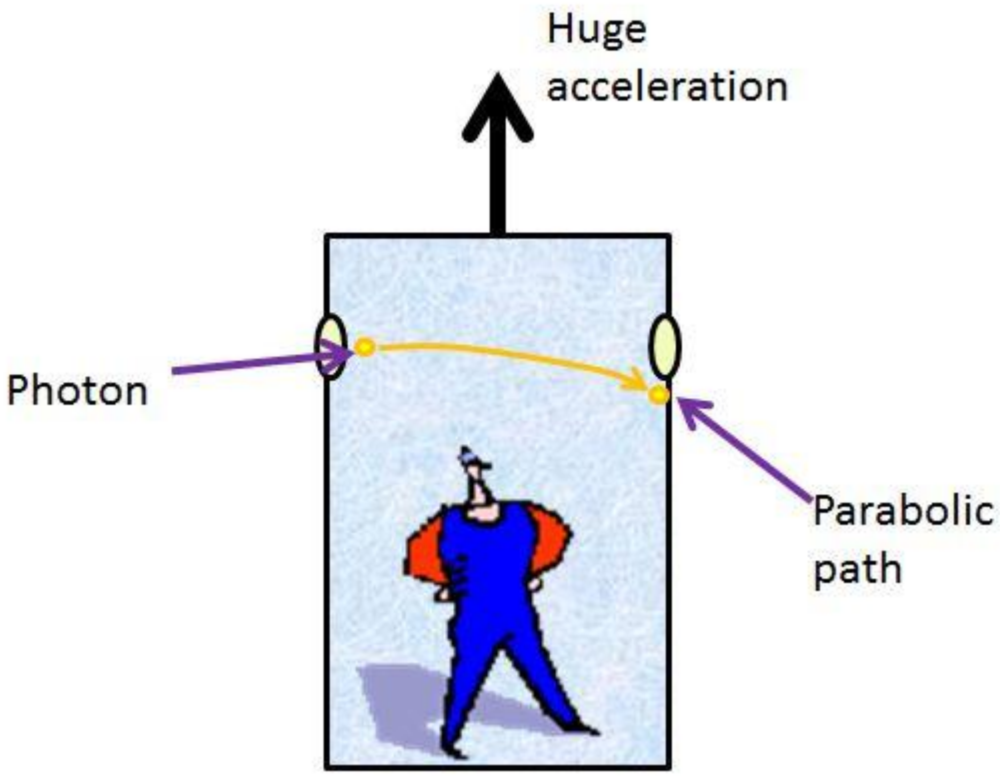

Figure 5 Photons travelling in a parabolic path through a lift accelerating at a huge rate

So, what can we conclude from this? Even though a photon is a particle with zero mass, from the point of view of the observer in the lift, its parabolic path suggests that its path is deviated by an acceleration. By Einstein's Equivalence Principle it should therefore be subject to gravity and can be bent by the gravity of a very large object, like a big star, or a black hole.

Now, of course, this thought experiment is impossible to reproduce in real life:

- The acceleration would be massive (about 5×10^{33} m s^{-2}) - or the lift impossibly wide.
- An individual photon, being a quantum being, is impossible to track.

Sorry to spoil a good story…

However, astronomers have found things that are explained by this argument.

15.013 Space Time

Newton considered gravity to be a force. And that is how we generally deal with it. However, Einstein produced a different model that treats gravity in a completely different way. He used a geometry that replaced the traditional (Euclidian) notion that forces act along straight lines of least effort. On a flat surface, we know that the shortest distance between two points is a straight line. On a curved surface, however, the shortest distance is a **geodesic**. This is important for pilots making long journeys in aeroplanes. Consider the way that pilots fly from Australia to South America. On the traditional **Mercator projection**, the shortest line is in *Figure 6*:

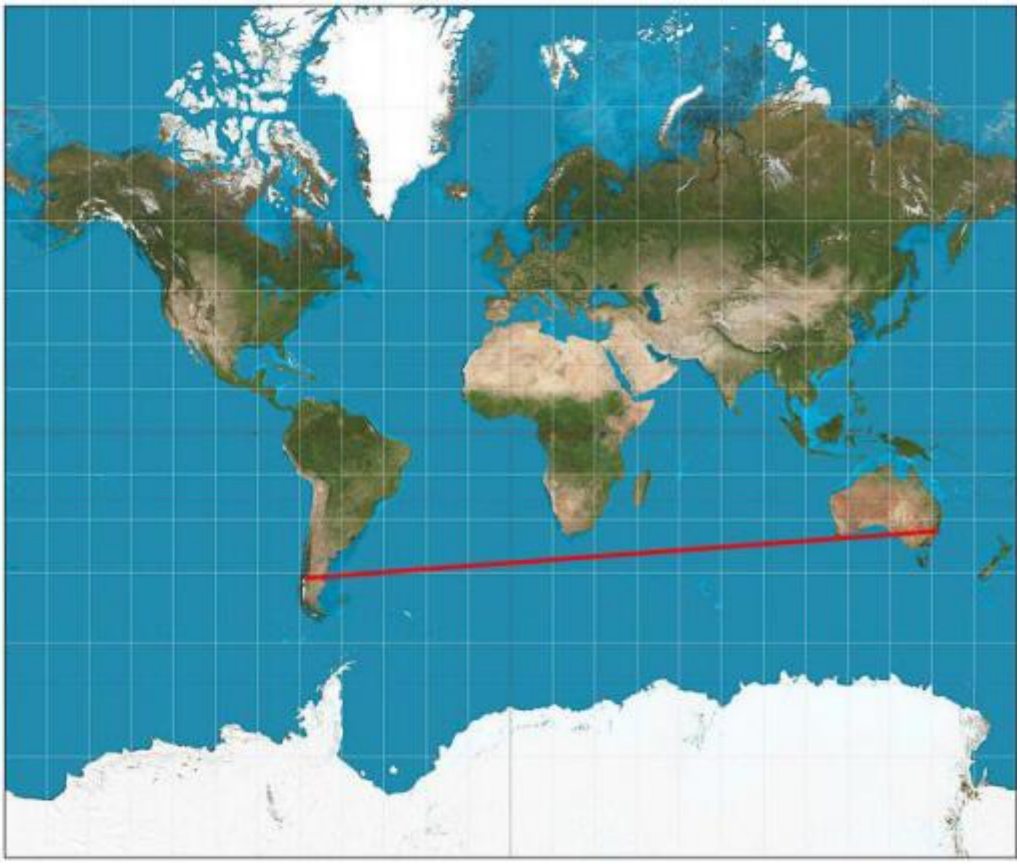

Figure 6 Flying from Australia to South America (Image from Wikimedia Commons)

Now if we plot the course across the spherical surface of the Earth we see *Figure 7*:

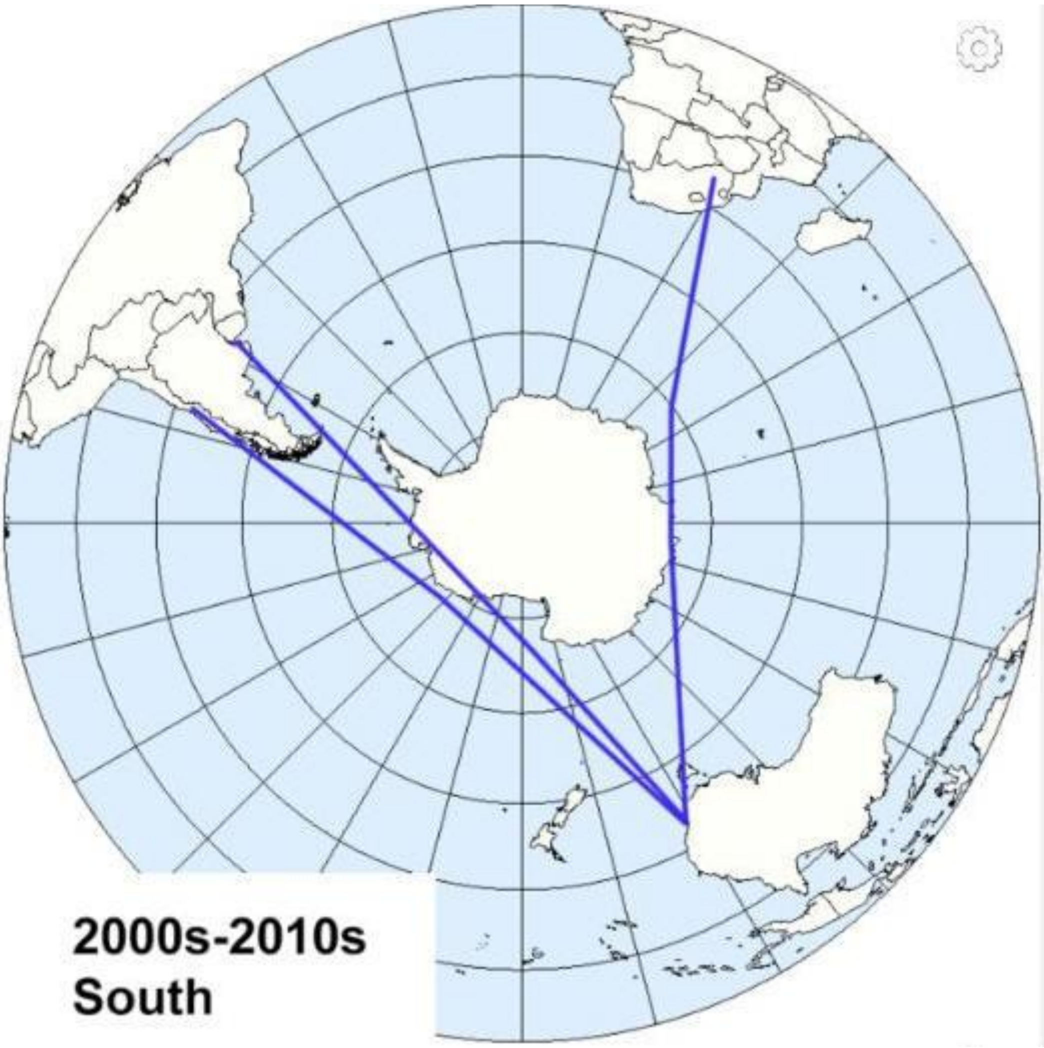

Figure 7 Geodesic flight paths between Australia and South America (Image: Rolypolyman, Wikimedia Commons)

Such courses are called **Polar Routes**.

Einstein considered that space was **curved**. He proposed the idea of space-time, with a **fourth** dimension of time. The diagram below (*Figure 8*) shows the idea of dimensions.

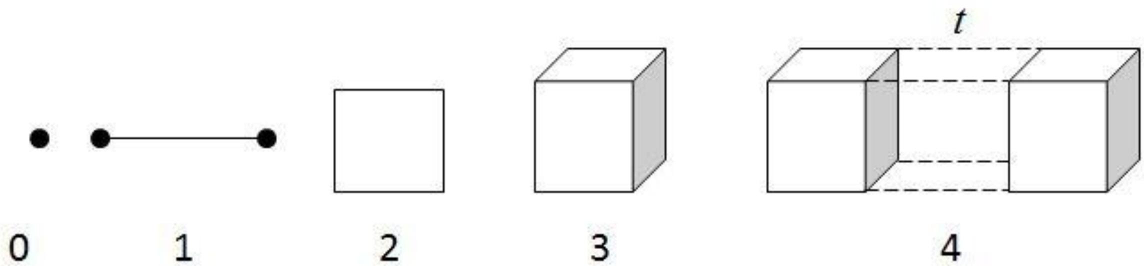

Figure 8 Dimensions

The shortest line between two points on the curved surface of space-time is a geodesic.

Gravity could be explained by depressions in space-time. The best way of explaining this to someone who (like me) finds it hard to visualise is to model space-time as a rubber sheet. Consider a rectangular trampoline frame (*Figure 9*):

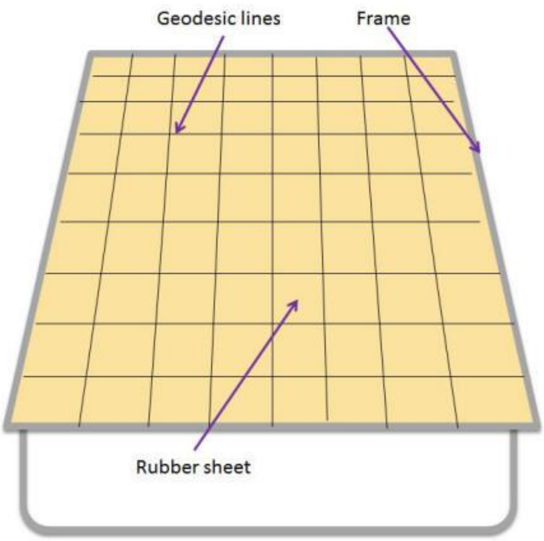

Figure 9 Space-time can be modelled as a rubber sheet on a trampoline frame

We can put large objects onto the rubber sheet like this (*Figure 10*):

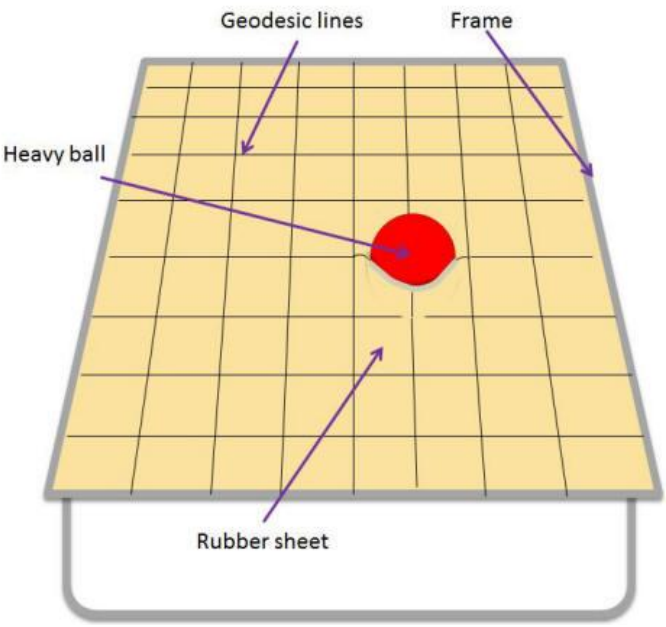

Figure 10 Large objects placed on the rubber sheet

Viewed from the side (*Figure 11*):

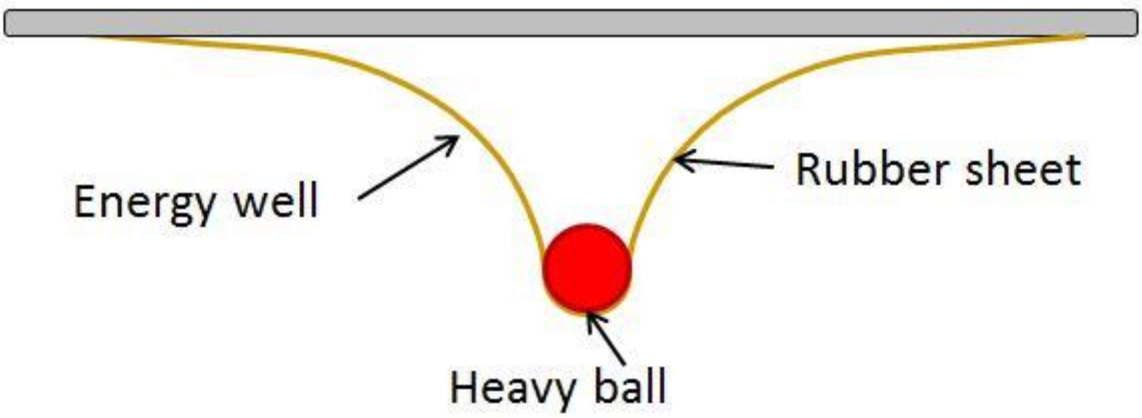

Figure 11Side view

If you push a smaller ball at an angle into the depression (or **energy well**) made by the heavier ball, you will find that the ball will make a **spiral path** into the depression. The depression is sometimes called a **warp**. If you fire the ball fast enough, it will go around the depression in an orbit, which is the path that uses the least energy as it runs along an **equipotential**. If the speed is high enough, the small ball will come out of the depression.

The mathematical analysis is complex and will not be considered here.

There is no central frame of reference for the universe. The most logical one to use is the frame of reference that is based on the Earth and its immediate environs.

So far, we have discussed space-time as a flat surface. The exact shape is matter of debate among cosmologists. Some consider it to be flat, others curved, while others say that it's a sphere. Theoretical Physicists consider that there are even more dimensions than the fourth suggested by Einstein.

For a more detailed discussion of Space-Time, see Tutorial 18.

15.104 Evidence

While we cannot see space-time directly, there are observations that have been made that cannot be explained without reference to space-time.

GPS

The Global Positioning System is a set of 24 satellites that was originally intended for military navigation. Now it's used worldwide, not just for navigation in the car, for aeroplanes, but also for accurate positioning, vital for surveying for construction projects. You can buy a hand-held device that will give you a precise positioning of latitude and longitude down to the nearest ten metres. Each satellite has a very accurate and precise atomic clock, accurate to 50 nanoseconds a day. However:

- Each satellite is 20 200 km above the surface of the earth.
- Each satellite has an orbital speed of 3900 m s^{-1}.
- As a result of special relativity, each clock ticks slower by about 7 microseconds a day when compared to a clock on the ground.
- Also, due to curvature of space and time as predicted by Einstein' theory, the clocks gain about 45 microseconds a day.

Therefore, in total the clocks gain 38 ms every day. This corresponds to a distance of 11 km. Accumulated uncertainties to this extent would lead to the system rapidly becoming useless, so the time shift is compensated for.

The Orbit of Mercury

Like all plants, Mercury's orbit is elliptical, in accordance with Kepler I. Its point of closest approach to the Sun is called the **perihelion**. It had been noted by astronomers that the perihelion of Mercury's orbit advanced slightly with each orbit. The change in position is about 0.67° per century. The idea is shown below (*Figure 12*):

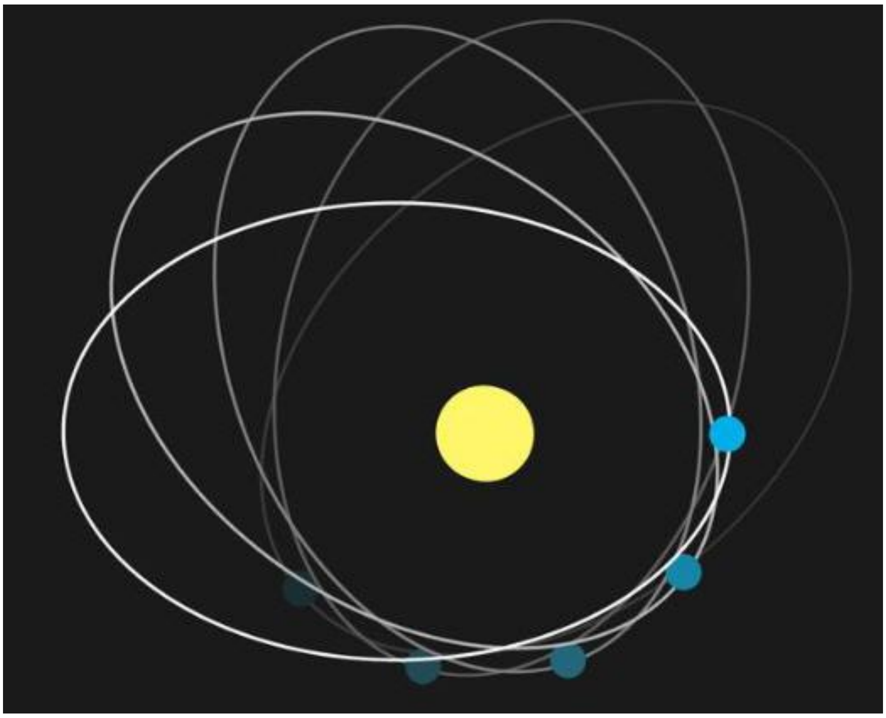

Figure 12 Precession in the orbit of Mercury (Image by Rainer Zenz - Wikimedia Commons).

The change in position is called a **precession**. The orbit is anticlockwise and the movement of the perihelion is anticlockwise. This happens because the orbit of Mercury is within the warp (curvature) made by the Sun in space-time.

Gravitational Red Shift

Light being emitted by stars has been observed as being slightly red shifted as it leaves the gravitational depression in space-time caused by a star. This was first identified in 1925 by the astronomer Walter Sidney Adams (1876 - 1956) on the star Sirius B. There were difficulties in the measurement due to interference of light from Sirius A. The first accurate measurements were carried out in 1954 on the white dwarf star 40 Eridani B, which indicated a red shift of 21 km s^{-1}. The red-shift from Sirius B was finally worked out as 89 km s^{-1} using the Hubble telescope (*Figure 13*).

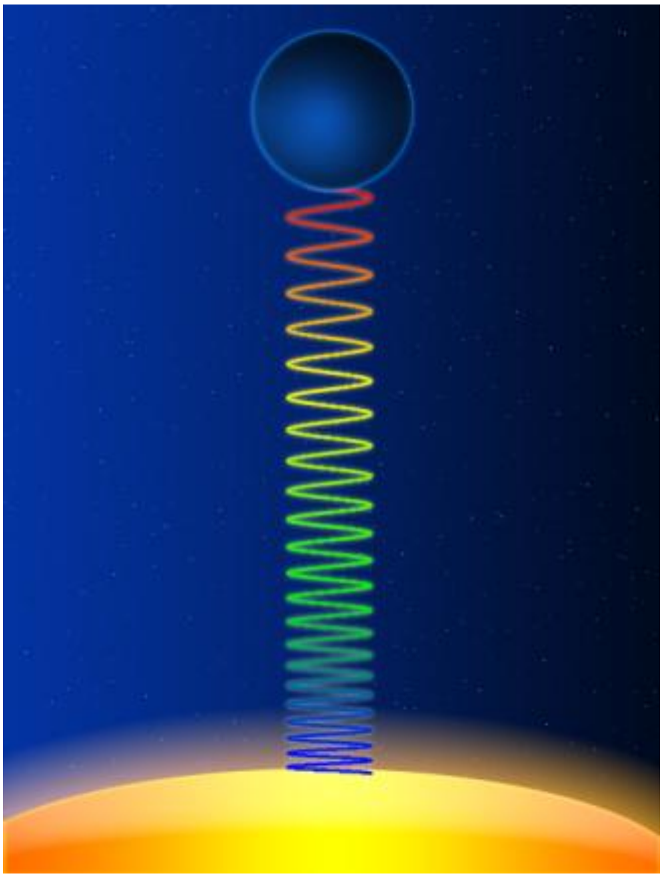

Figure 13 Red shift from Sirius B

An experiment to demonstrate "red-shift" in gamma rays was carried out by the physicists R V Pound (1919 - 2010), and G A Rebka (1931 - 2015). Gamma rays from an iron-57 source fixed to a loudspeaker were transmitted vertically up a 22.5 m tower from the basement. Gamma rays from an identical source fixed to a second loudspeaker placed at the top of the tower were transmitted down the tower. The results suggested that a red shift with an uncertainty of about 1 % was detected.

A more detailed account of the Pound-Rebka experiment can be found at Tutorial 20.

Gravitational Lensing

After Einstein had made his predictions about space-time, the British physicist, Arthur Eddington (1883 - 1944) observed an eclipse of the Sun on the island of Principe (off the West coast of Africa) in the year 1919. He predicted that the positions of several stars behind the Sun would change slightly as a result of the action of gravity on light. And the predictions were supported by the results.

In 1936 Einstein developed his theory of space-time further. He argued that light could follow the gravitational warp made by a sufficiently large object. Therefore, the light would follow a path rather like the path of light through a glass lens. Therefore, gravity could have a lensing effect on light. In 1979, astronomers were observing a distant galaxy when they noticed multiple images of a quasar around the galaxy. This was evidence to support Einstein's idea.

Gravitational lensing works like this (*Figure 14*):

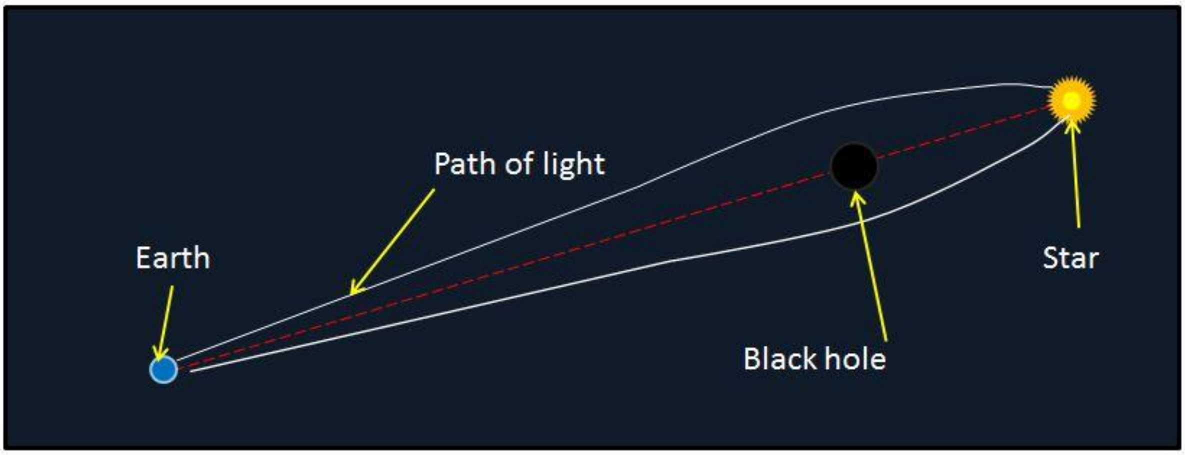

Figure 14 Gravitational lensing

Suppose we are observing a distant large black hole behind which was an even more distant star. The black hole would completely obscure the star, of course. However, the **warp** caused by the **gravity** of the black hole in space-time would guide the light around the black hole and bring it to a focus at the Earth. We would see **multiple images** of the star, like this (*Figure 15*):

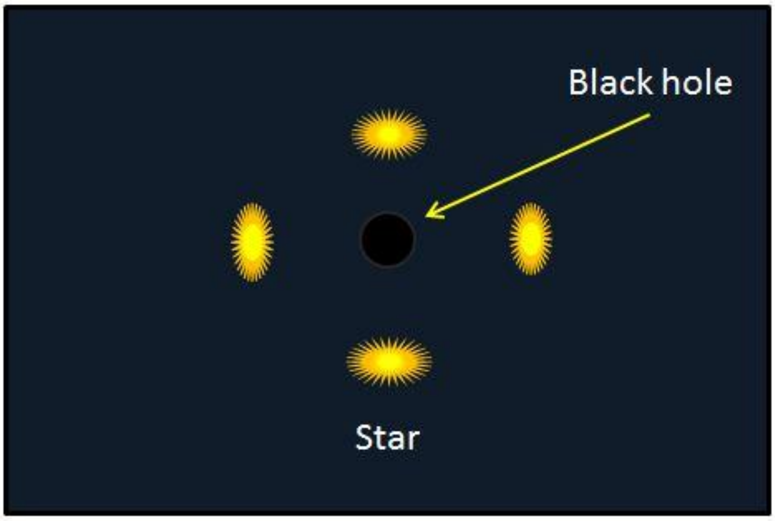

Figure 15 Effect of gravitational lensing

By measuring the amount by which the light has been deflected, astronomers can calculate the mass of the object that is causing the deflection. However, when the lensing caused by galaxies was analysed, the masses of the galaxies have been found to be much greater than the mass that could be accounted for using calculations from the visible light. This suggested the presence of **dark matter**, named such for the simple reason it could not be seen. Dark matter is considered in Topic 14A Tutorial 7.

15.015 World Lines

A **world line** is a unique line that is traced through space-time by an object. The path can be plotted on a time - space-time graph like this (*Figure 16*):

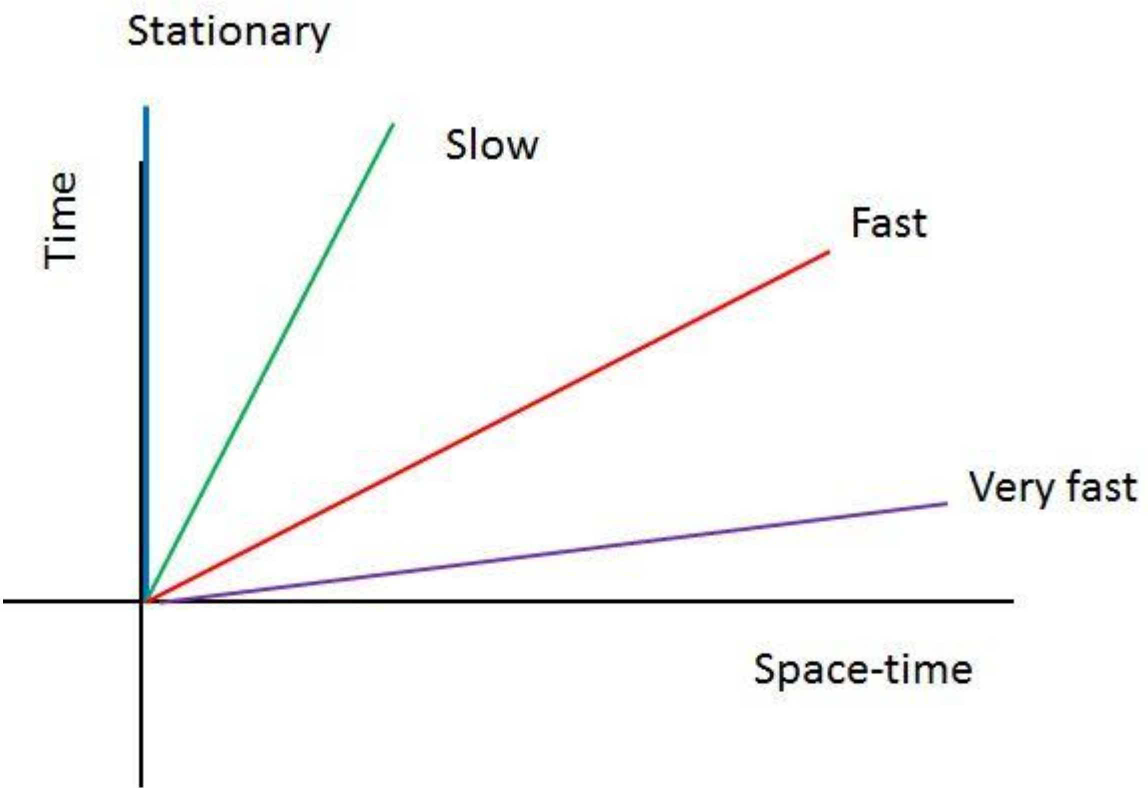

Figure 16 A space-time time graph

The world lines are shown on a two-dimensional graph here, but there can be a second space-time axis to give a three-dimensional graph. They can also be curved. This graphs are widely used in **string theory**. Some theoretical physicists have suggested that these lines might give the ability to travel forward and backward in time - every science fiction writer's dream.

Further material on Space-time can be found at Tutorial 18.

Questions

Tutorial 15.01

There are no questions for this tutorial.

2. Quantum Theory (SQA)

Tutorial 15.02 The Bohr Model

SQA Syllabus

Contents

15.021 Excited Atoms

The electron layout of an atom is usually presented as electrons orbiting a nucleus. It gives us an easy to understand arrangement of the nucleus and the electrons. Consider the **lithium** atom (*Figure 17*):

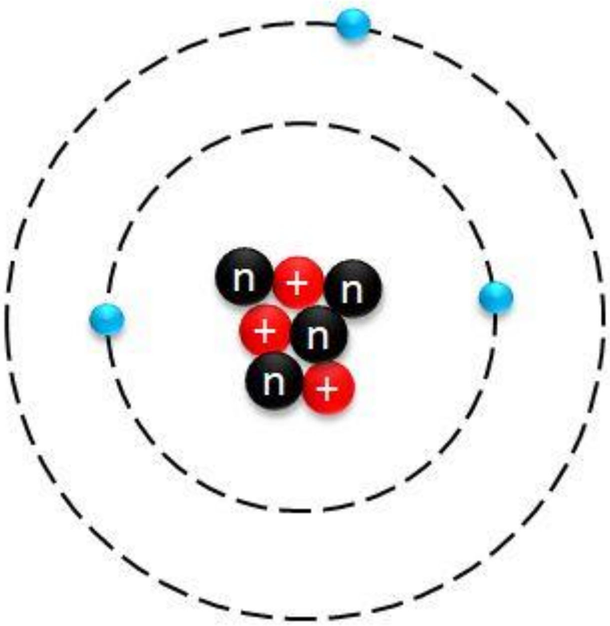

Figure 17 Lithium atom

Here we see the **nucleus** in the centre, with the electrons arranged in **shells**. This was the structure of the atom as proposed by Neils Bohr (1885 - 1962). He linked together the spectral behaviour of the hydrogen atom with his model of the atom. While his explanation was not complete, it is still a good representation and useful for introducing quantum theory to students.

Bohr's Model gives more than the simple structure of an atom. It can be used to explain how excited atoms give out spectral lines. You can look this up in Quantum 4 for the hydrogen atom.

The hydrogen atom is the simplest atom, consisting of a single proton and a single electron (*Figure 18*).

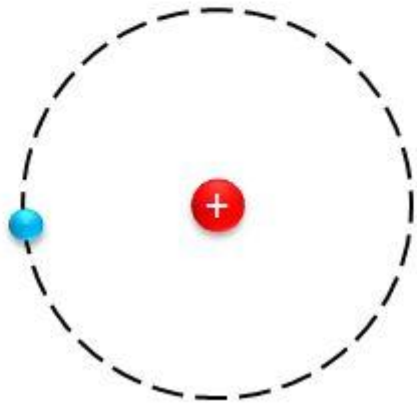

Figure 18 Hydrogen atom

Bohr's model went on to show how an excited electron could occupy higher orbits like this (*Figure 19*):

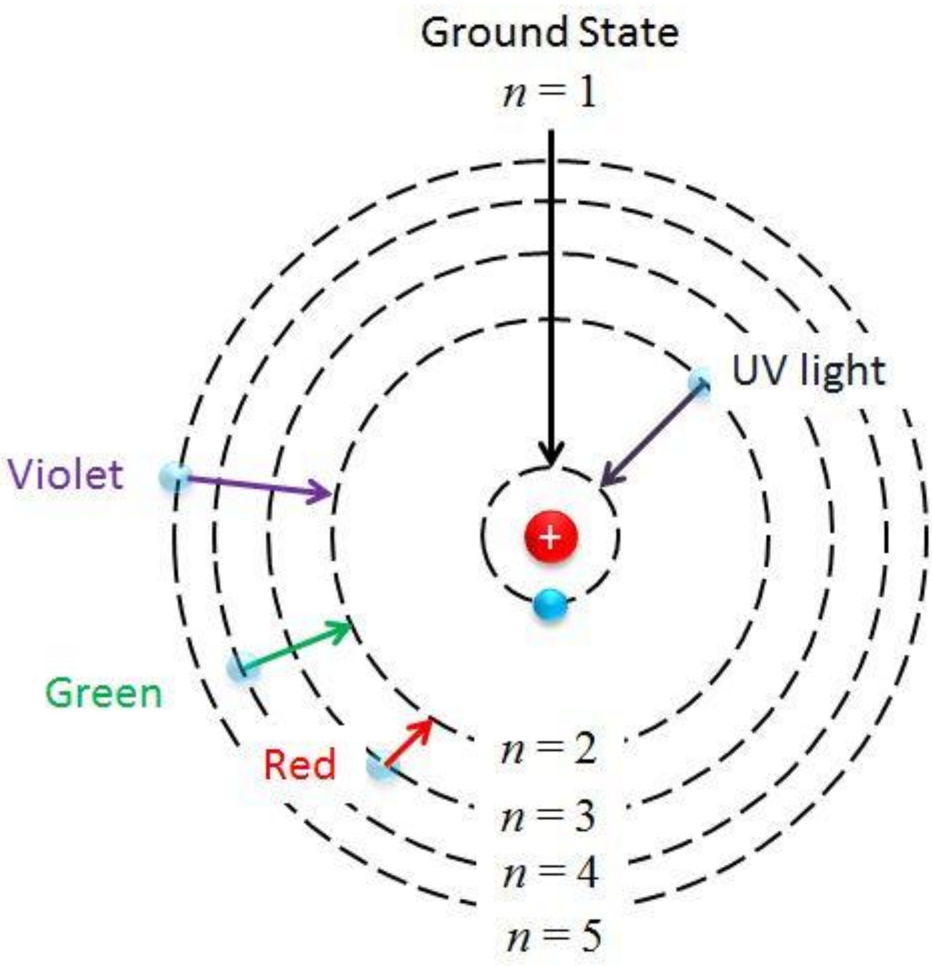

Figure 19 Excitation levels of a hydrogen atom

The electron normally occupies the **ground state** (n = 1) but when excited, it occupies a higher level. We can show the different states as higher orbits in this model. If the electron has sufficient energy, it can go to one of these higher orbits. It almost immediately falls back to the ground state, which it can do by dropping the whole way back to the ground state in one leap, or by dropping down the orbits one (or two) at a time. So, if we have an electron at n = 3, the photons emitted are:

- Red as the electron falls from n = 3 to n = 2.
- UV as the electron falls from n = 2 to n = 1.
- Shorter wavelength UV as the electron falls straight from n = 3 to n = 1.

We can quantify this in an equation. If an electron is at an excited level (E_1) and makes a transition to a lower level (E_2), then the energy ΔE of the photon given out can be worked out with the equation:

$$\Delta E = E_1 - E_2$$ .... *Equation 1*

The strange looking symbol Δ is Delta, a Greek capital letter 'D'. It is physics code for 'change in' or 'difference in'.

Since

$$\Delta E = hf$$ .... *Equation 2*

we can rewrite this as:

$$hf = E_1 - E_2$$ .... *Equation 3*

15.022 Energy of an Electron in Orbit

Bohr used classical physics to describe the behaviour of the electron in an orbit.

Consider an electron of mass m and charge $-e$ orbiting a nucleus of charge of $+Ze$ at a linear speed of v and a radius of r. We know that the centripetal force is:

$$F = \frac{mv^2}{r}$$ .... *Equation 4*

Strictly speaking, we should write:

$$F = -\frac{mv^2}{r}$$ .... *Equation 5*

This is because centripetal force is an attractive force.

We also know from Topic 9 (Fields) that for the force in an electric field:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

.... *Equation 6*

Where Q is the charge.

In the hydrogen atom, the proton number $Z = 1$. So, we have one electron and one proton, both of which carry the charge of magnitude $1e$. However the proton has a positive charge, and the electron has a negative charge. The equation becomes:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{-e^2}{r^2}$$

.... *Equation 7*

The minus sign tells us that the force is attractive.

We can equate these two equations (*Equations* 6 and *7*) and write:

$$-\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{-e^2}{r^2}$$

.... *Equation 8*

This is handy because we can get rid of the minus signs (which can be a damned nuisance at times). So, we write:

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

.... *Equation 9*

The r term on the left cancels with the r^2 term on the right to give:

$$mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

.... *Equation 10*

The kinetic energy of the orbiting electron can also be found:

$$E_\mathrm{k} = \frac{mv^2}{2}$$

.... *Equation 11*

This gives us:

$$E_\mathrm{k} = \frac{mv^2}{2} = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$

.... *Equation 12*

Since there is an electric field, there is potential energy The potential energy in an electric field is discussed in Topic 9. We can work out the potential energy using:

$$E_\mathrm{p} = \frac{-e^2}{4\pi\varepsilon_0 r}$$

.... *Equation 13*

The potential energy is negative, because work is obtained by bringing in the negative charge from infinity to the radius r. The total energy is the sum of the kinetic and potential energies. Therefore:

$$E_\mathrm{tot} = \frac{e^2}{8\pi\varepsilon_0 r} + \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{e^2 + -2e^2}{8\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$$

.... *Equation 14*

If there are Z protons, we can rewrite this as:

$$E_{\text{tot}} = \frac{-Ze^2}{8\pi\varepsilon_0 r}$$

.... *Equation 15*

This is the energy of a **single electron orbiting a bare nucleus**. We would achieve this with a hydrogen atom, an He^+ ion, or an Li^{2+} ion. So, this treatment is limited.

The radius of the ground state electron orbit in a hydrogen atom is 5.29×10^{-11} m. This is called the **Bohr radius**.

The minus sign is important. It means that when an electron comes in from infinity, work is got out. Similarly, work has to be put in to raise the electron to infinity, i.e. to ionise the atom. In the case of hydrogen, this is 13.6 eV. We use the idea of an **energy well**, in which work has to be done to remove the electron (*Figure 20*).

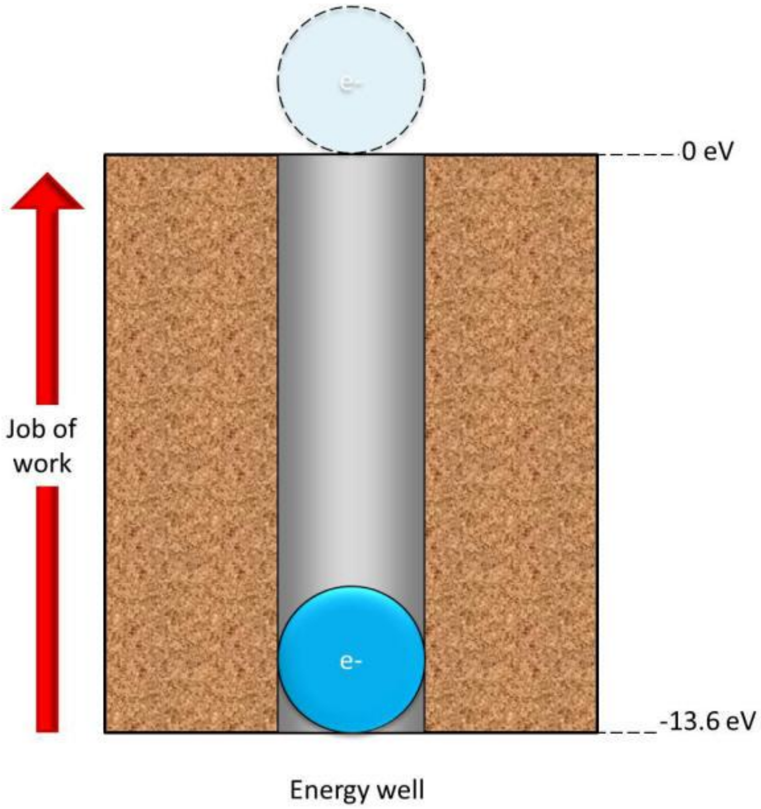

Figure 20 Energy well.

15.023 Angular Momentum in an Orbit

In Topic 3 we saw that the de Broglie relationship was:

$$\lambda = \frac{h}{mv}$$ *............... Equation 16*

When we discussed the de Broglie relationship, we did so in the context of an electron travelling in a straight line. However, any particle with mass moving in any path with a velocity is going to have momentum.

An electron making a circular orbit will have an **angular momentum**:

$$L = I\omega$$ *............. Equation 17*

Since the electron is a single particle, we can write an expression for the **moment of inertia**.

$$I = mr^2$$ *............ Equation 18*

So, we can write an expression for angular momentum:

$$L = \omega mr^2$$ *............ Equation 19*

We also know from circular motion that:

$$v = \omega r$$ *........... Equation 20*

Therefore:

$$\omega = \frac{v}{r}$$

.... *Equation 21*

So, we can substitute for ω:

$$L = \frac{v}{r} mr^2 = mvr$$

.... *Equation 22*

This is true for any **circular orbit**. In particle physics, this is called **orbital spin**.

15.024 de Broglie Wavelength and the Bohr Model

An electron orbiting a nucleus forms a de Broglie standing wave. The de Broglie wavelength of an electron wave at its fundamental frequency is the circumference of its orbit (*Figure 21*).

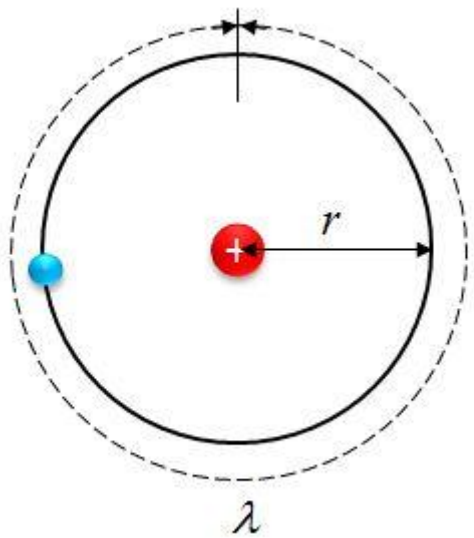

Figure 21 Electron orbits a proton

The orbiting electron forms a standing wave. For a standing wave to be formed under these circumstances, there needs to be a whole number, n, of waves. At the fundamental frequency (or first harmonic), $n = 1$.

We know that for each orbit, the electron will travel a distance of $2\pi r$. So, we can write:

$$n\lambda = 2\pi r$$ ....*Equation 23*

From the de Broglie equation:

$$\frac{nh}{mv} = 2\pi r$$ .... *Equation 24*

This further rearranges to:

$$mvr = \frac{nh}{2\pi}$$ .... *Equation 25*

From *Equation* 22, we know that the mvr term is the angular momentum, L. The radius of the electron orbit in a hydrogen atom is 5.29×10^{-11} m. This is called the **Bohr radius**.

We can use the photon energy formula to work out the wavelength of the photon emitted when the electron drops from the ionised state to the ground state. Question 15.02.3 gets you to do this.

The de Broglie wavelength is NOT the wavelength of emitted photons.

The orbits which are occupied by an electron raised to higher energy levels are a **whole number multiple** of the de Broglie Wavelength.

15.025 Application of the Bohr Model to Excited Atoms

In Topic 3, we saw that the energy ladder for the different energy levels was not even. The jump from the ground state ($n = 1$) to ($n = 2$) is -3.41 eV - -13.6 eV = 10.19 eV. When it's excited, the diameter of the atom will change. Let's see how:

Worked Example At the $n = 2$ level, the potential energy of the excited electron of a Hydrogen atom is -3.41 eV. What is the diameter of the excited atom? How does this compare with the diameter of the atom at ground state? Electronic charge = 1.6 × 10^{-19} C Permittivity of free space = 8.85 × 10^{-12} F m^{-1}.
Answer Formula first: $$r = \frac{-1e^2}{8\pi\varepsilon_0 E_{tot}}$$ E_{tot} = -3.41 eV × 1.60 × 10^{-19} J eV^{-1} = -5.456 × 10^{-19} J r = (-1 × (1.6 × 10^{-19} C)^2) ÷ (8 × π × 8.85 × 10^{-12} F m^{-1} × -5.456 × 10^{-19} J) = **2.11 × 10^{-10} m** Ratio of the radii = 2.11 × 10^{-10} m ÷ 5.29 × 10^{-11} m = **3.99 times** The atom swells to about four times its original size.

So, what happens to the speed of the electron? To find out, we use:

$$mvr = \frac{nh}{2\pi}$$

.... *Equation 26*

15.026 Drawbacks of the Bohr Model

Bohr's model works well for hydrogen, the simplest atom. It was worked out using classical physics. At the time, quantum physics was only just being worked out. Now we know that there are limitations:

- It assumes that electrons orbit in well-defined orbits - and this contradicts Heisenberg's Uncertainty Principle.
- The electrons do not orbit like satellites. They will crash into the nucleus.
- It is possible for the electrons to occupy any orbit according to the potential energy. This does not comply with the quantum nature of energy levels in atoms, where an electron rises to a new energy when exactly the right energy is given to the electron. If the exact energy is not given, the electron will not absorb it.
- It does not explain the Zeeman or Stark effects where spectral lines get split into components in magnetic fields or electric fields respectively.

We will look at the models of the atom that superseded the Bohr Model in the next tutorial.

Niels Bohr is said to have an impish sense of humour that enraged the Professor of Physics at Copenhagen University during his final oral exam (not recommended). However, it failed him when Richard Feynman introduced his Feynman diagrams - he was thunderously angry and was ready to take a swing at Feynman. As well as being a brilliant physicist, Bohr was a good football player and was the goalkeeper for a League Football team in Copenhagen. Bohr's brother, Harald, played in the Danish Olympic Football team in 1902, and played Hockey in the London Olympics in 1948.

Questions

Tutorial 15.02

15.02.1

(a) What is the total potential energy of the electron in the ground state in a hydrogen atom?

(b) What is this in eV?

(c) Comment on what this represents.

Electronic charge = 1.6×10^{-19} C

Permittivity of free space = 8.85×10^{-12} F m^{-1}.

15.02.2

An electron is orbiting a hydrogen nucleus at the ground state. Calculate:

(a) the angular momentum and give the correct unit.

(b) the linear speed of the electron.

(c) the de Broglie wavelength of the electron, to 2 significant figures.

(d) the angular velocity of the electron.

(e) the frequency of electron orbit.

Planck constant = 6.63×10^{-34} J s

Mass of electron = 9.11×10^{-31} kg

15.02.3

The ionisation energy of the hydrogen atom is 13.6 eV.

(a) Calculate the wavelength of photons emitted when the electron falls from the ionised state to the ground state.

(b) Is this the same as the de Broglie wavelength?

Planck constant = 6.63×10^{-34} J s

15.02.4

Calculate the speed of the electron as it orbits the nucleus at level n = 2, if the radius of the excited atom is 2.11×10^{-10} m. Compare your answer to Question 2 (b)

Planck constant = 6.63×10^{-34} J s

Mass of electron = 9.11×10^{-31} kg

Tutorial 15.03 Heisenberg's Uncertainty Principle	
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15.031 Early Quantum Physicists	15.032 Uncertainty
15.033 Particles and Probability	15.034 Quantum Model of the Atom

15.031 Early Quantum Physicists

Classical physics is sometimes referred to as **Newton's Clockwork Universe**. You apply simple and well-defined rules, and you get the result that was predicted. For most physical phenomena, those involving very large things, it works rather well. However, once we get to the very small, strange things start to happen. The quantum world is weird in the way that things happen that cannot be explained by the laws that govern the behaviour of large objects. A new way of thinking is needed. It is often said that if we think that we understand the quantum world, we don't.

The principles of the quantum world explain things like why atoms don't implode, how the Sun continues to shine, why space is not a complete vacuum, and how particles can be seen to be in two places at once. The key concept to quantum phenomena is that energy is not continuous, but in discrete amounts call **quanta**. (**Quantum** is a Latin word meaning *How Much*. The plural is quanta.) Anything that is smaller than an atom behaves in a quantum manner. If we have a fine enough pair of tweezers, we can just about pick up an atom, although it is rather fuzzy. Once we start to look at sub-atomic particles, like electrons, neutrons, and protons, we cannot do that. We have seen neat pictures of protons (usually red) and neutrons (usually black), but these are really a gross simplification. The situation is a lot more complex.

Much of the work that has led to the explanation of quantum concepts was due to the work of Werner Heisenberg (1901 - 1956), Wolfgang Pauli (1900 - 1958), and Erwin Schrödinger (1887 - 1961). They were German and Austrian theoretical physicists who developed the observations of Einstein and Planck. Much of what they worked on involved complex theoretical arguments. Although they met from time to time, Pauli and Schrödinger left Germany to escape from the Nazis. Pauli went to America, and Schrödinger stayed in Ireland. Heisenberg remained in Germany and was involved with the attempts to produce a fission bomb during the Second World War. Fortunately, the attempts were thwarted due to attacks on the laboratories which were moved several times. Eventually Heisenberg was captured with several other physicists shortly before the end of the war and debriefed in a farm house near Huntingdon. Not long after, in January 1946, Heisenberg returned to Germany, having demonstrated that the German physicists had not come close to producing a fission bomb.

Quantum Physics has a lot of complex mathematics, which involves **probability**.

15.032 Uncertainty

The principle of uncertainty says that there are two properties of a subatomic particle. Firstly, there is the position (x) of the particle, and secondly the momentum (p). The more we know one, the less we know of the other. In other words, if we are chasing an electron, the closer we get to catching the little brute, the less likely we are to get it. There is uncertainty in the position, if we know the momentum. Similarly, if we know the position, we find there is uncertainty in the momentum. This is called **Heisenberg's Uncertainty Principle**. In the Bohr Model of the hydrogen atom (see Tutorial 2), we could predict the position and the momentum of the electron. However, it only gave predictable results in very specific conditions.

The minimum uncertainty is the product of the position and the momentum. This product is **greater than** the **Planck Constant divided by 4π**. We write this as:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

.... Equation 27

- Δx - the uncertainty in the position (m).
- Δp - the uncertainty in the momentum (kg m s^{-1}).
- h - Planck's constant (= 6.63×10^{-34} J s).

Sometimes you will see the equation written as:

$$\Delta x \Delta p > \frac{\hbar}{2}$$

.... Equation 28

The term $\hbar$ (with the slash through it ("h-bar")) is sometimes called the **shortened Planck Constant**, and is related to h by:

$$\hbar = \frac{h}{2\pi}$$ .... *Equation 29*

We can, of course, give a value for $\hbar$:

$\hbar = 1.06 \times 10^{-34}$ J s .... *Equation 30*

In the SQA syllabus, we will use the Planck Constant divided by 4π, which gives us a value of **5.28 × 10^{-35} J s**. The reason for this is because due to large numbers of particles in the same state, there is **uncertainty** in their positions and momenta given by the relationship:

$$\Delta x \Delta p = \hbar / 2$$ .... *Equation 31*

There is also a minimum for the products of the uncertainty for energy and uncertainty for time. In other words, if you know the energy of the electron, you will have a lot of uncertainty in predicting the time for the little brute:

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$ .... *Equation 32*

Worked example A neutron has a mass of 1.67 × 10^{-27} kg and a speed of 1.56 × 10^{6} m s^{-1}. Calculate the minimum uncertainty for the position.
Answer Momentum: p = 1.67 × 10^{-27} kg × 1.56 × 10^{6} m s^{-1} = 2.605 × 10^{-21} kg m s^{-1}

Equation:

$$\Delta x = \frac{h}{4\pi \Delta p}$$

Minimum uncertainty in position = Δx

= (6.63 × 10^{-34} J s) ÷ (4 × π × 2.605 × 10^{-21} kg m s^{-1})

= **2.03 × 10^{-14} m**

Although 2 × 10^{-14} m does not seem that far, it is about ten times the diameter of the neutron (*Figure 22*).

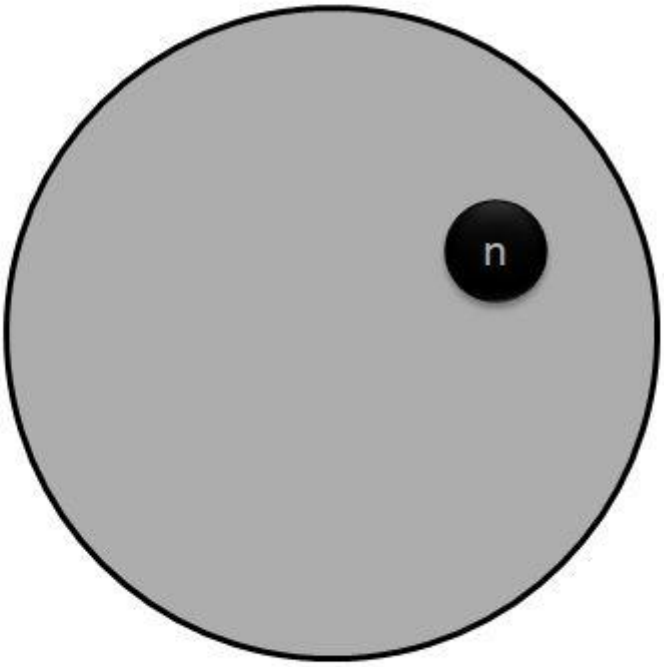

Figure 22 Uncertainty field compared with the size of a neutron

Remember also that the neutron is not a neat black snooker ball with an 'n' written on it. It is fuzzy. And it exists as a **probability**.

The distance travelled by the electron in the time you worked out is about the diameter of an atom. We cannot express it in terms of the diameter of an electron, as the electron is a **point charge** with mass and charge only.

Position and momentum go together, and energy and time. You cannot put together position and time, or, for that matter momentum and energy.

15.033 Particles and Probability

This is where things get really strange and we won't go into too much detail. In the exam, you will only need to give a qualitative description.

The key point is that particles like electrons, proton, and neutrons exist as **probabilities**. Consider a **normal distribution curve**, say for the mass of students in a secondary school (*Figure 23*).

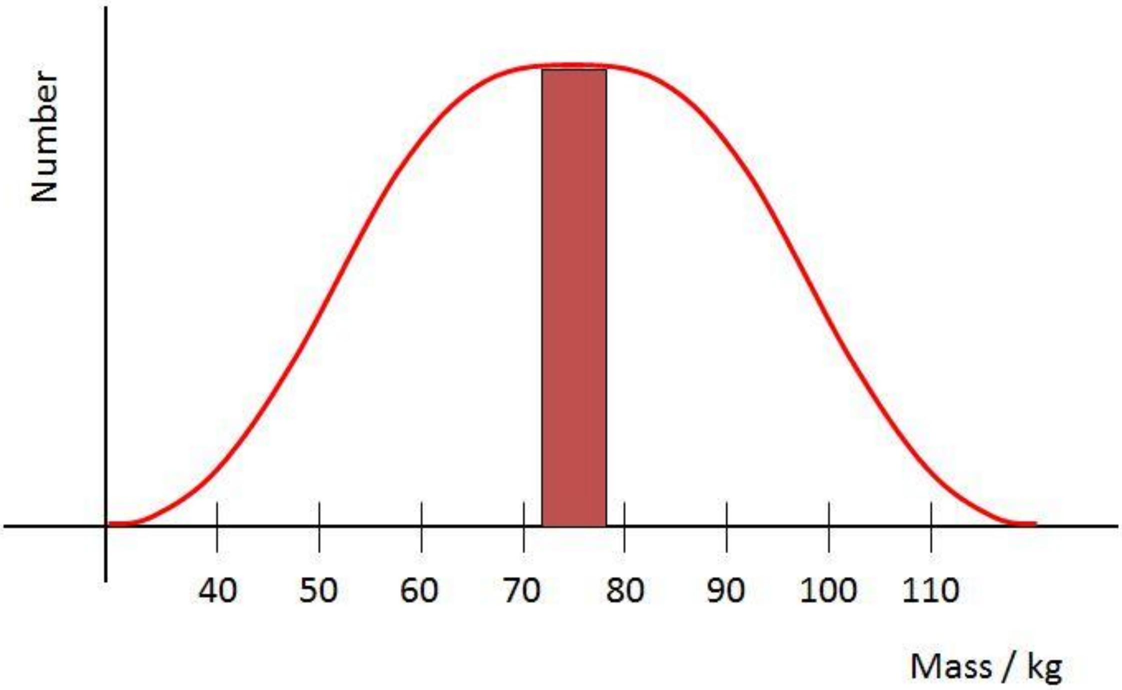

Figure 23 A normal distribution curve

If we were looking for students of mass between 72 kg and 78 kg, the probability that they will be found in the shaded box is 1 (i.e. they will all be there). If we took a single student at random, without knowing its mass, we would know that it would be somewhere under the normal distribution curve. However, we would need to take an educated guess, looking at the student's height, body build, etc, before we could place it on the normal distribution curve. There is a probability that we will get the answer right, but a greater probability that we will get the answer wrong.

Suppose we consider the energy of electrons. The energy of electrons is given as 9.0 eV. We can show this on the normal distribution curve (*Figure 24*):

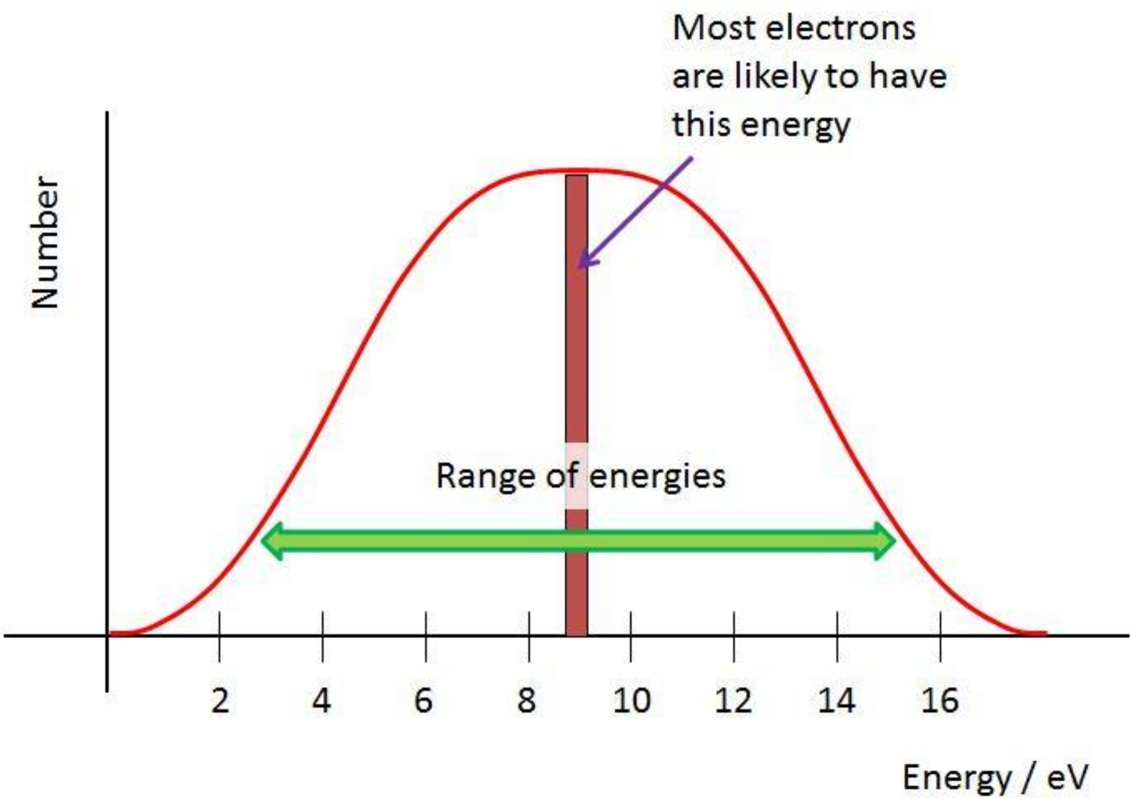

Figure 24 Normal distribution of electron energies

The probability of finding the electron under the graph is 1. In other words, we know that all the electrons have an energy of some value (really?). The most likely value for the energy is going to be 9.0 eV. You may well consider that this is a thesis from the University of the Truly Obvious, but it does have an important implication, in the context of quantum tunnelling.

When chemical reactions occur, there needs to be a certain energy to set it going. This is called the **activation energy**. To start that reaction, the electrons need to have a certain energy level, to get over the energy hill. Sometimes the energy hill is quite low, so the electrons can easily vault over it.

Now consider this situation, where the electron has a certain amount of energy, but it is much lower than the energy hill (*Figure 25*).

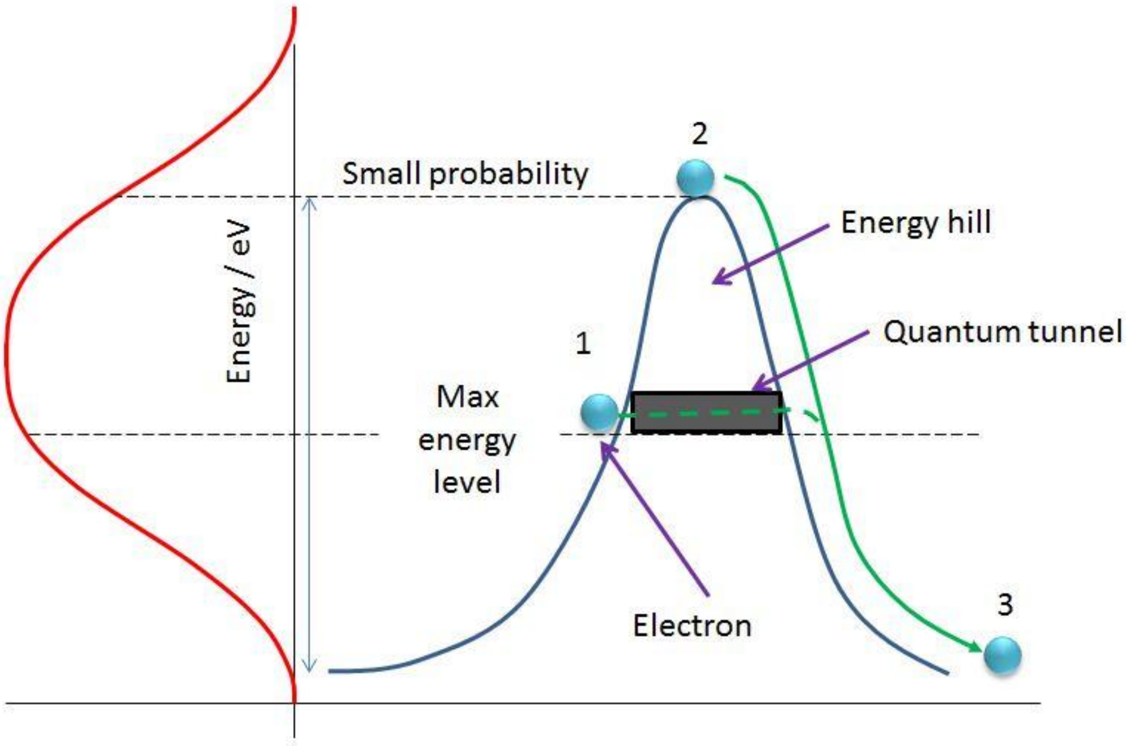

Figure 25 Quantum tunnelling

On the left hand side, we have the distribution curve of electron energy (turned on its side). In classical physics, the electron has achieved it maximum energy level at position 1. It cannot go over the energy hill. In quantum physics, there is a small, but definite probability that the electron can be at position 2, and so can roll down the energy hill to position 3. We imagine that it has burrowed through the hill using a **quantum** tunnel.

This model can be used to explain how electrons in a semiconductor can jump the forbidden zone into the conduction band, when in classical physics, they could not do so. It also helps to explain how unstable atoms decay.

Erwin Schrödinger quantified the probability functions in an equation:

$$H\Psi = E\Psi$$

.... *Equation 33*

The strange looking symbol (that looks like a candle holder that you would find on the dining room table of a posh house) is Ψ, Psi, a Greek capital letter 'Ps'. (This letter gives us the spelling of words like "psycho".) It is the physics code for probability. It combines the quantum aspects of a particle, and the classical aspects. You will see more of this when you study quantum physics at university.

15.034 Quantum Model of the Atom

We are used to the Bohr shell model, in which the electrons are arranged in **shells**. For example, consider neon, which has 10 electrons (as its proton number is 10). The configuration in the shell model is:

2, 8

The idea of **quanta** has led to a more sophisticated model of the atom than the Bohr Model. In this case the electrons occupy **orbitals**, where there is a high probability of finding the electron. The idea of this is shown in the picture (*Figure 26*):

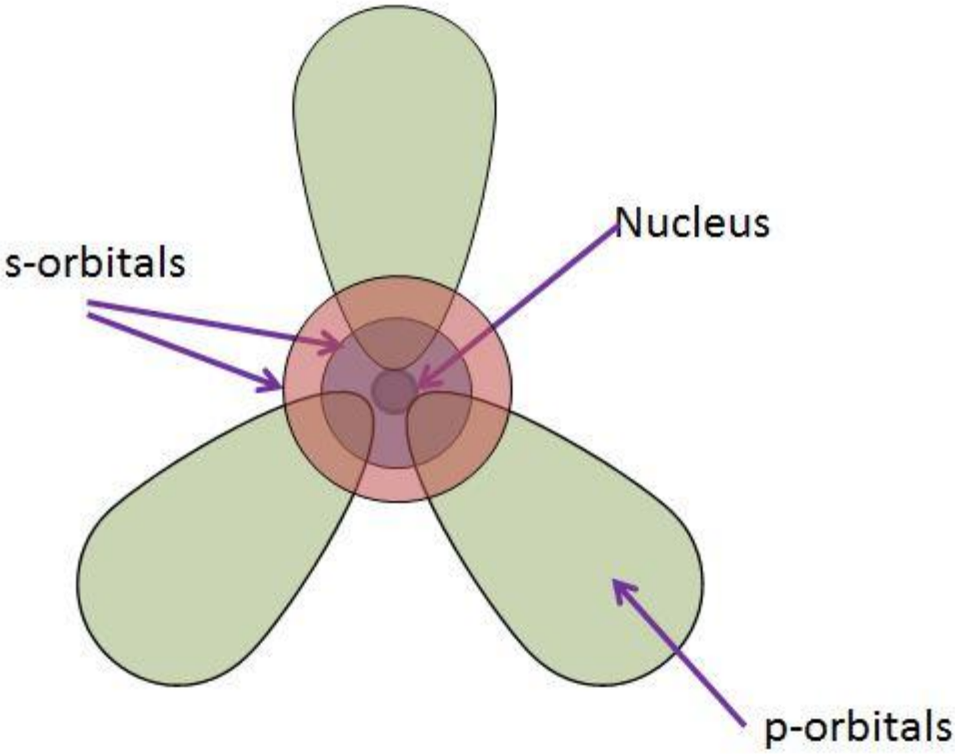

Figure 26 Quantum model of an atom

The orbitals are probability clouds. You know that the electron is there in the orbital, but the closer you are to catching the little brute, the less likely you are to catch it.

There are different types of orbitals shown here, the s-orbitals that are spherical, and the p-orbitals that are elongated. The s-orbitals carry 2 electrons, while the p-orbitals carry up to 6 electrons. Therefore, for neon, the quantum electron configuration is:

$$1s^2\ 2s^2\ 2p^6$$

Note that $1s^2$ is pronounced "one-ess-two", NOT "one-ess-squared".

There is more discussion on Quantum concepts at Tutorial 17.

Questions

Tutorial 15.03

15.03.1

An electron moving in a straight line has an energy of 13.6 eV. What is the minimum uncertainty in measuring the time it travels a certain distance?

What is the distance covered by the electron in this time, if its speed is 3.50×10^6 m s^{-1}?

15.03.2

Position and momentum go together, and energy and time. You cannot put together position and time, or momentum and energy.

Why not? Explain your answer.

Tutorial 15.04 Cosmic Rays

SQA Syllabus

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15.041 Cosmic Rays

Cosmic rays are not well understood but are thought to be fragments of atoms that originate from space beyond the Solar System. They travel at close to the speed of light. They are NOT photons of electromagnetic radiation. They have been known to damage electrical and electronic equipment. They have the following properties:

- They have mass.
- They have kinetic energies.

Some cosmic rays have huge amounts of kinetic energy due to relativistic effects as they approach the speed of light, and the energy is much higher than is possible with photons.

The maximum energy of an ultra-high energy gamma ray photon is reckoned to be 1.0×10^{19} eV.

Contrast this with the *Oh-My-God* particle (yes, it was called that) that was recorded in 1991. It had an energy of 3×10^{20} eV equivalent to 48 J. This high energy was due to the very high relativistic kinetic energy due to the speed which was exceptionally close to the speed of light. Its Lorentz factor was about 3.2×10^{11}. It would take a photon racing it about 200 000 years to get a 1 cm lead on it. Its energy was about 40×10^{6} times higher than the energy of the highest energy accelerated protons.

A simple kinetic energy calculation will show you that 48 J is equivalent to a 5 kg dumbbell hitting you at a speed of about 4 m s^{-1}. It would thump you hard.

15.042 Discovery

The first observation was made by Charles-Augustin de Coulomb in the 1780s. He had a charged sphere that was insulated from the ground by air, which was considered to be an insulator. The charge on the sphere was spontaneously lost and there was no explanation for it. Later discoveries suggested that air could be ionised by charged particles or X-rays and become conducting as a result.

Later experiments showed the loss of charge even if the charged spheres were kept behind thick lead walls that would stop X-rays and any charged particles from radioactive sources.

In 1909, Victor Hess (1883 - 1964) demonstrated the existence of intense radiation at a height of 5 km above the ground by using very sensitive electrometers. The radiation at 5300 m was about 4 time as intense than the radiation experienced at 1 km above the ground, and this led him to believe that this radiation came from space. Similar findings were made by Werner Kohlhörster in 1913. Hess' results were confirmed by Robert Millikan (Millikan's Experiment) in 1925. Millikan first used the expression "Cosmic Rays".

Work to discover the interaction with particles in atmosphere was carried out in 1937. This led to the discovery of the first anti-particle, the positron. Also, a range of short-lived particles such as pions and muons were discovered. These were the result of interactions between the cosmic rays and molecules in the atmosphere. Physicists still did not know the precise nature of the cosmic ray. It was only recently that the nature of cosmic rays was finally worked out, almost a century after physicists started to study them.

15.043 Nature of Cosmic Rays

Cosmic rays are misnamed. They are not electromagnetic radiation at all. They gain their huge energies by travelling at close to the speed of light, and their Lorentz factors are very high.

The majority (90 %) are **protons**. 9 % are **helium nuclei** (alpha particles). The remaining 1 % include electrons and other nuclei of elements. Some of these are radioactive, and the half-lives of the radionuclides and their products can be used to date the particles. One example is iron-60, a radioisotope of iron.

Iron 60 has a half-life of 2.6×10^6 years. It decays to Cobalt-60. Look at Topic 12 to revise exponential decay.

Studies of the iron-60 and its decay products suggested that the source of iron-60 cosmic rays was about 3000 light-years, about the distance to a spiral arm of the galaxy.

15.044 Sources of Cosmic Rays

It is thought that cosmic rays have their source in the explosion of **supernovae**. When a giant star collapses in itself, vast amounts of energy are released in the shock wave that bounces back as a result of the collapse (see Topic 14A). Often gamma rays accompany the cosmic rays, as a result of neutral pions, which are antiparticles to themselves, annihilating. The pions arise to collisions of protons that are caught up in the intense magnetic fields produced by the shockwave of a supernova.

15.045 Interaction of Cosmic Rays with the Earth's Atmosphere

When a cosmic ray particle enters the atmosphere, it will interact with molecules in the atmosphere to produce a shower of a range of different particles including:

- **Pions** (positive, negative, and neutral).
- **Kaons** (positive and negative).
- **Muons**.
- **Muon neutrinos**.
- **Electrons**.
- **Electron neutrinos**.

As the neutral pions annihilate themselves almost immediately (8×10^{-17} s) there are gamma rays that then interact with each other close to the nuclei of air molecules to produce electrons and positrons in pair production events. The electrons may pass through the electron shells of other atoms to produce **bremsstrahlung** (braking radiation) events to produce further gamma photons, which may go on to be involved in pair production. See Topic 2 for annihilation and pair production.

Remember that pair production <u>only</u> happens when there is interaction between gamma photons in the **presence of a nucleus**.

Where there is no nucleus present, the gamma photons simply superpose and carry on as before.

Charged pions have a longer lifetime, about 26 × 10^{-9} s, while kaons have a lifetime of about 12 × 10^{-9} s. Muons have a life time of 2.2 × 10^{-6} s, which gives them time to reach the Earth's surface, before they decay to electrons. This covered by Topic 2.

The diagram shows a shower of particles produced by the interaction of a cosmic ray particle with molecules in the atmosphere (*Figure 27*):

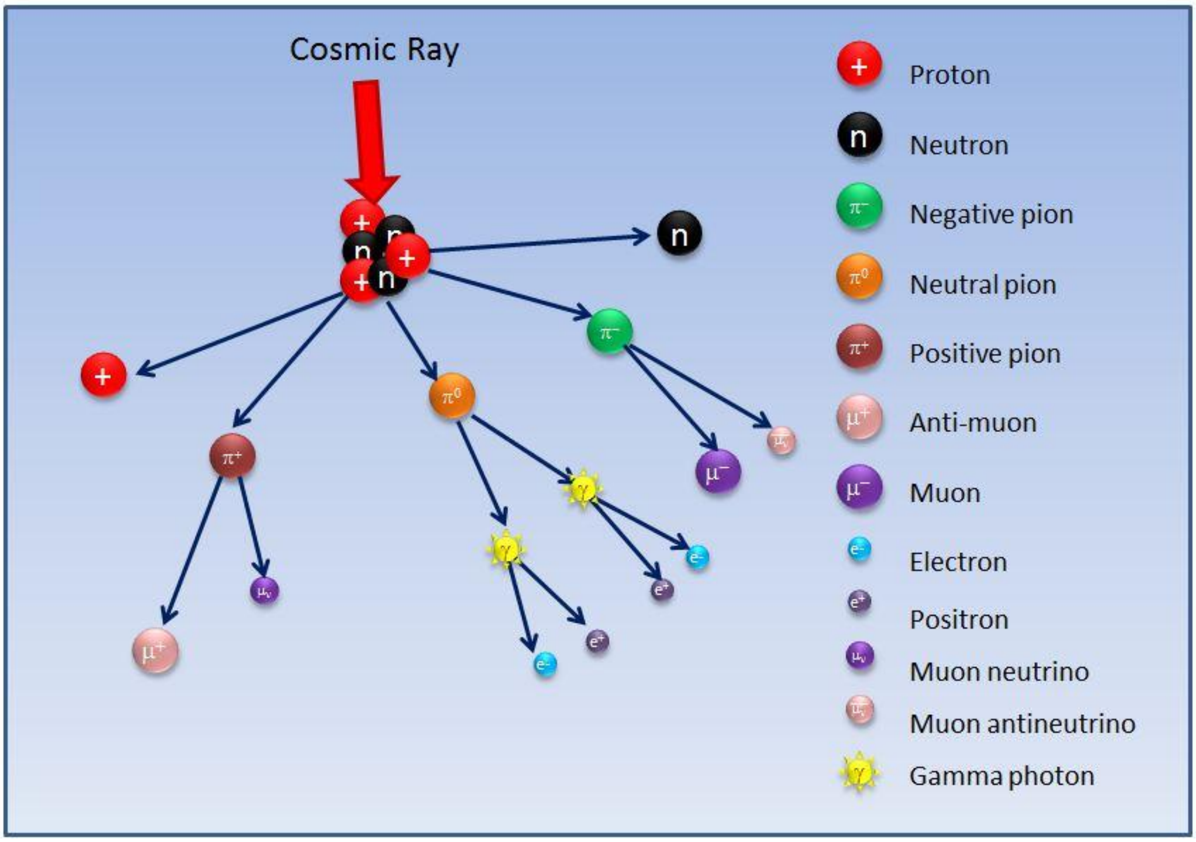


Figure 27 Shower of particles caused by the interaction of cosmic rays with molecules in the atmosphere

If a high speed electron interacts with a nitrogen atom by **electron collision**, the electron can be captured by the nucleus to turn a proton into a neutron, emitting an electron neutrino:

$$^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{14}_{6}\mathrm{C} + \nu_\mathrm{e}$$

This is the source of Carbon-14 in the atmosphere.

The table below shows typical cosmic ray **fluxes**. The extreme energy events are rare.

Particle Energy /eV	Flux / m^{-2} s^{-1}
1×10^{9}	10 000
1×10^{12}	1
1×10^{16}	1×10^{-7} (A few times a year)
1×10^{20}	1×10^{-15} (Once a century)

It is thought that cosmic rays are instrumental in setting of **lightning strikes**. Air loses its insulating properties at an electric field strength of about 3×10^{6} V m^{-1}. However, such a field strength has never been detected even in the most violent thunderstorm. Therefore, there needs to be another explanation and this puzzled meteorologists for many years.

Figure 28 Lightning earth strike

It is thought now that lightning is set off by cosmic ray particles. As we have seen above, the interaction of the cosmic ray particles with molecules in the atmosphere causes intense local ionisation, which makes the air conductive. Electrons then tend to pool, leading to localised intense electric fields. The lightning stroke propagates in a series of small steps (called a **stepped leader**) until it reaches close the ground. The electric field is then intense enough to attract ions from nearby objects to complete a conducting path. Then the main stroke occurs, allowing a current of about 25000 A to flow. The intense heating effect on the air causes a loud bang, if you are close to the strike. The subsequent rumble is because the source of the sound wave is not a point source, but along a long and irregular front. The low frequency sound is because the higher frequency sounds from the distant sources are absorbed by the atmosphere. In the picture above, you can see the stepwise propagation of the earth strike.

15.046 Interaction with Magnetic Fields

Cosmic ray particles are mostly hydrogen and helium nuclei. They are positively charged. Therefore, they will interact with magnetic fields of any planet. The Earth's magnetic field will protect us from them.

Consider two cosmic rays. Ray 1 strikes the Earth's magnetic field at point 1 at 90°. Ray 2 strikes the Earth above the North Pole (*Figure 29*).

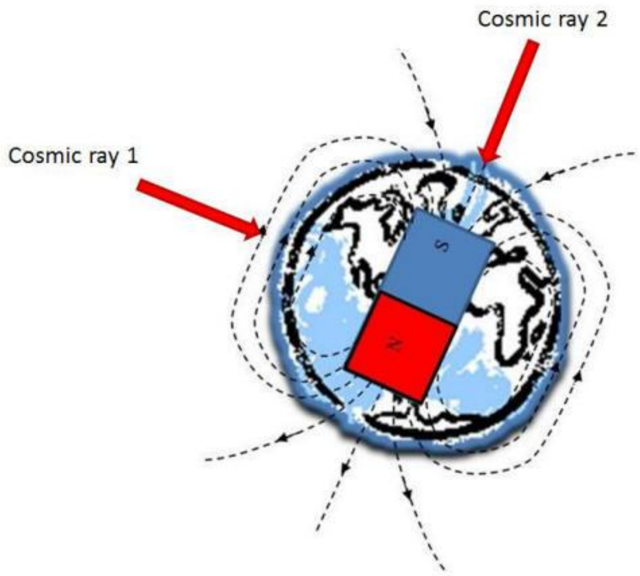


Figure 29 Two cosmic rays interacting with the Earth's magnetic field

We will look at the behaviour of Cosmic Ray 1. To understand how a positively charged particle interacts with a magnetic field, you need to review Topic 11.

We find that in a magnetic field, the force acts on a stream of electrons always at **90°** to the direction of the movement. Therefore, the path is **circular** (*Figure 30*).

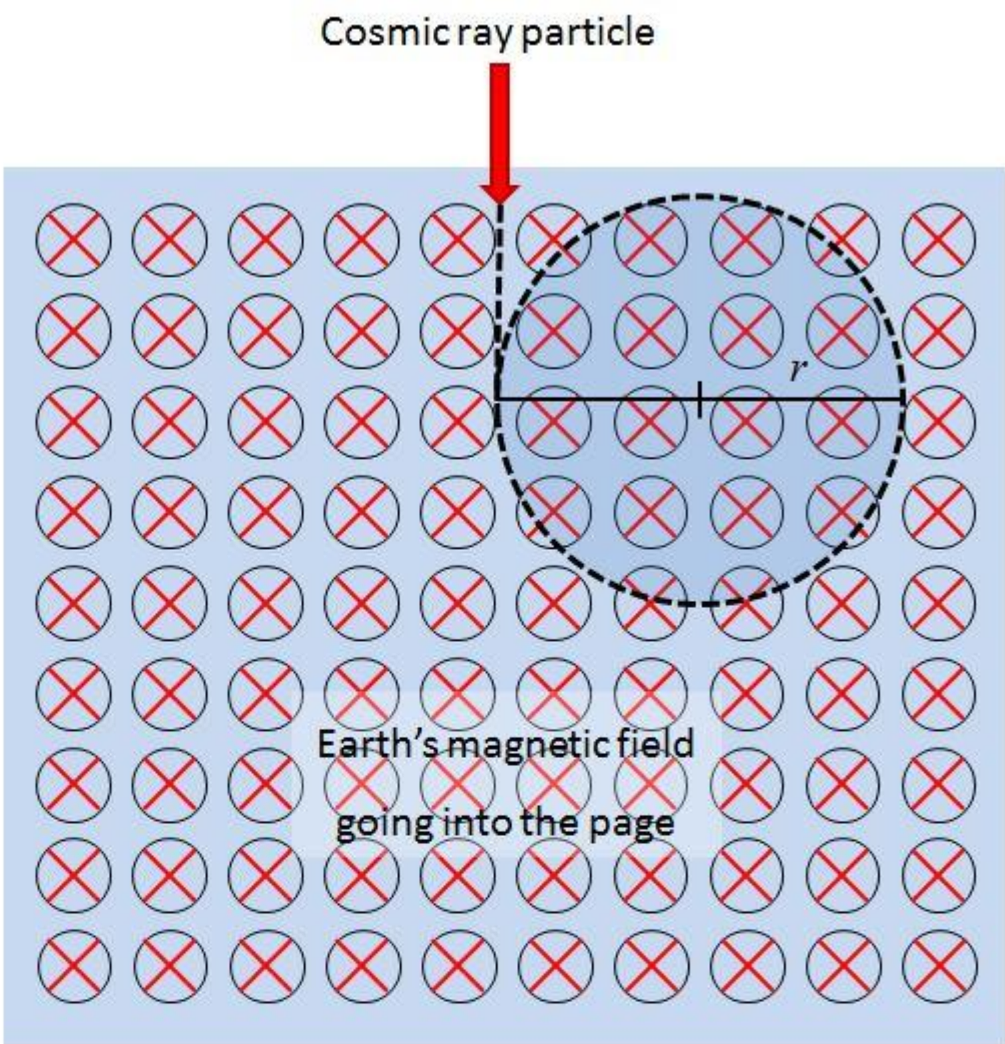


Figure 30 Interaction between a cosmic ray and the Earth's magnetic field

The magnetic force always acts on the charged particles at 90°, and that gives us the condition for **circular motion**.

We can combine the relationship

$$a = v^2/r$$ .... *Equation 34*

with Newton II to give us:

$$F = \frac{mv^2}{r}$$ .... *Equation 35*

Since

$$F = Bqv$$ .... *Equation 36*

Therefore:

$$Bqv = \frac{mv^2}{r}$$ ....*Equation 37*

The v on the left cancels to get rid of the v^2 term on the right:

$$Bq = \frac{mv}{r}$$ .... *Equation 38*

This rearranges to give us:

$$v = \frac{BQr}{m}$$ .... *Equation 39*

We assumed that the cosmic ray particle struck the magnetic field at a perfect right angle and tracked a perfectly circular path. This would be rare. The side view of the path would look like this (*Figure 31*):

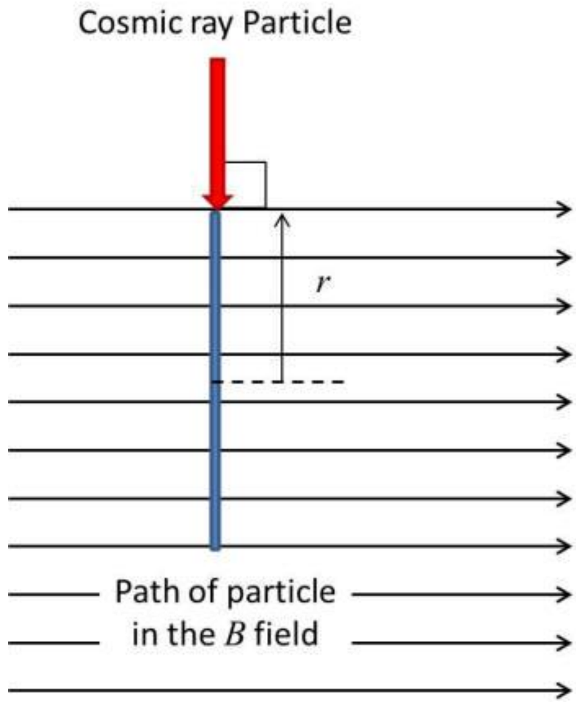


Figure 31 Side view of a cosmic ray striking a magnetic field at 90 degrees

The majority of particles, though, would strike at an angle other than 90 degrees, so there would be a component of velocity along the magnetic field. So let us think about the particle hitting the magnetic field with a velocity v at an angle of θ. The magnetic field is horizontal and has a flux density of B (*Figure 32*).

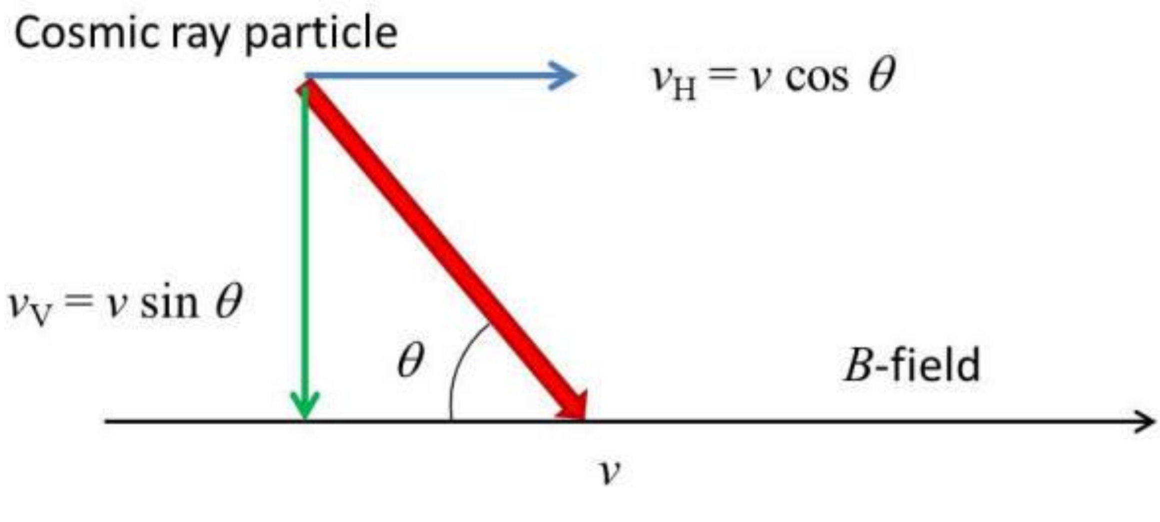


Figure 32 Path of a cosmic ray NOT at 90 degrees

The velocity vector can be split into its vertical component and its horizontal. As with all motion in two directions, we treat the two components **separately**. So, for the **horizontal component**:

$$v_H = v \cos \theta$$ .... *Equation 40*

Since the horizontal motion is parallel to the magnetic field, the force acting on it is **zero**. Therefore, there is no horizontal force acting on it. Newton I applies, so the horizontal velocity remains the same.

The vertical component will be affected by the magnetic field. The **vertical component is**:

$$v_V = v \sin \theta$$ .... *Equation 41*

So, we can write:

$$v \sin \theta = \frac{BQr}{m}$$

.... *Equation 42*

Therefore, we can rewrite this for the radius:

$$r = \frac{mv \sin \theta}{BQ}$$

.... *Equation 43*

Therefore, the radius of the circular path is smaller than if the particle struck the field at right angles.

Because there is horizontal motion, the path of the particle is **helical** (like the coil of a spring), not circular. This is shown side-on in the diagram (*Figure 33*):

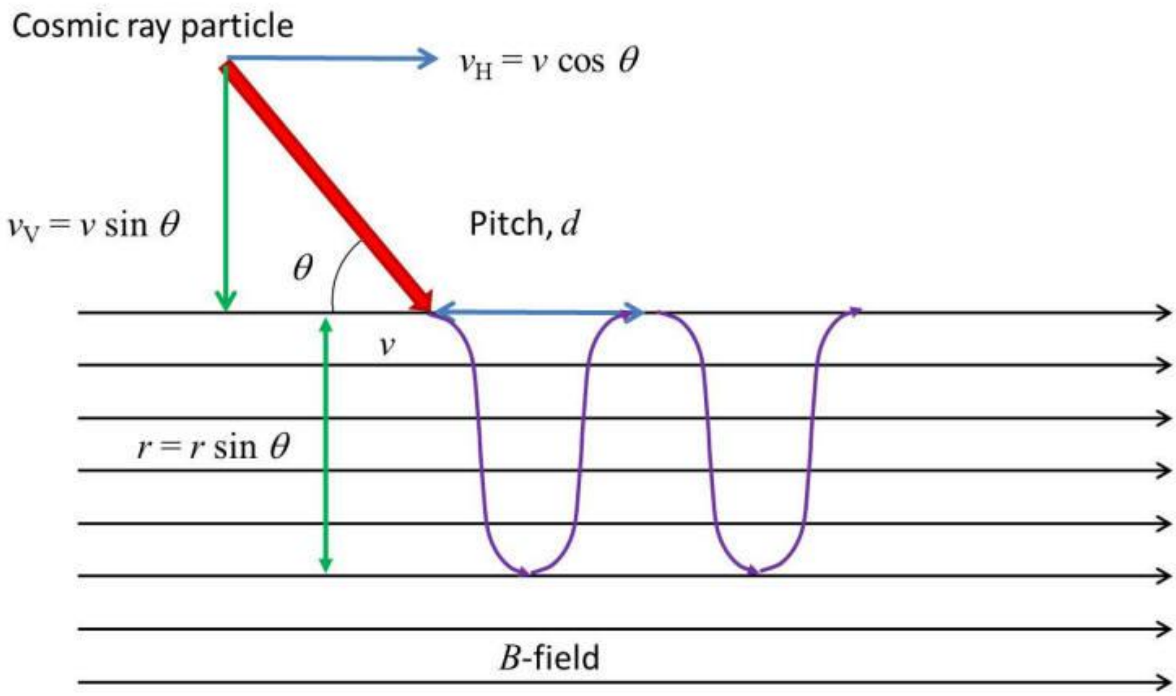


Figure 33 Helical path of a cosmic ray

If viewed end-on, the cross section of the path is **circular** (*Figure 34*).

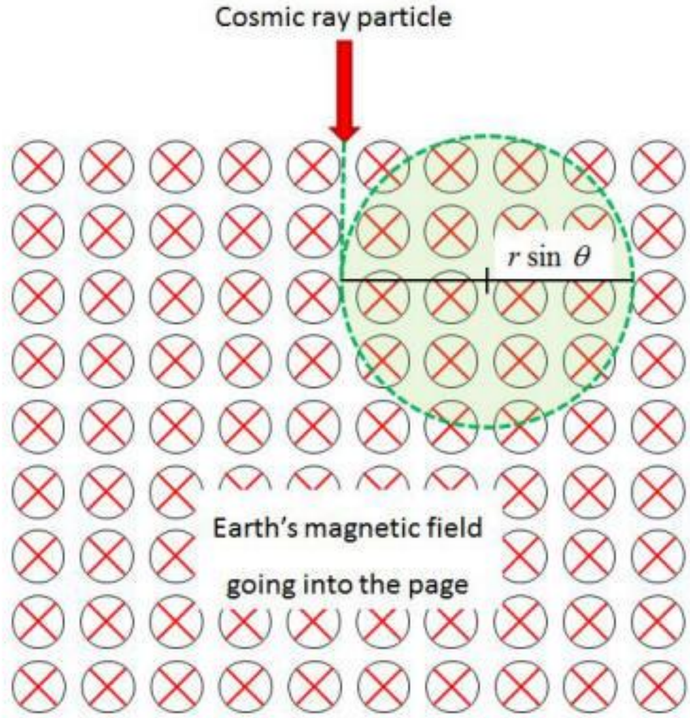


Figure 34 End on view

The **pitch**, d, of the coil can be worked out simply by multiplying the magnitude of the horizontal velocity by the time, T, it takes the particle to complete one revolution of the circular cross section. Therefore:

$$d = v_H T$$ .... *Equation 44*

The time taken to complete one revolution is the circumference of the circle divided by magnitude of the vertical velocity:

$$T = \frac{2\pi r \sin\theta}{v \sin\theta}$$ .... *Equation 45*

It doesn't take a genius to write:

$$T = \frac{2\pi r}{v}$$ .... *Equation 46*

This tells us that the rate of rotation is NOT affected, as the radius and the magnitude of the vertical velocity are both changed by the same amount

The path of the cosmic ray follows the field lines of the Earth's magnetic field. It is guided towards the North Pole, where it will interact with molecules high in the atmosphere. If the particles produced by the interaction are charged, they too will spiral with the magnetic field. If they are uncharged, they will go off in random directions.

15.047 Path through the Magnetosphere

We treated the Earth's magnetic field as **uniform** in the argument above. The Earth's magnetic field (the **magnetosphere**) extends to about 90 000 km from the Earth. At its limit, the magnetic field strength has a low value, much lower than the figure given in Question15.04.4. The radius of the curved path traced by a lower energy cosmic ray will be large at the limit of the magnetosphere but will decrease as the magnetic field strength gets bigger. The idea is shown in the picture below (*Figure 35*):

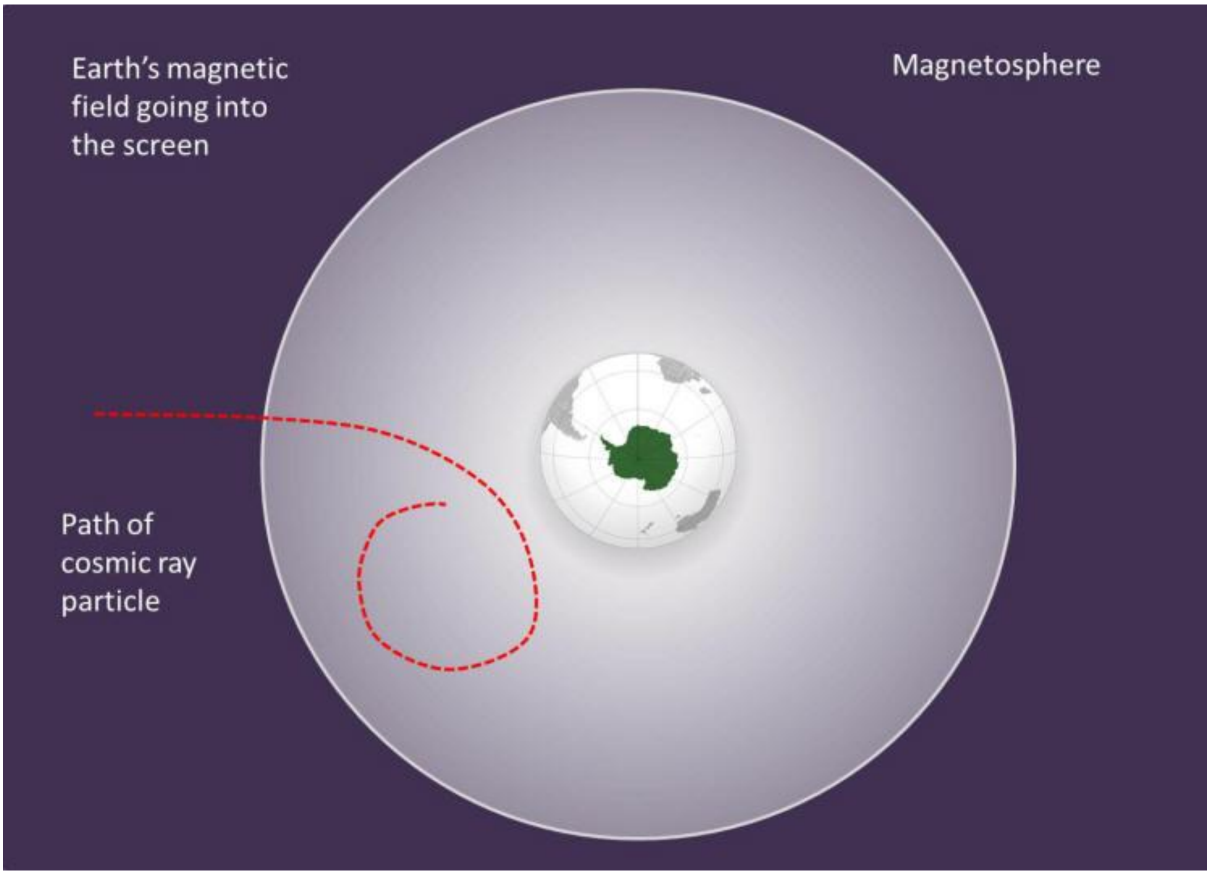


Figure 35 Path of cosmic ray particle (Image of Antarctica - Wikimedia Commons)

So, the cosmic ray particle **spirals** as shown.

Very high energy cosmic ray particles would not be affected by the Earth's magnetic field to the extent shown in this diagram. The path would be slightly curved, but would strike the atmosphere and interact with molecules, or even strike the ground.

The interactions produce high energy particles, such as muons and pions, which in turn form ions. These should, in theory, produce visible photons that could be seen near the poles in the form of the **aurora borealis** (and the **aurora australis**). In practice, the effect is weak. The effect is more marked due to the **Solar Wind**, which we will consider in the next tutorial.

15.048 Exposure to Cosmic Rays

The magnetosphere protects us from many cosmic rays, although some cosmic ray particles get through. The dose received on the ground is quoted about 3.0×10^{-8} Sv per hour. This is increased to about 5×10^{-6} Sv h^{-1} when flying at 10 000 m in a commercial aeroplane. Even so, a passenger flying across the Atlantic will receive of dose of no more than 3.2×10^{-5} Sv, about the dose that you would get from an X-ray at the dentist.

Cosmonauts travelling to Mars will not have the protection afforded by magnetosphere, so will receive significantly more exposure to cosmic rays. Additionally, Mars has no magnetic field. It is possible that such cosmonauts could be exposed to significant damage to the DNA.

Cosmic ray particles can also cause damage to electronic components, which is not desirable on a long distance spaceflight.

Questions

Tutorial 15.04

15.04.1

From where does a photon get its energy?

15.04.2

What is the energy 1.0×10^{19} eV in joules?

15.04.3

What kind of radiation would iron-60 emit? Explain your answer.

15.04.4

A cosmic ray particle consists of a helium nucleus. It is travelling at a speed of 0.98 c. It then enters the Earth's magnetic field where it is parallel to the surface.

(a) Show that the Lorentz factor of the helium nucleus at this speed is about 5.

(b) Calculate the relativistic mass of the helium nucleus.

(c) Calculate the radius of the circular path, assuming that the helium nucleus has not collided with any molecules in the atmosphere.

Mass of a helium nucleus at rest = 6.64×10^{-27} kg.

Speed of light = 3.00×10^{8} m s^{-1}.

Magnetic field of the Earth = 43.5×10^{-6} T.

15.04.5

A cosmic ray particle is identical to the one in Question 15.04.4 but strikes the Earth's magnetic field at an angle of 60° to the horizontal. Calculate the radius of the circular path made by the particle now.

15.04.6

Use your answers to Question 5 to:

(a) work out the time taken for the particle to make one revolution.

(b) to calculate the pitch of the coiled path.

Tutorial 15.05 Solar Wind

SQA Syllabus

Contents

The Sun is a middle sized and middle-aged star. It has shone for about 4500 million years and is expected to do so for another 4500 million years. We know that it provides energy to the Earth at a rate of about 500 W m^{-2}. Observations on the star reveal it to be a very dynamic system. The structure of the Sun is shown here (*Figure 36*):

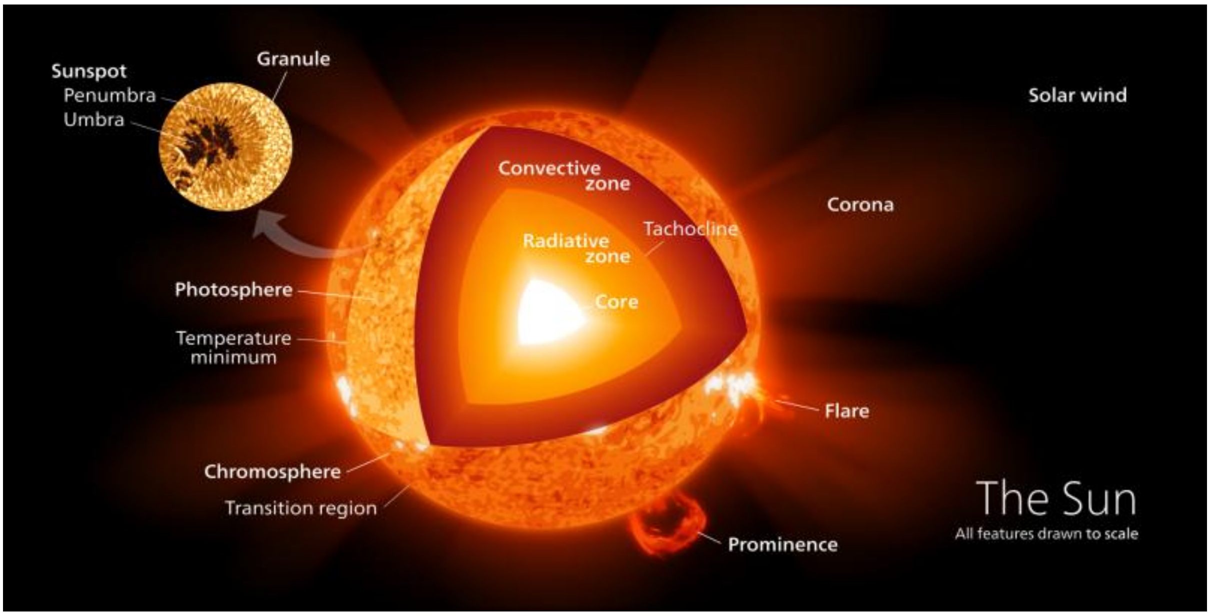


Figure 36 Structure of the Sun (Image by Kelvinsong - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=23371669)

A lot of information has been observed during solar eclipses, as well as viewing of the star through special telescopes. The surface of the Sun, the top part of the **photosphere**, has a temperature of about 4100 K at its coolest point. The surface is under the atmosphere of the Sun. The particle density of the upper part of the photosphere is about 10^{23} m^{-3}, about 0.4 % of the atmospheric density at sea-level on Earth.

NEVER attempt to view the Sun with the naked eye.

To do so will cause **permanent** damage to your eyesight.

The atmosphere of the Sun consists of 4 different layers:

- the **chromosphere**, a layer about 2000 km deep with a temperature of 20 000 K.
- the **transition region**, a layer about 200 km deep where the temperature rises from 20 000 K to about 1×10^6 K.
- the **corona**, a region of very low particle density (about 10^{15} m^{-3}) and very high temperatures, about 1 - 2×10^6 K.
- the **heliosphere**, a tenuous layer that extends about 50 astronomical units from the Sun. (1 AU = 150×10^6 km)

Why the atmosphere is so much hotter than the surface is not well understood, although physicists are sure that the magnetic field of the sun has some bearing on it.

15.051 The Solar Wind

In the late nineteenth and early twentieth centuries astronomers had observed that the Sun was at times a lot more active than normal, giving out intense **flares** that went deep into space. They associated this activity with the occurrences of intense activity of the **Aurora Borealis** (Northern Lights) and the **Aurora Australis** (Southern Lights). These events became known as **geomagnetic storms**, causing magnetic disturbances on the ground. The most severe of these could also affect telegraph lines, and later on, electricity distribution systems.

Observations of **comets** revealed that the tails always pointed away from the Sun, regardless of whether the comet was approaching or going away.

The first direct observations were made by the Soviet satellite Luna 1 in 1959.

15.052 Nature of the Solar Wind

The solar wind is formed by bulk movement of charged particles, mostly protons and electrons, but also other small atoms. At the very high temperatures, the atoms are stripped of their electrons and exist as naked nuclei. This state is called a **plasma**. The particles in the corona move at speeds that are quite a bit above the solar escape speed, 618 km s^{-1}.

The particles in the **fast solar wind** travel at about 750 km s^{-1}, while those of the **slow solar wind** travel at 300 - 500 km s^{-1}. The temperature of the slow solar wind particles is about 10^6 K, suggesting that they have their source in the corona. The fast solar wind particles have a temperature of about 80 000 K, suggesting an origin in the photosphere.

Physicists cannot at the moment explain fully how the particles are accelerated. Their high speed was originally put down to their high temperatures, although this could not fully account for their speed. It is now thought that they are accelerated by effects of the solar magnetic field. The **fastest** of the solar winds appear to originate from coronal holes near the magnetic poles where small magnetic fields confine the plasma in magnetic fields and funnel it out into space. As the sun rotates on its axis, the streams of charged particles from these sources are hurled in a helical pattern, rather like water drops from a rotating lawn sprinkler. The idea is shown in this rather simplified diagram (*Figure 37*):

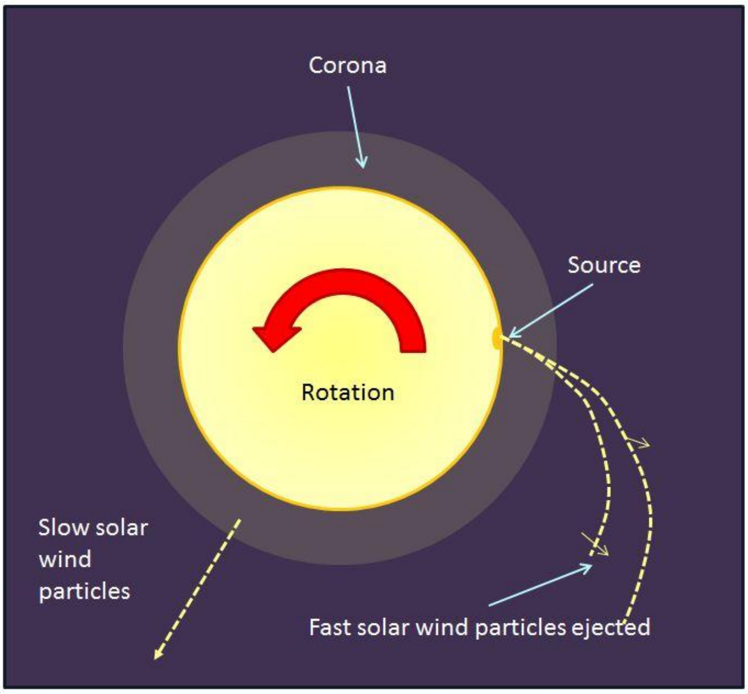


Figure 37 Origin of the solar wind

15.05.3 Energy of the Solar Wind

The energy of the solar wind particles is much lower than those of cosmic ray particles, between 1.5 keV and 10 keV.

This energy in the answer to Question 15.05.1 is equivalent to the energy carried by a soft (low energy) X-ray photon. Contrast this with the very high energy of cosmic particles.

15.054 Interaction with Magnetic Fields

The Earth has a magnetic field, which is similar to the field of a bar magnet. The region of the Earth's magnetic field is called the **magnetosphere**. The boundary of the magnetosphere is called the magnetopause. This is distorted by the solar wind as shown in this diagram (*Figure 38*):

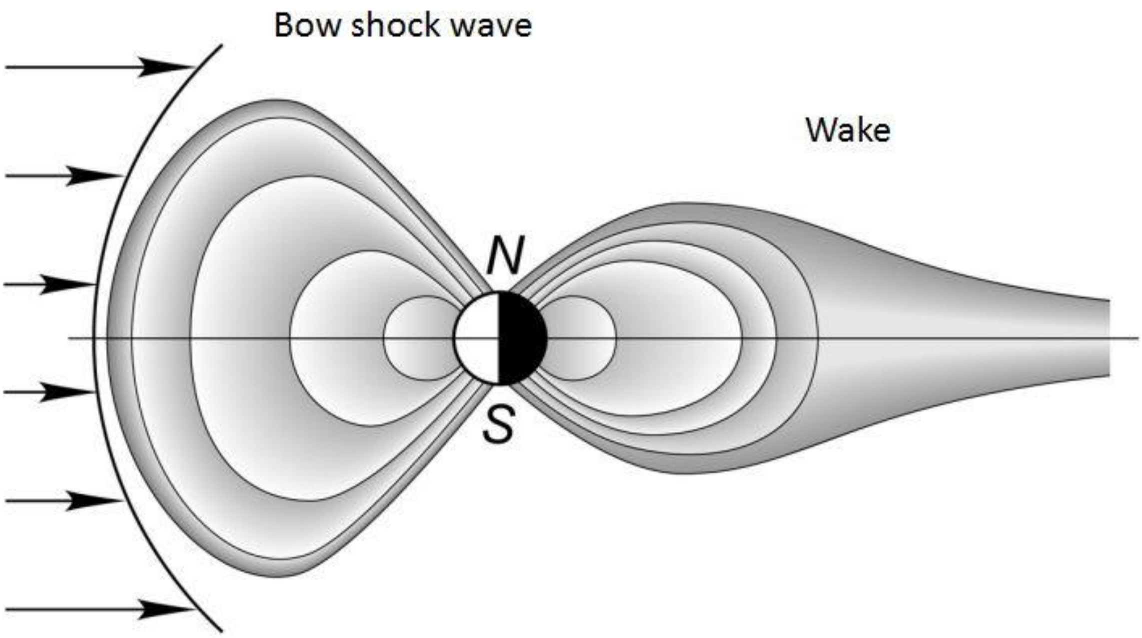


Figure 38 The effect of the magnetosphere on the solar wind (Image by Alec Baravik (adapted), Wikimedia Commons)

The charged particles are deflected by the magnetic field to form a bow shock wave, rather like the way air molecules pile up on the leading surfaces of a supersonic aeroplane. The magnetic field is pulled outwards to form a wake on the right hand side (the night side)

The Earth's magnetic field protects us from these particles, by trapping particles that penetrate the magnetosphere in the magnetic field. The way the charged particles interact with the magnetic field is identical to the way the particles of cosmic rays do, except that the radii of the circular paths are smaller. We will look at this in more detail. The important relationship is:

$$r = \frac{mv}{BQ}$$ .... *Equation 47*

If the angle is not 90° then we have to take the angle into account. The charged particle follows a helical path (*Figure 38*):

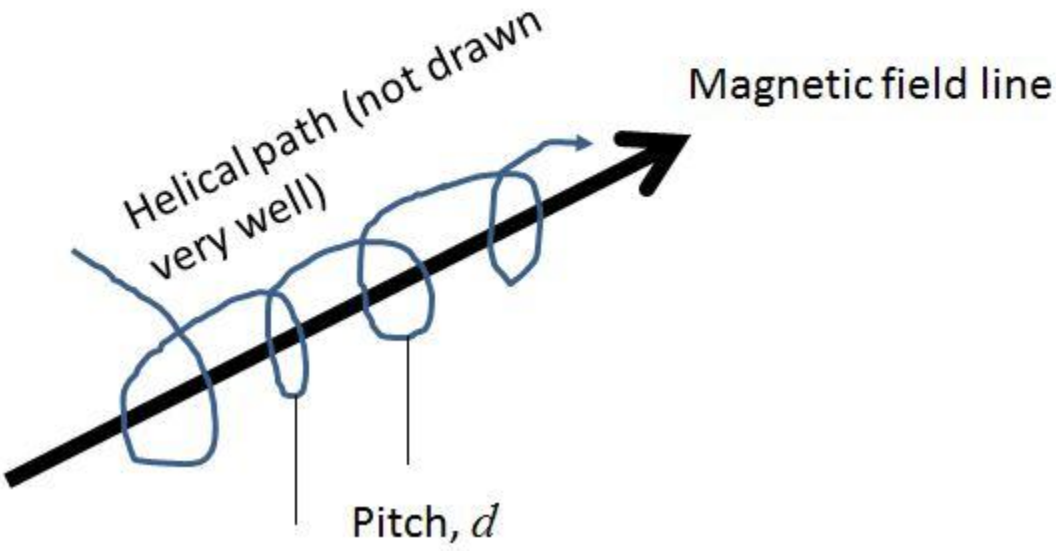


Figure 39 Helical path of a charged particle in the Earth's magnetic field

We use exactly the same argument that we used in the previous tutorial, to give:

$$r = \frac{mv \sin \theta}{BQ}$$ .... *Equation 48*

Remember that:

- The vertical velocity component gives the radius of the helical path.
- The horizontal velocity component gives the pitch of the helix.
- The horizontal component and the vertical component are treated separately.

The charged particles move down the field lines towards the poles. They interact with particles to form ions. The positive ions attract electrons, and energy is lost in the form of photons. We see this as the glow of an **aurora**.

15.055 Auroras

The **Aurora Borealis** (Northern Lights) is seen in northern regions around the north pole. They can be seen occasionally in latitudes as far south as northern England. Charged particles from the solar wind from the Sun is captured by the Earth's magnetic field. The particles are guided along the field lines until they come low enough to collide with molecules in the atmosphere. The collisions cause ionisation. This picture shows the formation of the **Aurora Australis** (Southern Lights *Figure 40*).

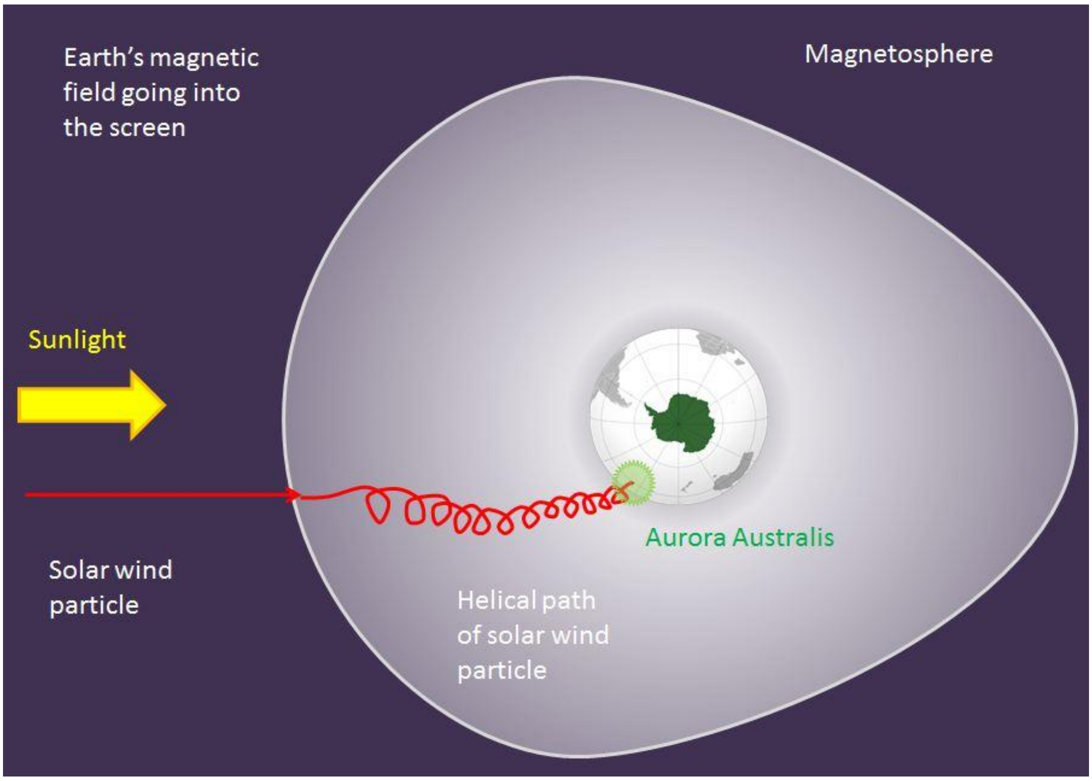


Figure 40 Formation of an aurora

When the ions lose their charge, they lose energy by emitting photons of particular energy, which results in different colours being seen. Molecular nitrogen glows purple, while atomic nitrogen glows blue. Ionisation events with oxygen lead to yellow and green lights. UV is also emitted, as well as red light (*Figure 41*).

Figure 41 Spectacular colours of an aurora

Altitude also affects the colours:

- Reds are seen above 240 km.
- Greens up to 240 km.
- Violet and purple above 100 km.
- Blue up to 100 km.

Auroras can be observed on other planets. The planets that have a magnetic field have auroras at the poles, for example, Jupiter (*Figure 42*).

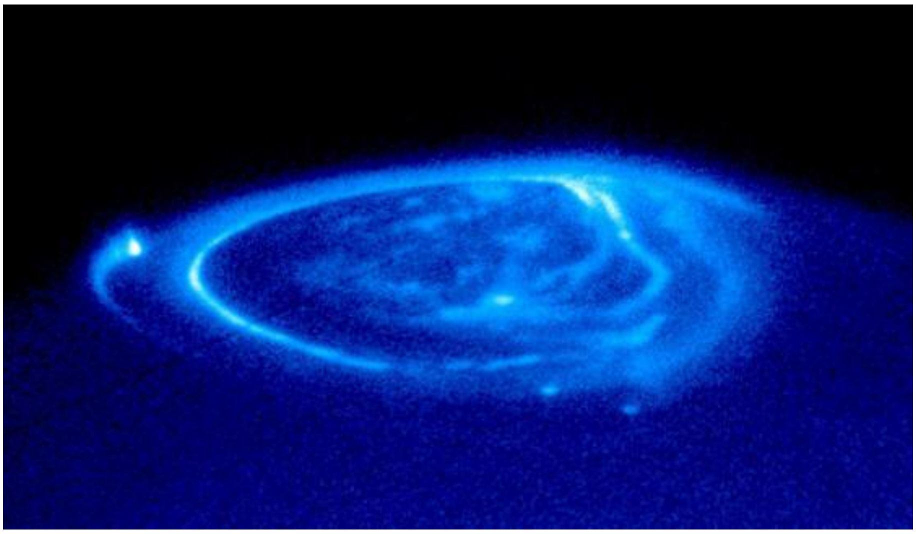


Figure 42 Aurora on Jupiter (Image J T Clarke (University of Michigan), Wikimedia Commons)

Planets that have no magnetic fields still have auroras, but these are distributed across the surface of the planet.

Auroras have been observed on a nearby brown dwarf star.

15.056 Effects of the Solar Wind

We are protected from high energy particles from the Sun by the Earth's Magnetic field. This is just as well, for the particles could interact with molecules in our bodies to cause genetic damage. During particular intense periods of solar activity, the bow shock wave can be pushed closer to the Earth, and solar wind particles can penetrate to the surface. This can interfere with telecommunications equipment and power networks. Outside the magnetosphere, the particles can cause damage to electronic equipment on satellites and space probes.

Where there is no magnetic field, the particles of the solar wind can strip away the molecules that form an atmosphere by collisions. This is thought to be how Mars has lost most of its atmosphere. A **reversal** in the Earth's magnetic field (which happens from time to time) will result in a period in which the magnetic field is zero. This will leave us exposed to the solar wind, and some of the atmosphere will be lost.

A wind in the atmosphere is the result of bulk movement of particles. The change in momentum of the particles acting on a surface results in a **force**. This can be used to propel a ship, or the blades of a wind-turbine. The solar wind can be harnessed in a similar way. A sail 800 m × 800 m is thought to be able to produce a force of about 5 N.

The idea of using space sails has been considered as a way of propelling probes into interplanetary space. The force may be low, but space is a zero friction environment.

While the acceleration from such sails is low, probes have all the time in the universe to accelerate.

Questions

Tutorial 15.05

15.05.1

A particle has an energy of 3.2 keV. What is this in joules?

15.05.2

An alpha particle is travelling at 500 km s^{-1}. Explain whether or not there will be any relativistic effects.

15.05.3

An alpha particle is travelling at 500 km s^{-1}. Where it interacts with the Earth's magnetic field, the magnetic field strength of the horizontal component is 2.3×10^{-5} T. It strikes the magnetic field at 90°.

(a) Calculate the kinetic energy of the alpha particle. Express your answer in keV as well as J.

(b) Work out the radius of the circular path it forms. Give your answer to an appropriate number of significant figures.

Mass of a helium nucleus = 6.64×10^{-27} kg.

15.05.4

An alpha particle is travelling at 500 km s^{-1}. Where it interacts with the Earth's magnetic field, the magnetic field strength of the horizontal component is 2.3×10^{-5} T. It strikes the magnetic field at 70° to the horizontal

(a) Work out the radius of the helical path it forms.

(b) Work out the time it takes to make one revolution.

(c) Work out the pitch of the helical path.

Mass of a helium nucleus = 6.64×10^{-27} kg.

15.05.5

What is the pressure on a sail 800 m × 800 m that produces a force of 5 N?

15.05.6

Two sails as described above are used to propel a space probe of total mass 1500 kg.

(a) Calculate the acceleration of the space probe using the two sails.

(b) The probe is initially accelerated by a rocket. If the initial speed of the probe is 5000 m s^{-1}, calculate the speed after 1 day.

3. Waves (SQA)

Tutorial 15.06 Mathematical Treatment of Waves

SQA Syllabus

Contents

In Waves Tutorials 1 and 2 (Topic 7), we used a **graphical** approach to describe progressive waves. As an extension, we modelled a sinusoidal wave, primarily to show how sine waves can be drawn. In this tutorial, we are going to take a more mathematical approach to study waves.

15.061 Sinusoidal Waveforms

The simplest type of wave is called a **sine wave**. This is because the displacement varies with the sine of the time. The equation is shown below:

$$x = A\sin(\omega t)$$

.... *Equation 49*

The terms are:

- x - the **displacement** (m).
- A - the **amplitude** (m) which is the maximum displacement.
- ω - the **angular velocity** (rad s^{-1}).
- t - the **time** (s).

Note that x is often used for the displacement in waves. The code s can be used as well. The symbol ω is omega, a Greek lower case letter long 'o' (ō). It represents the frequency of the wave, and is linked to the frequency by the equation:

$$\omega = 2\pi f$$

.... *Equation 50*

So, we can also write:

$$x = A\sin(2\pi f t)$$

.... *Equation 51*

Be careful about how you input *wt* into your calculator:

$$\sin(\omega \times t) \neq \sin(\omega) \times t$$

Velocity of the particles that form the wave can be worked out using the gradient of the sine wave. In calculus notation, we can write:

$$v = \frac{dx}{dt} = A\omega\cos(\omega t)$$

.... *Equation 52*

Maths Window

The derivatives of trigonometrical functions are as follows:

$$\frac{d}{dx}\sin x = \cos x$$

And

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

We won't look at the tangent functions here.

If there is a constant being processed by the function, then the derivative is multiplied by the constant. In this case, we'll make the constant B. Therefore:

$$\frac{d}{dx}\sin(Bx) = B\cos(Bx)$$

And:

$$\frac{d}{dx}\cos(Bx) = -\sin(Bx)$$

For acceleration, we can write:

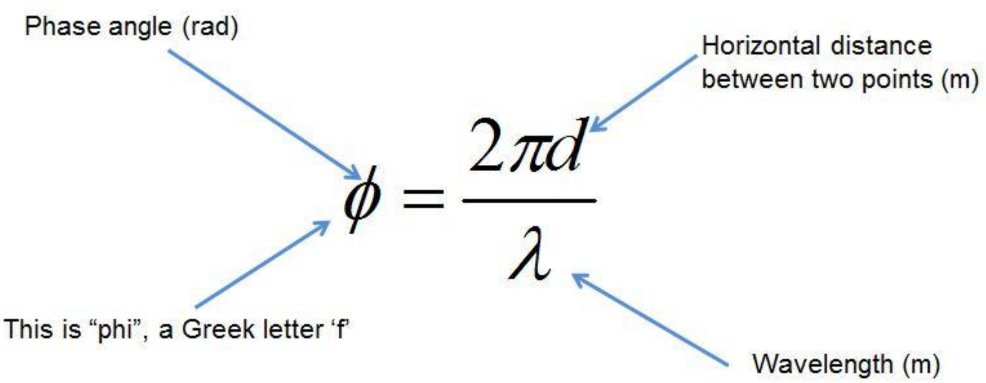
$$a = \frac{dv}{dt} = \frac{d_2x}{dt^2} = \frac{d}{dt}(A\omega\cos(\omega t)) = -A\omega^2\sin(\omega t)$$

.... Equation 53

The minus sign tells us that the acceleration is towards the average level (zero displacement).

15.062 Phase Equation

Phase tells us the relative displacement of two points on a wave. In Topic 7 Tutorial 1 we saw the equation for the phase difference between two points in a wave:



.... Equation 54

The phase angle is in radians.

When you use radians, you must ensure that your calculator is set to **radians**. The easiest way to lose marks is put your angle in **radians** while you are set to degrees...

It is up to you to make sure how to work your calculator.

15.063 Phase Difference

Consider two surfers, P and Q on a water wave. The water wave has a wavelength, λ, and an amplitude A. This is a **displacement-distance graph** (*Figure 43*).

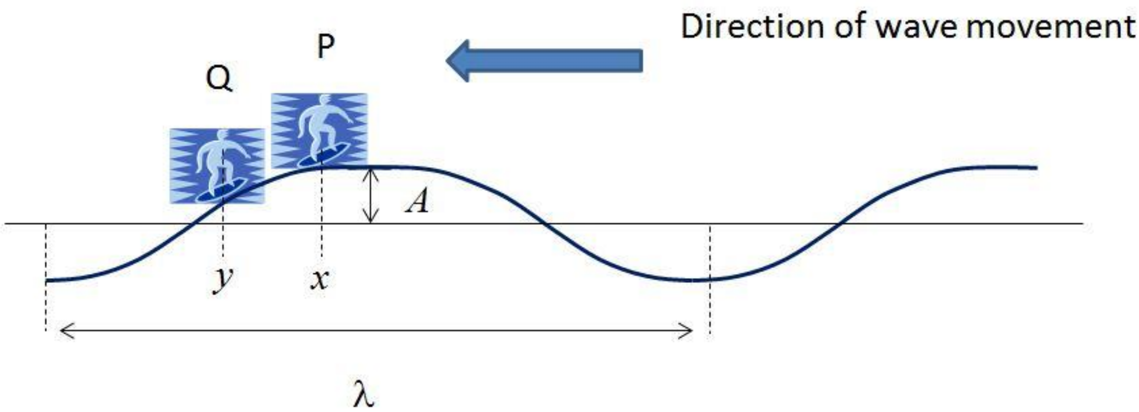


Figure 43 Surfers on a water wave

The wave is travelling from right to left. Surfer P is further up the wave than his mate, Q. This means that P **leads** Q. Or we can say that Q is behind P, Q **lags** P.

(Note that we are treating the water wave as transverse. This is true only if the amplitude is much smaller than the wavelength. A water wave is technically a **roller**, and it has feature that are both transverse and longitudinal.)

On a displacement-time graph at a certain time, t, this looks like (*Figure 44*):

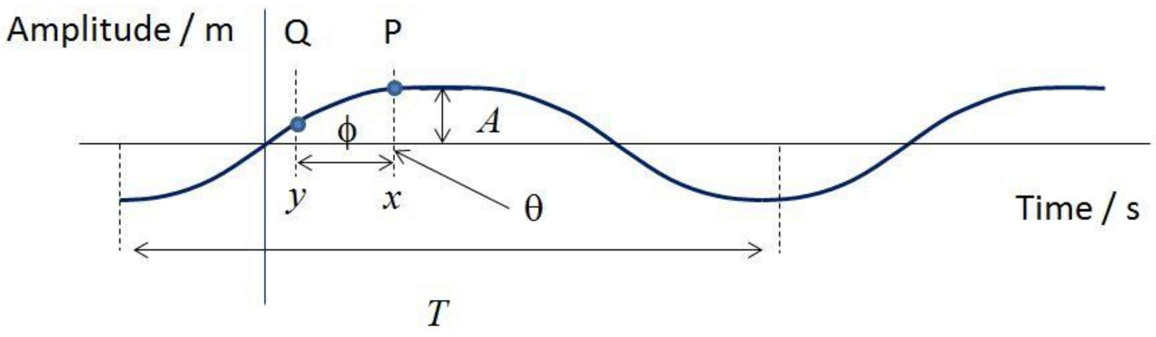


Figure 44 Displacement-time graph for Figure 43

On the displacement time graph, surfer P is at angle θ. The amplitude of the wave at angle θ is x, where:

$$x = A\sin(\theta)$$

.... *Equation 55*

Surfer Q is at angle (θ - ϕ). The amplitude of the wave here is:

$$y = A\sin(\theta - \phi)$$

.... *Equation 56*

We know that:

$$\theta = \omega t$$ .... *Equation 57*

So, we can rewrite this expression as:

$$y = A \sin (\omega t - \phi)$$ .... *Equation 58*

Or:

$$y = A \sin (2\pi f t - \phi)$$ .... *Equation 59*

From above (*Equation 54*), we see that:

$$\phi = \frac{2\pi d}{\lambda}$$

So, it doesn't take a genius to write:

$$y = A\sin\left(2\pi f t - \frac{2\pi d}{\lambda}\right)$$

.... *Equation 60*

The expression tidies up to give:

$$y = A\sin 2\pi\left(f t - \frac{d}{\lambda}\right)$$

.... *Equation 61*

Let's put some numbers in. Answer Question 15.06.3.

15.064 Superposition of Waves

We have looked at how waves can superpose. See Topic 7 Tutorial 3. In this case, we will look at how two waves add using their formula. Normally this would be highly tedious, but a spreadsheet makes light work of it. We can use an Excel Spreadsheet to model the superposition of two waves. In this case we have used the basic relationship for amplitude:

$$x = A\sin(\omega t)$$

.... *Equation 62*

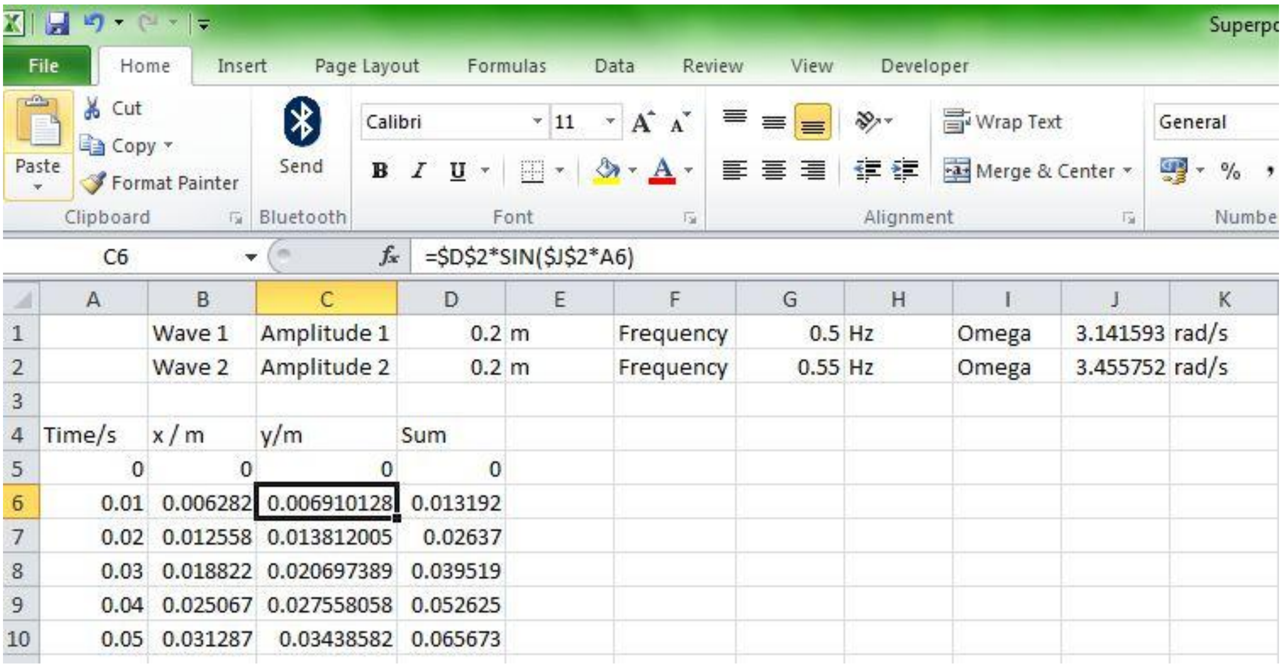
For Wave 1, the numerical parameters are:

- Amplitude = 0.20 m.
- Frequency = 0.5 Hz.

For Wave 2, the numerical parameters are:

- Amplitude = 0.20 m.
- Frequency = 0.55 Hz.

A screenshot shows the spreadsheet data (*Figure 45*):



	A	B	C	D	E	F	G	H	I	J	K
1		Wave 1	Amplitude 1	0.2	m	Frequency	0.5	Hz	Omega	3.141593	rad/s
2		Wave 2	Amplitude 2	0.2	m	Frequency	0.55	Hz	Omega	3.455752	rad/s
3											
4	Time/s	x / m	y/m	Sum							
5	0	0	0	0							
6	0.01	0.006282	0.006910128	0.013192							
7	0.02	0.012558	0.013812005	0.02637							
8	0.03	0.018822	0.020697389	0.039519							
9	0.04	0.025067	0.027558058	0.052625							
10	0.05	0.031287	0.03438582	0.065673							

Figure 45 Screenshot of data to draw sine waves

The formula in Cell 6 is:

=D2*SIN(J2*A6)

The dollar signs ($) are to lock the reference cells. The * sign is the multiplication operator for Excel. The graph shows the two waves (*Figures 46* and *47*):

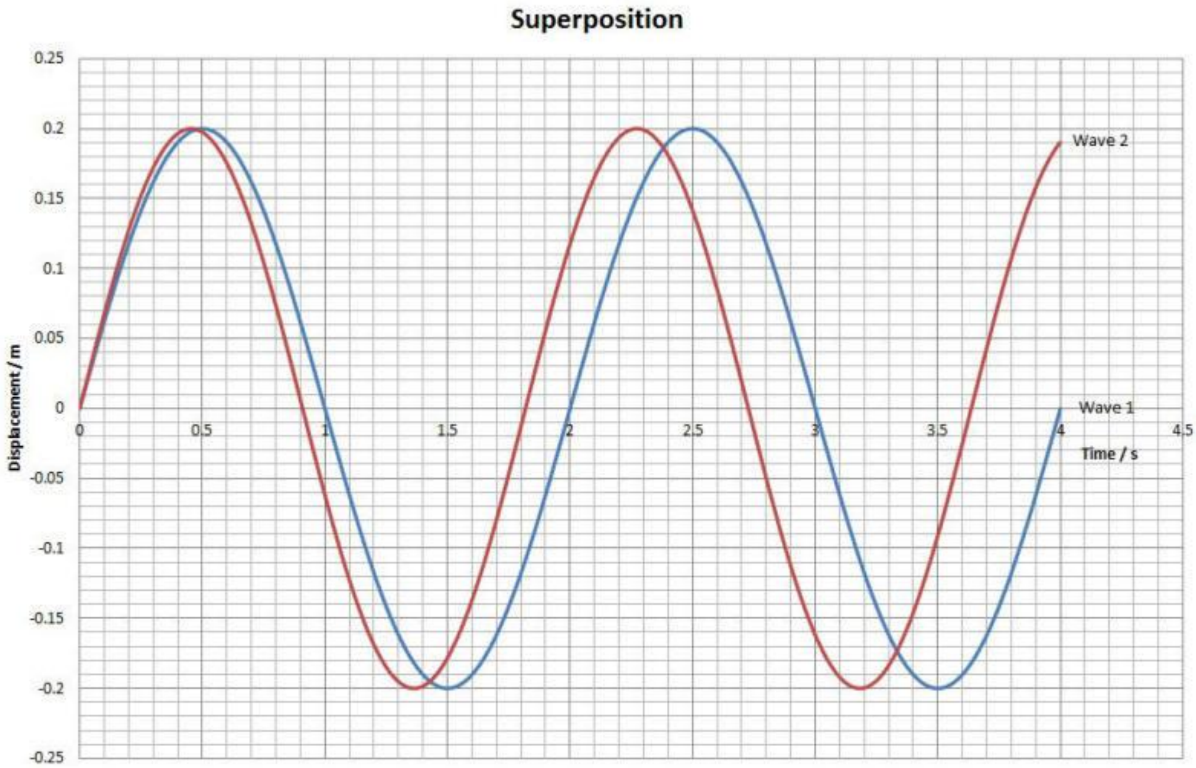


Figure 46 Two waves of slightly different periods

And the resultant wave is:

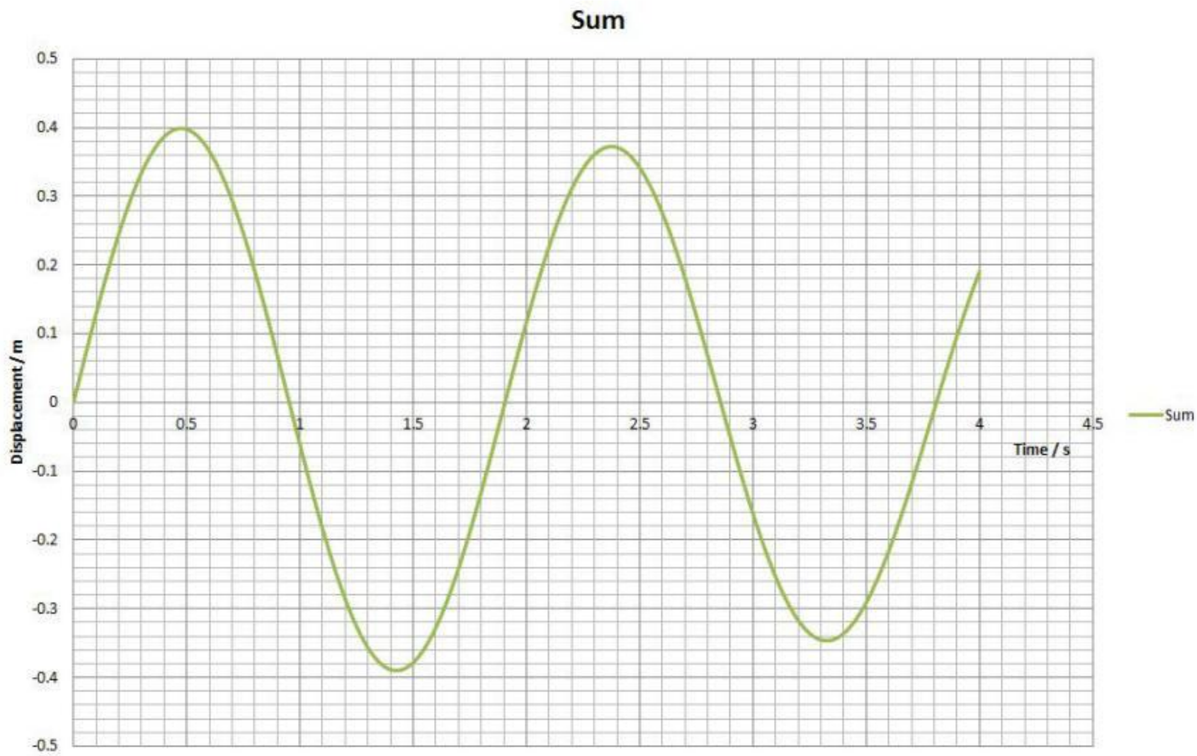


Figure 47 And their sum

Remember that the displacement is a **vector**. If the two displacements have the same sign, the result is **reinforcement** or **constructive** interference. If the two displacements have opposite signs, the result is **cancellation** or **destructive** interference.

15.065 Beats

If we extend the time axis, we get (*Figure 48*):

Figure 48 Beats

In this picture we see that there is a periodic rise from a minimum of 0 to a maximum of 0.4 m and back to 0. These are called **beats**.

The effect is most noticeable if the two (sound) waves have a frequency that is almost the same. The beat frequency is simply the difference of the two frequencies:

$$f_{\text{beat}} = f_2 - f_1$$

.... *Equation 63*

15.066 Musical Waveforms

The sound of a musical instrument is determined by:

- The **pitch** or **frequency** of the note.
- The **volume** or **amplitude** of the note.
- The **quality** or **waveform** made by the instrument.

The pitch is NOT the volume.

Pure sine waves are very dull to listen to. The waveform of an instrument is quite complex. Musical instruments have their own characteristics that depend on the waveform that they produce. For example (*Figure 49*):

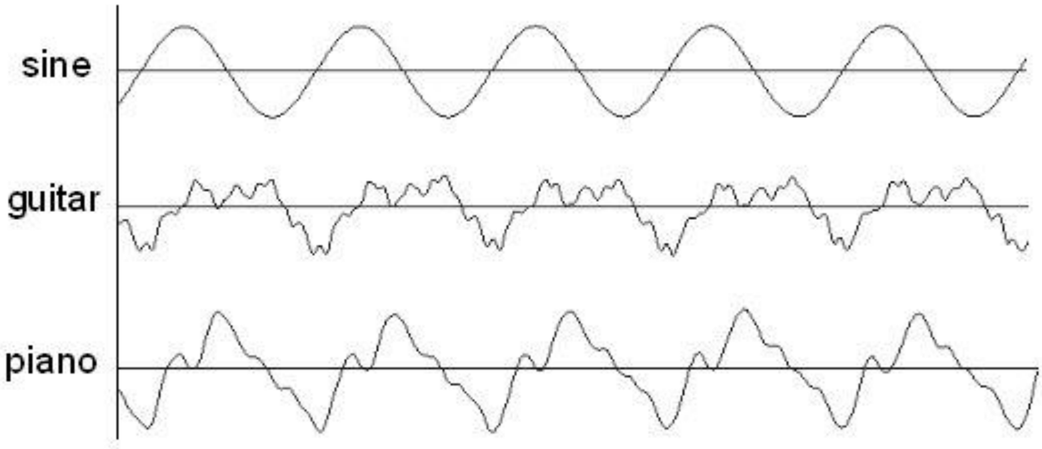


Figure 49 Waveforms of musical instruments (Image from Yuval Nov)

The complex waveforms shown in the diagram are actually made up of lots of **sine waves** that are superposing.

The factors that produce the distinctive sounds are complex and may even vary between instruments of the same type. For example, the *Stradivarius* violin has a particular tone that is much sought after by professional classical musicians over other violins. It is not just the way the sound-box and other characteristics of the instrument are made; the tone can be influenced by things like the grain of the wood, or even the varnish used by the craftsman. Antonio Stradivari (1644 - 1737) had his own techniques that were used in his family, which are not known, and cannot be copied. His instruments are priceless.

One of the most important features of the sound of an instrument is the production of the **harmonics**. If the note A_3 (A below Middle C) is played, its fundamental frequency is 220 Hz. Its harmonics are shown in the table:

Harmonic	Frequency / Hz
1	220
2	440
3	660
4	880
5	1100
6	1320

Each frequency is 220 Hz apart. **Beats** of 220 Hz are also set up, which the ear can pick up. If the fundamental frequency is removed by filtering, it is still possible for a listener to "deduce" the fundamental frequency. This is called the **missing fundamental** effect.

Old-fashioned police whistles have two separate frequencies that are close to each other. These produce beats, which gave the whistles their characteristic warble.

Woodwind and brass instruments use beats to give a strong sense of pitch. Two flutes played together make beats, and this gives the impression of a third instrument ("a trio of two flutes"). A brass instrument player can hum another frequency to make a third tone. This is called **multiphonics**.

15.067 Fourier Analysis

This is a mathematical technique that is applied to complex waveforms, first worked out by the French mathematician Jean-Baptiste Joseph Fourier (1768 - 1830). The idea is that a complex waveform can be broken down or **decomposed** into its component sine waves. This is done using advanced and complex mathematical functions. Consider a bass guitar playing the note A_2 (55 Hz). The waveform looks like this (*Figure 50*):

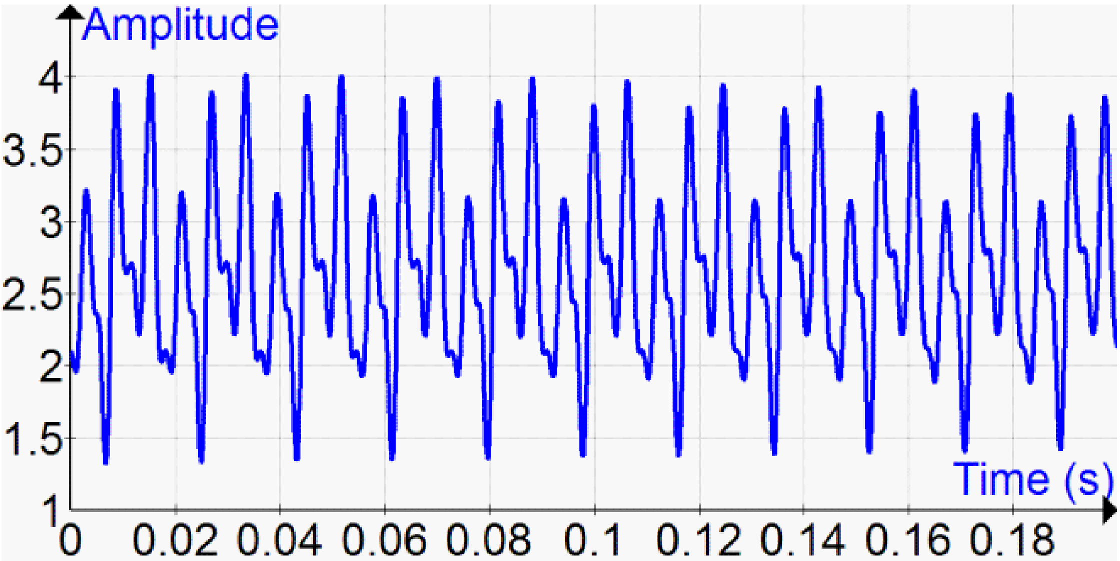


Figure 50 A complex waveform Image by Fourier1789 - Wikimedia Commons)

Once Fourier Analysis (or Transform) is applied, we can see the components that make up the complex wave:

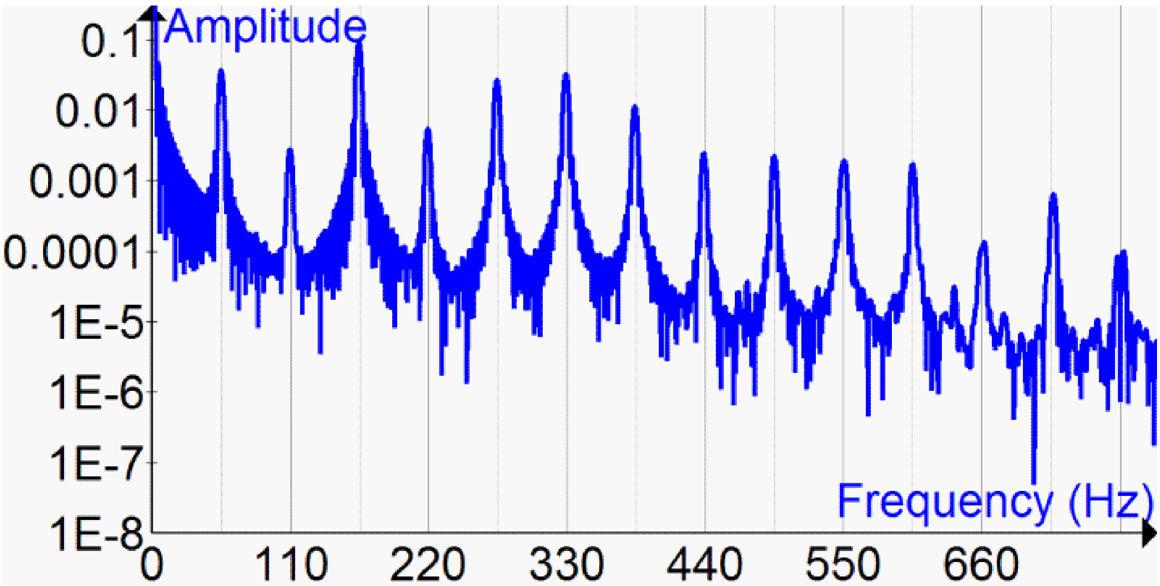


Figure 51 Waveform split into its components (Image by Fourier1789 - Wikimedia Commons)

A Fourier transform in reverse can be used to **synthesise** musical sounds. This enables musicians to connect a synthesiser box to an instrument like a guitar to make a range of musical (and non-musical sounds). You can buy keyboards at low cost that have a variety of synthesised musical sounds that you can play.

Fourier analysis can be applied to a range of oscillating systems, for example, the processes within a large chemical plant. If there are several linked steps in a continuous process, and one process is slowed down, then the others will be as well. If we try to speed the first process up to compensate, we get a build-up of the intermediate product. This means that the next stage needs to be speeded up, and so on. The plant starts to **oscillate**, which can lead to problems (including the risk of an explosion). The oscillations can be complex but can be more easily understood using a Fourier analysis. The interpretation of the analysis enables chemical engineers to see where the oscillations start and apply measures to keep the oscillations to a minimum.

Questions

Tutorial 15.06

15.06.1

A wave has an amplitude of 0.25 m and a frequency of 6.0 Hz. Calculate:

a. The angular velocity.

b. The displacement at 1.1 s.

c. The velocity at 1.1 s.

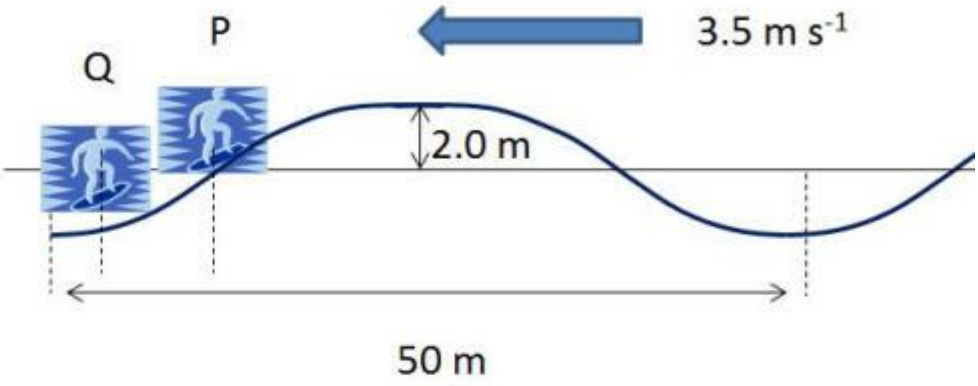
d. The acceleration at 1.1 s.

15.06.2

A wave of wavelength 3.5 m has two points that are 20 cm apart. Calculate the phase difference.

15.06.3

The surfers in the argument above are riding a wave that has a wavelength of 50 m, amplitude 2.0 m and is travelling at a speed of 3.5 m s^{-1}. At time $t = 0$, Surfer P is about to ride the wave as shown in the diagram:



Surfer Q is 6.5 m behind Surfer P.

(a) Show that the period of the wave is about 14 s.

(b) Calculate the displacement *x* of Surfer P after 2.5 s.

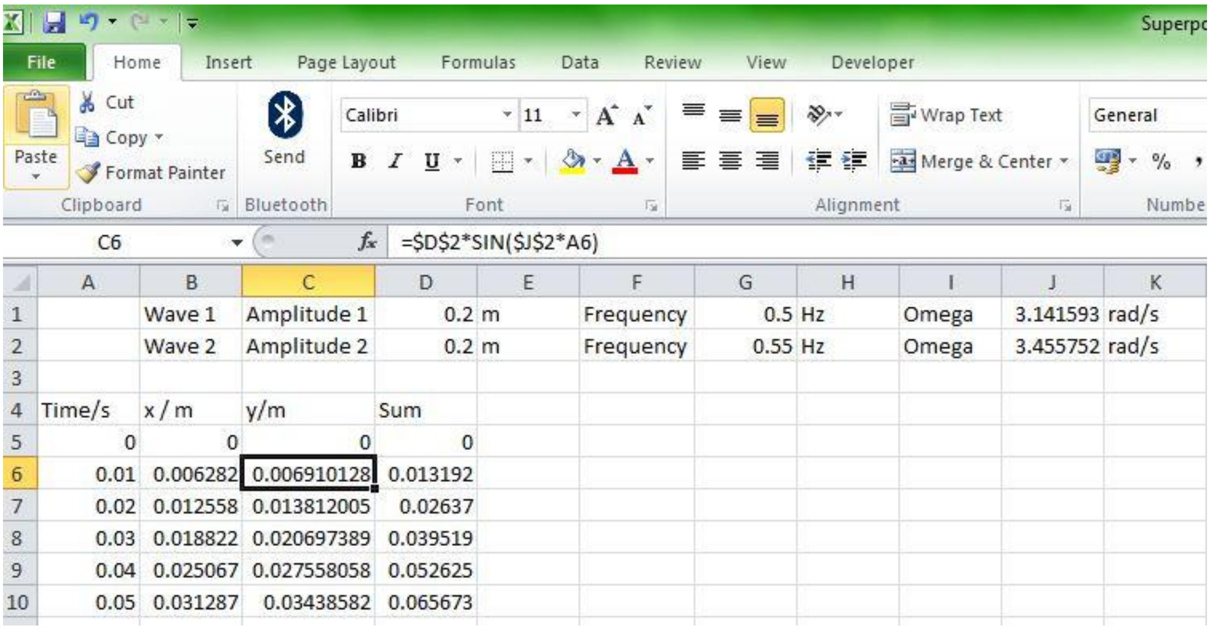
(c) Calculate the displacement *y* of Surfer Q after 2.5 s.

15.06.4

Look at *Figure 48* on page 76. What is the period of the beats? What is the frequency of the beats?

15.06.5

Show that the beat frequency you worked out in Question 4 is consistent with the data that are given in the Excel Spreadsheet model.



	A	B	C	D	E	F	G	H	I	J	K
1		Wave 1	Amplitude 1	0.2	m	Frequency	0.5	Hz	Omega	3.141593	rad/s
2		Wave 2	Amplitude 2	0.2	m	Frequency	0.55	Hz	Omega	3.455752	rad/s
3											
4	Time/s	x / m	y/m	Sum							
5	0	0	0	0							
6	0.01	0.006282	0.006910128	0.013192							
7	0.02	0.012558	0.013812005	0.02637							
8	0.03	0.018822	0.020697389	0.039519							
9	0.04	0.025067	0.027558058	0.052625							
10	0.05	0.031287	0.03438582	0.065673							

15.06.6

Loof at *Figure 51*. What do you notice about the peaks?

Tutorial 15.07 Interference

SQA Syllabus

Contents

This is quite a long and challenging tutorial. Work through it slowly and carefully. If you don't understand it, ask for help from your tutor.

Before you attempt this tutorial, you should review Topic 7 Waves Tutorial 7 and Waves Tutorial 8.

Remember also that **coherent** waves have:

- the **same frequency**,
- (nearly) the **same amplitude**,
- and a **constant phase relationship**.

They do not have to be in phase, as long as the phase difference remains the same. Light waves of the same frequency are called **monochromatic**.

Note that the AQA Physics codes used in Waves Tutorial 7 are different to the SQA codes used here:

- **Wavelength** is the same (λ).
- **Distance** to the screen is the same (D).
- **Slit separation** is d here (instead of s).
- **Fringe separation** is Δx (instead of w).
- **Number of wavelengths** or **orders** is m (instead of n).

17.071 Path Difference

We know that constructive interference results when two waves interact and their displacements are in the same direction. If the displacements are in the opposite directions, the result is destructive interference.

In this diagram (*Figure 52*) there are 8 blue waves from Source 1 and 12 red waves from Source 2. The waves are **coherent**. (They are different colours to show the different trains of waves.) The path difference is 4 wavelengths, and the two waves are **in phase** when they interact:

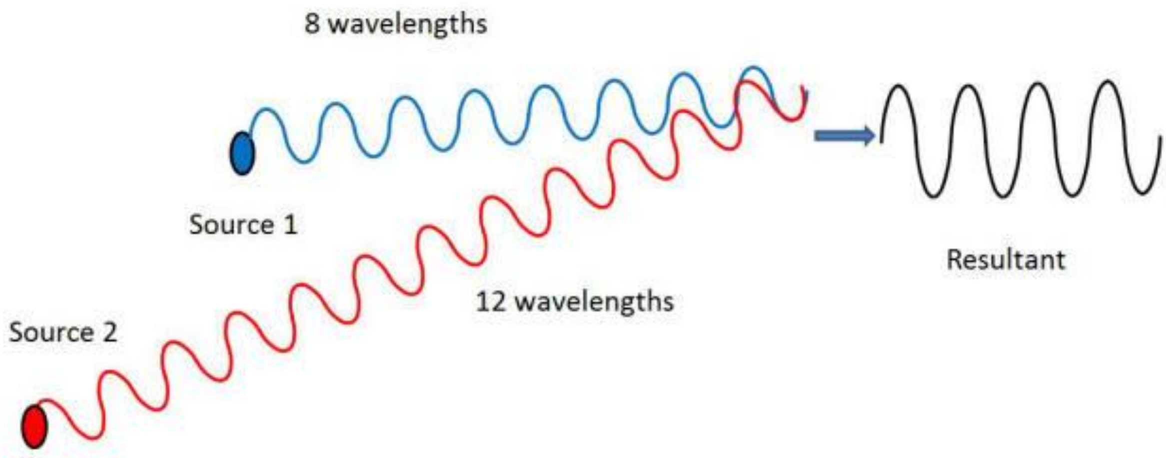


Figure 52 Path difference

In other words, the crests coincide with crests, and the troughs coincide with troughs. Therefore, they interfere **constructively** so the **resultant** has a large amplitude. The constructive interference is the result of the path difference being a **whole number** of wavelengths. Or we can say that the path difference is an **even** number of **half-wavelengths**.

Now in this case (*Figure 53*), the path difference is 4.5 wavelengths. Since the waves are π radians out of phase, there is **destructive interference**. In other words, the crests coincide with troughs.

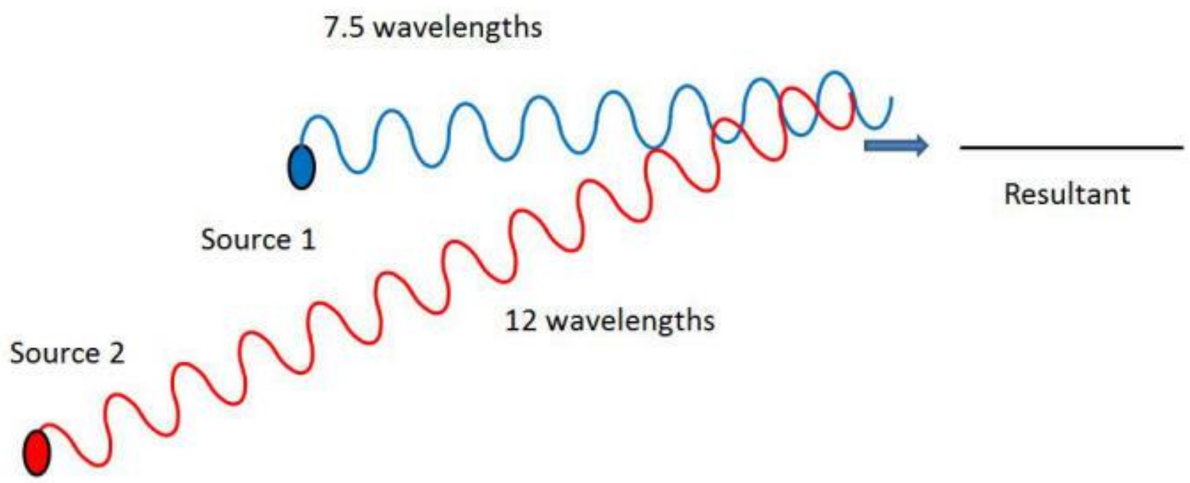


Figure 53 Path difference of an odd number of half wavelengths

The resultant has an amplitude of zero because the waves have the same amplitude, but the directions are opposite. We say that the path difference is an **odd** number of half wavelengths.

15.072 Path Length

We have seen how path difference is the difference between two distances travelled by two light rays from their sources. Consider these diagrams.

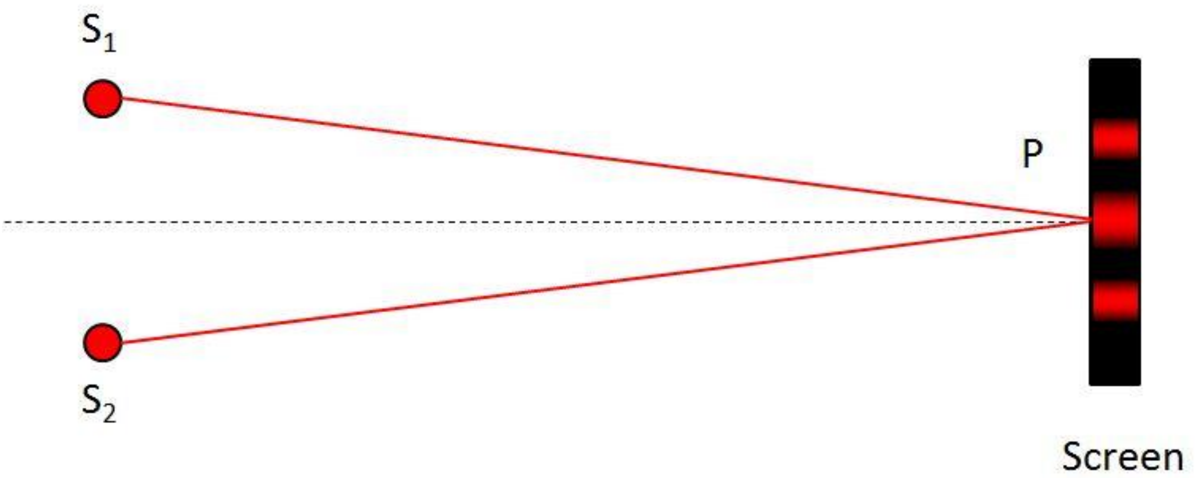


Figure 54 Light rays with zero path difference

It doesn't take a genius to see that the path length between S1 to P is the same as S2 to P. The path difference is zero. So, we can say:

$$S_2P - S_1P = 0$$ .... *Equation 64*

Now look at this (*Figure 55*):

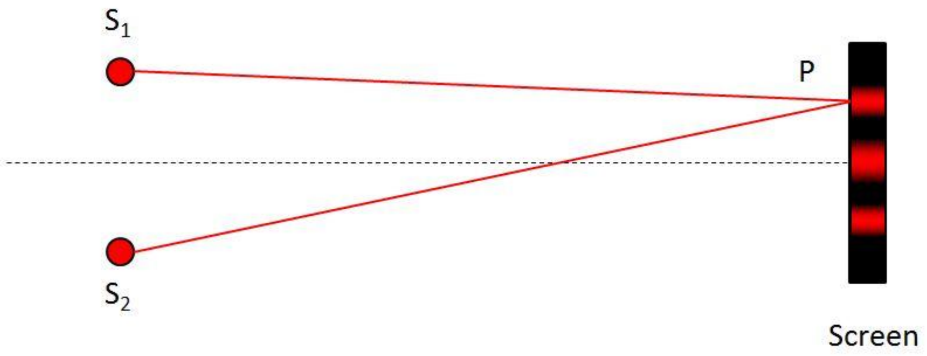


Figure 55 Path difference of one wavelength

We know that the difference between S_2P and S_1P is a whole number of wavelengths, *m*, because we have a **bright region** or **bright fringe** due to constructive interference. So:

$$S_2P - S_1P = m\lambda$$ ....*Equation 65*

The rays meet at the first bright fringe, because there is a path difference of 1 wavelength.

In the diagram below, the path difference is (*m* + ½) wavelengths, because we have a dark fringe due to destructive interference (*Figure 56*):

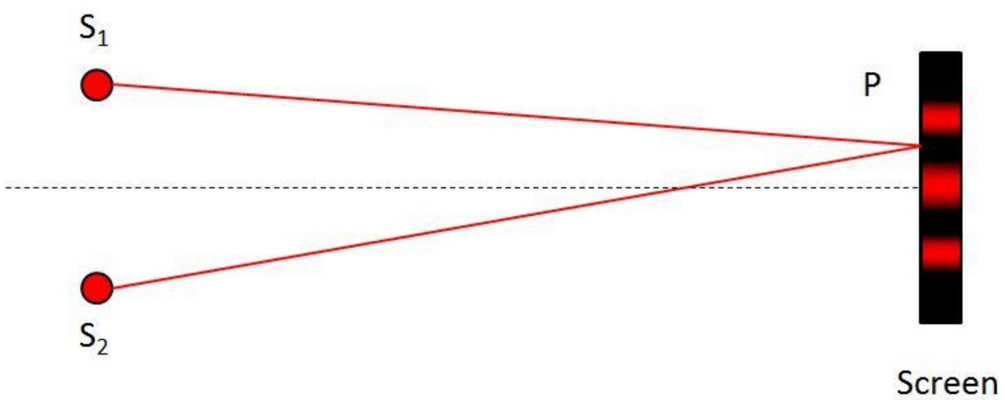


Figure 56 Destructive interference

So, we can write:

$$S_2P - S_1P = (m + ½)\ \lambda$$ .... *Equation 66*

In all these cases, the optical path difference is the same as the geometrical path difference. Now consider this (*Figure 57*):

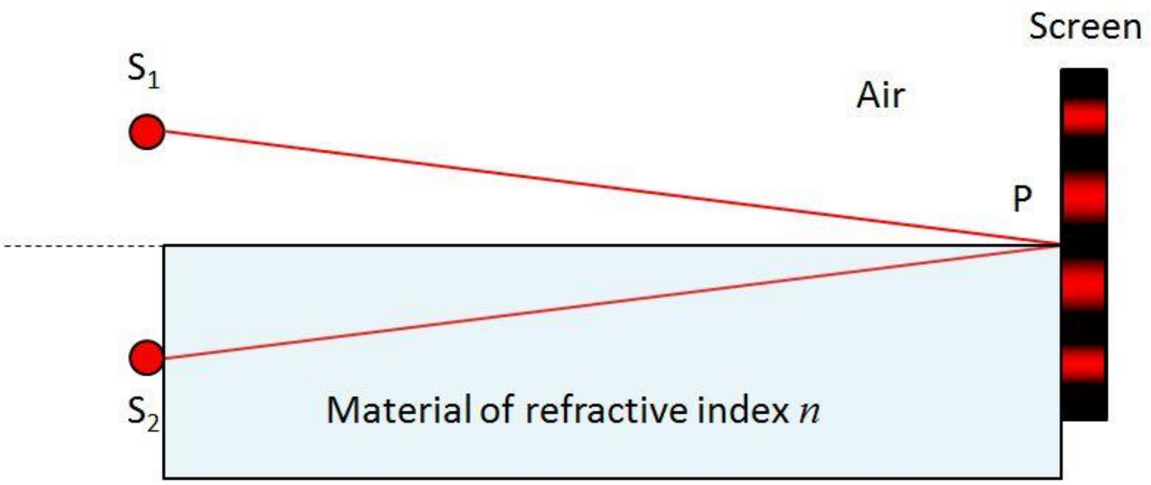


Figure 57 Passing light through a block of material of refractive index n

In this case, the **geometrical** path difference is **zero**, as we would expect. However, the **optical** path difference is $(m + ½)$ **wavelengths**, because we have **destructive interference** to give a dark spot. The waves are out of phase by π radians.

The reason for this is that light waves travel more slowly in optically denser materials. In a material with refractive index of 1.5, the speed of light is 2.0×10^8 m s^{-1}, as compared with 3.0×10^8 m s^{-1} for air. The key point to remember is that the **frequency remains the same**. The colour of the light, governed by frequency, does not alter. Therefore, to comply with the wave equation, the wavelength must be reduced by the same proportion. Therefore:

$$n = \frac{\lambda_{\text{air}}}{\lambda_{\text{glass}}}$$ .... *Equation 67*

Therefore, the optical path length can be worked out:

optical path length = geometrical path length × refractive index

So the optical path difference, G, in this case is:

$$\Gamma = (S_2P \times n) - S_1P = (m + ½)\,\lambda$$ .... *Equation 68*

The strange looking symbol, Γ, that looks like a gallows is "Gamma", a Greek capital letter 'G'. It is used in several sources.

We can now extend this to write a more general expression. Let's call S_2P distance d_2, and S_1P distance d_1:

$$\Gamma = (d_2 n_{glass}) - (d_1 n_{air}) = (m + ½)\,\lambda$$ .... *Equation 69*

If the two distances are the same, we can write for a **minimum**:

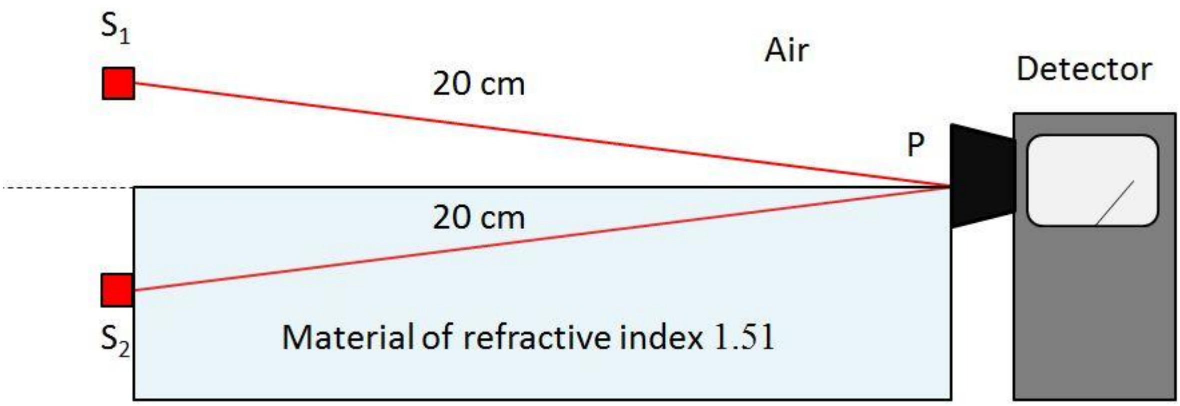
$$\Gamma = d\,(n_{glass} - n_{air}) = (m + ½)\,\lambda$$ .... *Equation 70*

If the result is constructive interference (leading to a **maximum**) the relationship is changed to:

$$\Gamma = d\,(n_{glass} - n_{air}) = m\,\lambda$$ .... *Equation 71*

Worked example

A coherent source of microwaves of wavelength 6.00 mm is split between two slits. Each microwave beam is 20 cm long. The top beam passes through air, while the bottom ray passes through a block of material of refractive index 1.51. A maximum reading is observed when the two rays meet.



What is the optical path difference in metres and wavelengths?

Answer

$\Gamma = d(n_{material} - n_{air})$

Γ = 0.20 m × (1.51 - 1.00) = 0.20 m × 0.51 = 0.102 m

$m \times 6.00 \times 10^{-3}$ m = 0.102 m

$m = 0.102$ m ÷ $(6.00 \times 10^{-3}$ m) = **17 wavelengths**.

In the last example we had the whole of one beam being transmitted by an optically denser material. Of course we don't have to have that. We could set up a laser with two slits and sneak in a thin piece of glass into the optical path - like this (*Figure 58*):

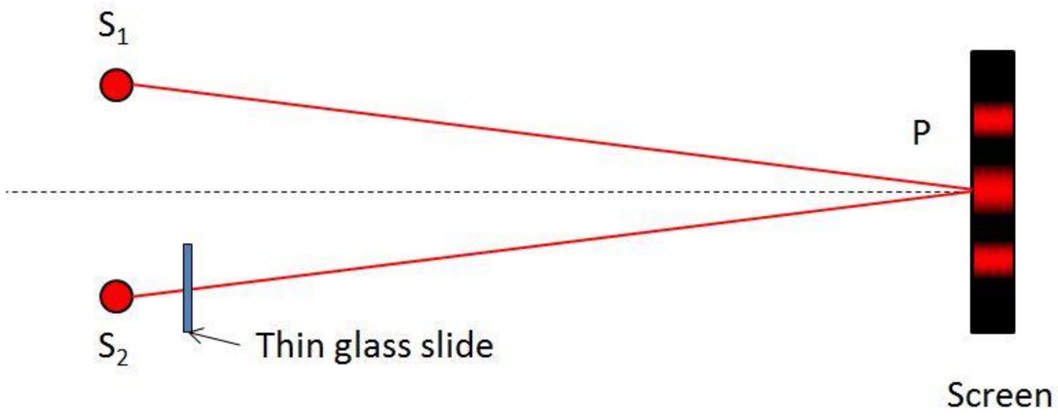


Figure 58 Inserting a glass slide into one of the optical paths

This is easier said than done.

15.073 Interference has to use a Single Light Source

We can easily get a sound interference pattern from two loudspeakers. However, getting coherent light from two light sources (including lasers) is impossible. This is because photon generation is **random**, not continuous. Light is produced by the excitation of individual groups of atoms in bursts lasting less than nanoseconds ($<1 \times 10^{-9}$ s). There is no constancy in the phase relationships, even from a small region of the light source. Although we need not go into the explanation for this, it has been found that the coherence length for two rays of light rarely exceeds 1 mm. The phase relationships between the many millions of photons produced every second will be entirely random.

If we place a light bulb behind the two slits through which light passes, we will not achieve coherence. So, we won't get an interference pattern. With a laser, we do, because the photons have a constant phase relationship.

If we place a single narrow slit in front or the light bulb, before the light passes to the two slits, the ray becomes more coherent, so an interference pattern can be seen (*Figure 59*).

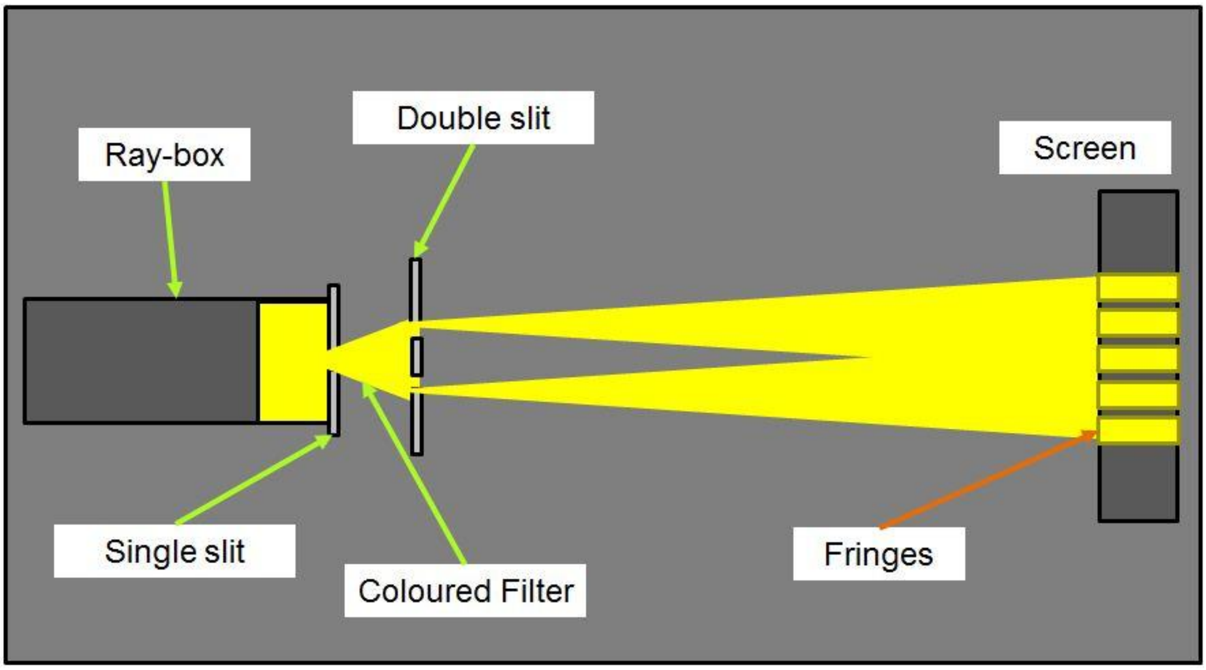


Figure 59 Placing a single slit before a light source increases the coherence

Interference patterns are still difficult to see with a set-up like this. How anything meaningful could be observed with a candle...

Interference patterns in light are made using a single ray of light that is split into two. We can produce interference patterns by two methods:

1. Interference due to division of amplitude.
2. Interference due to division of wavefront.

15.074 Interference due to Division of Amplitude

A single light beam is split into two rays. One ray is transmitted and one is reflected at a boundary between two materials of different refractive indices. Consider a thin film of oil floating on water, as shown below (*Figure 60*).

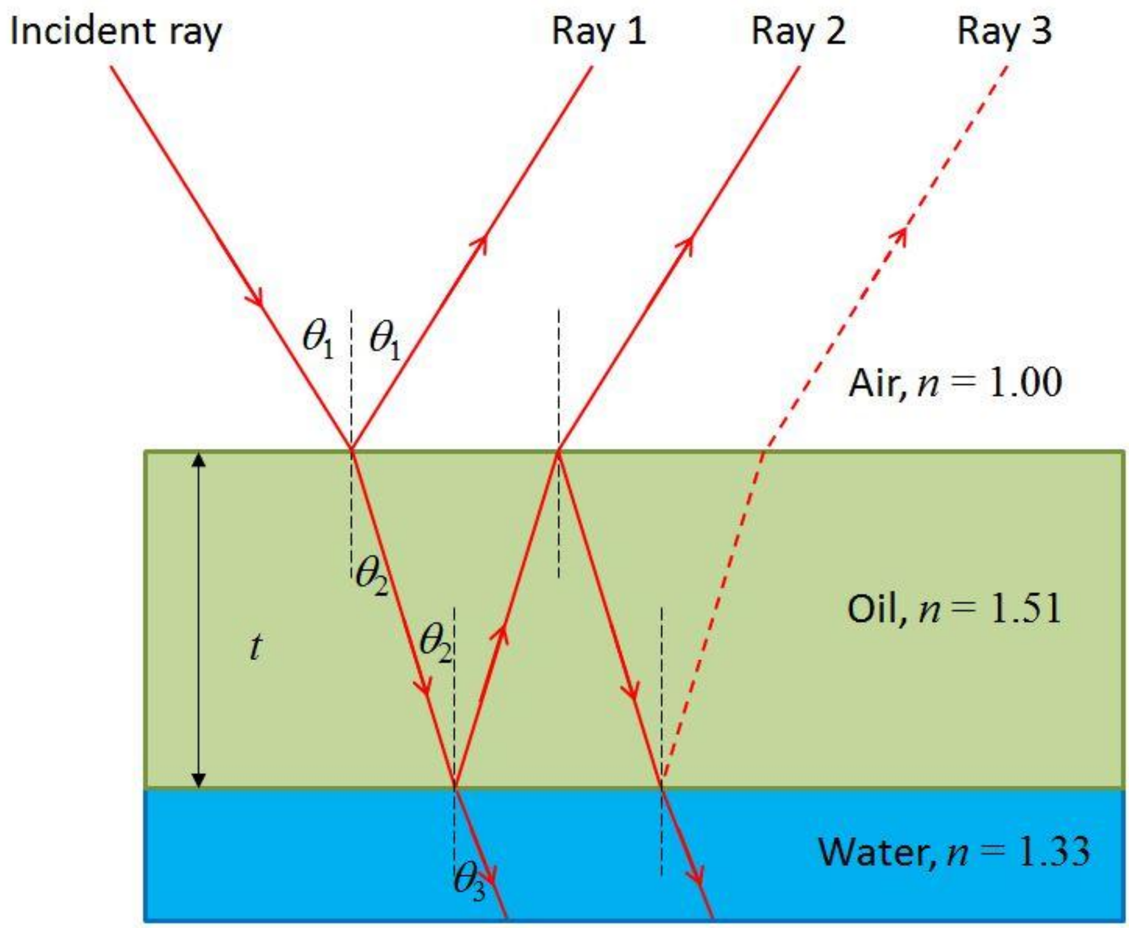


Figure 60 Thin film of oil on water

The layer of oil has a thickness, t. The incident ray strikes the surface of the oil at an angle of θ_1. It is refracted into the oil at an angle of θ_2. When it has passed through the oil, it is refracted into the water at an angle of θ_3.

Most of the light energy is transmitted into the water. However, a small proportion of the energy is **reflected** at each boundary to make a weak reflected ray. In the diagram we can see the way these rays are reflected. Let's suppose 5 % of the energy is reflected at each boundary. The Incident ray has an energy of 100 units.

- The first reflected ray, Ray 1, has an energy of 5 units.
- The first refracted ray has an energy of 95 units.

- At the boundary between the oil and the water, the second reflected ray has an energy of 0.05 × 95 units = 4.75 units.
- At the boundary between the oil and the air, the second refracted ray (Ray 2) has an energy of 0.95 × 4.75 = 4.51 units

The way Rays 1 and 2 interact depends on the thickness of the oil. If the optical path length of Ray 2 is one half wavelength different to the **optical** path length of Ray 1, we will get destructive interference. This explains why we get dark regions on a film of oil on water. We also get a **spectrum** of colours, because red light is refracted less than blue light (*Figure 61*).

Figure 61 Oil on water (Image by John - Wikimedia Commons)

A similar effect is seen with soap bubbles, or coatings on glass. We will now look at the optical path difference in a thin film.

Two important points before we start:

- If the light goes from an optically less dense material (lower refractive index) to an optically more dense material (higher refractive index), there is a **phase change** of π radians when the light is reflected.
- If the light goes from an optically more dense material (higher refractive index) to an optically less dense material (lower refractive index), there is no phase change when the light is reflected.

15.075 Optical Path Difference in a Thin Film

We will consider an incident ray of wavelength *l* that strikes an air-oil boundary at an angle of incidence of θ_1. Most of the ray is transmitted, being **refracted** through an angle of refraction of θ_2. However, a **weak reflected ray**, Ray 1, is observed. The angle of reflection is, of course, θ_1. It is important to note that the **phase of the reflected ray is changed by π radians (180°)**.

At the oil-water boundary, the ray is mostly refracted into the water. We are not interested in this, but we are interested in the ray that is reflected. Since the ray strikes a boundary where the water is optically less dense than the oil, there is **zero phase change**. The ray is reflected at an angle of θ_2. It then is refracted through the oil-air boundary. Some is reflected back, but we are not interested in that. The angle through which Ray 2 is refracted is θ_1 (*Figure 62*).

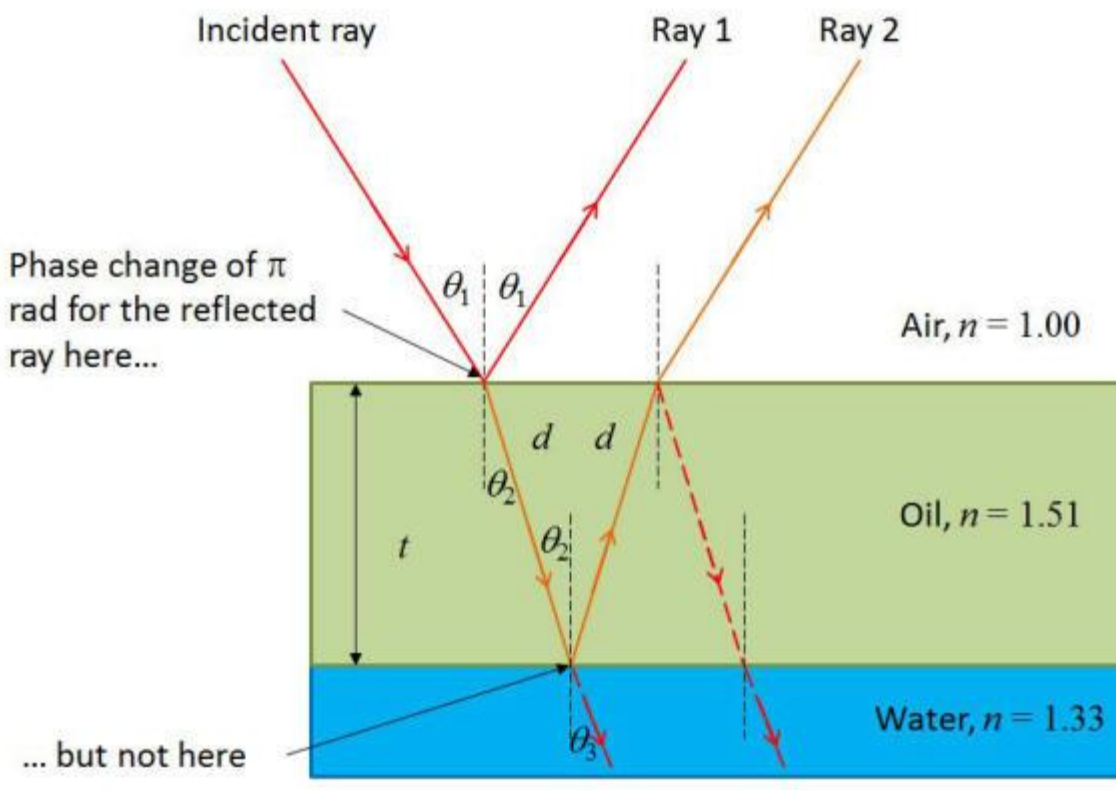


Figure 62 Light ray passing through oil on water

The incident ray and Ray 1 are in red, while the path followed by Ray 2 is shown in orange. The geometric path difference of Ray 2 can be seen to be $2d$. We can also express d in terms of the thickness of the film, t:

$$d = \frac{t}{\cos \theta_2}$$

.... Equation 72

For the sake of simplicity, we will keep this factor out of the argument until the end. However, we must also **take into account the phase change** which is ½ a wavelength:

$$\Gamma = 2n_{\text{oil}}d + \frac{\lambda}{2}$$

.... Equation 73

Let us consider what would happen for **constructive interference**. We need a whole number of wavelengths (or even number of half wavelengths) for the optical path difference. Therefore:

$$m\lambda = 2n_{\text{oil}}d + \frac{\lambda}{2}$$

.... *Equation 74*

We can rewrite this as:

$$2n_{\text{oil}}d = m\lambda - \frac{\lambda}{2}$$

.... *Equation 75*

Hence:

$$2n_{\text{oil}}d = \lambda\left(m - \frac{1}{2}\right)$$

.... *Equation 76*

Rearranging:

$$d = \frac{\lambda\left(m - \frac{1}{2}\right)}{2n_{\text{oil}}}$$

.... *Equation 77*

It is far more likely that we will know the thickness, t. No problem: we have an expression above linking d to t. Therefore:

$$\frac{t}{\cos\theta_2} = \frac{\lambda\left(m - \frac{1}{2}\right)}{2n_{\text{oil}}}$$

.... *Equation 78*

For destructive interference (a dark region on the film) we need the optical path difference to be an odd number of half wavelengths:

$$\Gamma = \left(m\lambda + \frac{\lambda}{2}\right)$$

.... Equation 79

We know that:

$$\Gamma = 2n_{\text{oil}}d + \frac{\lambda}{2}$$

.... Equation 80

Therefore:

$$m\lambda + \frac{\lambda}{2} = 2n_{\text{oil}}d + \frac{\lambda}{2}$$

.... Equation 81

Since the $\lambda/2$ terms cancel, we can therefore write:

$$m\lambda = 2n_{\text{oil}}d$$

.... Equation 82

And we can rearrange to give:

$$d = \frac{m\lambda}{2n_{\text{oil}}}$$

.... Equation 83

We can now bring in the expression linking d and t:

$$\frac{t}{\cos\theta_2} = \frac{m\lambda}{2n_{\text{oil}}}$$

.... Equation 84

15.076 Coated Lenses

If you have a good quality camera (*Figure 63*), you will see that it has a thin coat of **magnesium fluoride** (MgF_2) on the lens. The idea is to stop light reflecting from the lens. In the picture, you can see how the reflection from the bright flash has been reduced.

Figure 63 A good quality camera

The reason for the coating is to reduce flare and ghost images, which can spoil the picture. This camera also has a daylight filter, primarily to protect the lens. So, let's look at how this works. Remember:

- If the light goes from an optically less dense material (lower refractive index) to an optically more dense material (higher refractive index), there is a **phase change of π radians** when the light is reflected.
- If the light goes from an optically more dense material (higher refractive index) to an optically less dense material (lower refractive index), there is **no phase change** when the light is reflected.

If the lens of a camera is pointed towards a very bright source, you can see repeated reflections of the iris diaphragm.

Consider a ray that strikes the air to magnesium fluoride coating at an angle of θ_1 (*Figure 64*).

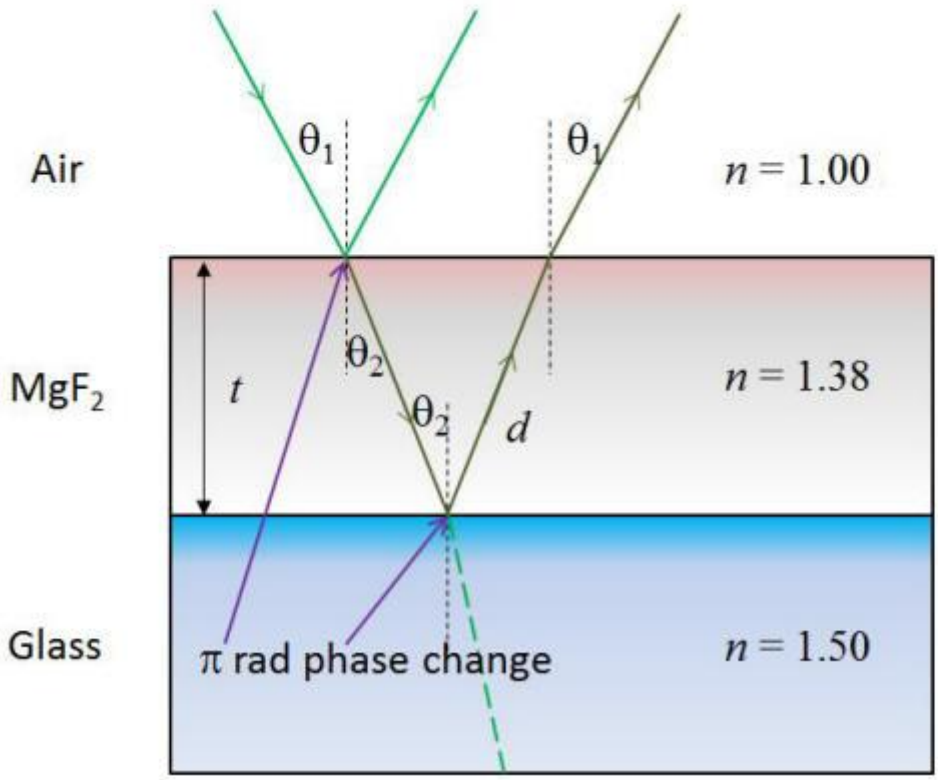


Figure 64 Coating on a lens

As before most of the ray is refracted to an angle θ_2 and passes into the magnesium fluoride coating. The weak reflected ray undergoes a phase change of π radians.

When the refracted ray reaches the magnesium fluoride-glass boundary, most is transmitted into the glass. We are not interested in this transmitted ray. We are interested in the weak reflected ray. There is a **second phase change of π radians**.

Therefore, the two reflected rays are **in phase**.

In the diagram, we showed the incident ray coming in at an angle. If we were to take into account the angle, we would need to bring in the relationship we saw with the oil film:

$$d = \frac{t}{\cos\theta_2}$$ *........... Equation 85*

However, for simplicity, we are going to make the incident ray strike the air-magnesium fluoride boundary **normally**, i.e. with an incident angle of zero. The transmitted ray is not deviated. Therefore:

$$d = t$$ *.......... Equation 86*

The optical path difference is the geometrical path difference multiplied by the refractive index of the magnesium fluoride coating (n_{coating}):

$$\Gamma = 2dn_{\text{coating}}$$ *........... Equation 87*

We want destructive interference, so:

$$\left(m + \frac{1}{2}\right)\lambda = 2dn_{\text{coating}}$$ *............ Equation 88*

Therefore:

$$d = \frac{\left(m + \frac{1}{2}\right)\lambda}{2n_{\text{coating}}}$$ *............ Equation 89*

Remember that m is a whole number, 0, 1, 2...

If we want the thinnest coating, we need to have the thickness as half a wavelength, i.e. when $m = 0$. Therefore, we can write:

$$d = \frac{\left(0 + \frac{1}{2}\right)\lambda}{2n_{\text{coating}}}$$

.... *Equation 90*

And we can rewrite this as:

$$d = \frac{\lambda}{4n_{\text{coating}}}$$

.... *Equation 91*

You may be asked to derive this equation in the exam.

The thickness of the coating is designed to stop green light (the middle of the visible spectrum) from being reflected. Blue and red light is reflected as their wavelengths are different. Therefore, a coated lens appears to be magenta (the mixture between blue and red).

If the ray comes in at an angle, we need to take that into account as we did above:

$$d = \frac{t}{\cos\theta_2}$$

.... *Equation 92*

So we write:

$$\frac{t}{\cos\theta_2} = \frac{\lambda}{4n_{\text{coating}}}$$

.... *Equation 93*

15.077 Wedge Fringes

If you put a couple of clean pieces of thin glass together, you may see interference patterns that look like this (*Figure 65*):

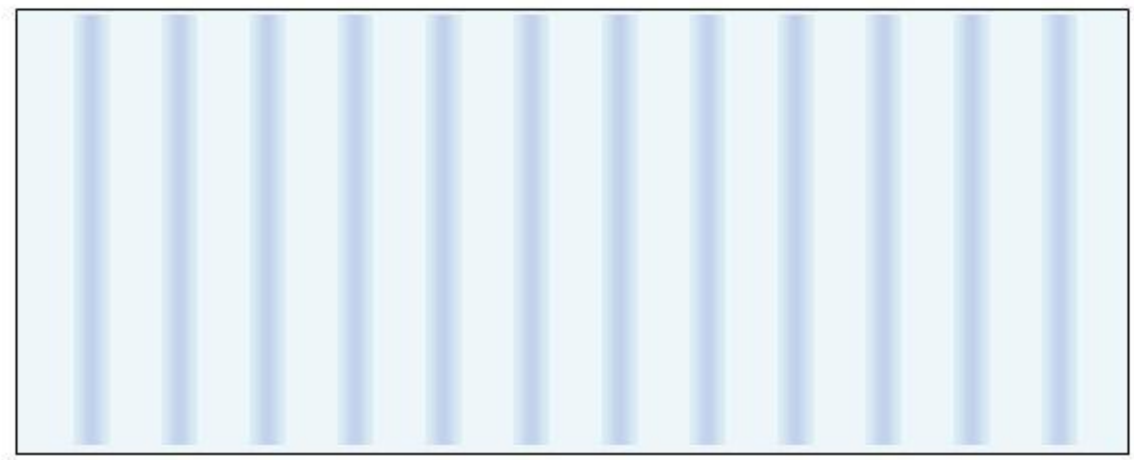


Figure 65 Interference pattern in two thin glass slides

If you press on the pattern, you can see it change. In a school lab, you will most likely see this with microscope slides or cover slips. Be careful, though. Microscope slides and coverslips can very easily break and leave nasty shards of glass that can puncture your fingers.

The reason for this is that the surfaces of the slides are not perfectly smooth, and there may be small bits of dirt. Therefore, there is an air gap, leading to interference patterns. We will look at this in more detail. In the diagram below (*Figure 66*), we can see two thin pieces of glass with a wedge of air between the two, as there is thin piece of paper between them at one end. Not shown are the paper clips that hold the slides in place. Monochromatic light is shone onto the slides.

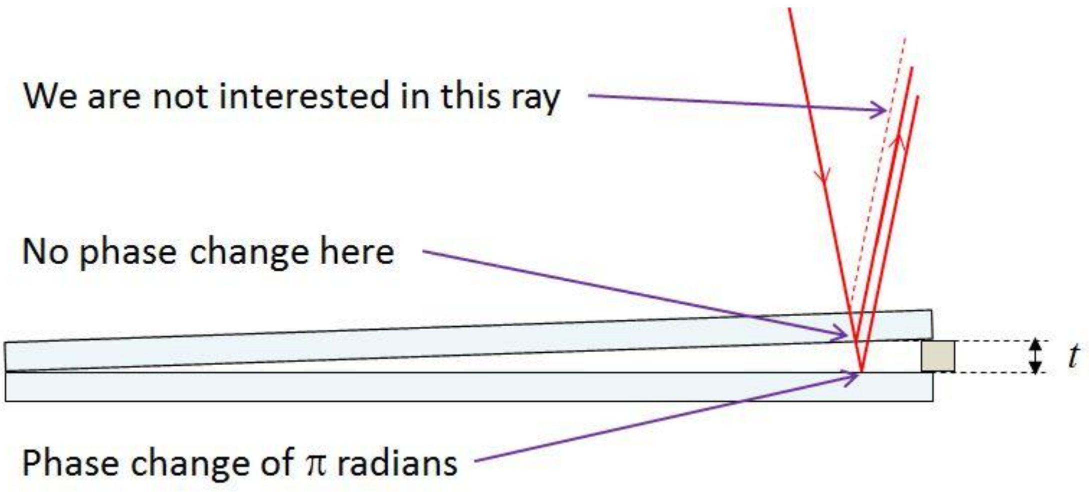


Figure 66 Light passing through thin glass slides

The angles and thickness of the piece of paper are exaggerated to show what is happening. (The angle in the diagram is 2 degrees while in reality, the angle is a small fraction of a degree.)

Notice also that we ignore the weak reflected ray, which does not take part in the interference pattern.

We know that as light passes from glass to air, the reflected ray is subject to zero phase change. This is because it's at a boundary from a high refractive index to a lower refractive index. When the refracted ray strikes the boundary between the air and the glass of the bottom slide, it undergoes a phase change of π radians. This is because the glass has a higher refractive index than the air gap.

The optical path difference is the air gap between the two slides. It is also the geometric path difference. So, we can write:

$$\Gamma = 2t + \frac{\lambda}{2}$$

.... *Equation 94*

For **constructive interference** to occur:

$$2t + \frac{\lambda}{2} = m\lambda$$

.... *Equation 95*

Rearranging:

$$2t = m\left(\lambda - \frac{\lambda}{2}\right)$$

.... *Equation 96*

Therefore:

$$t = \frac{m\left(\lambda - \frac{\lambda}{2}\right)}{2}$$

.... *Equation 97*

For **destructive interference**:

$$2t + \frac{\lambda}{2} = \left(m + \frac{1}{2}\right)\lambda$$

.... *Equation 98*

Rearranging and tidying up, we get:

$$t = \frac{m\lambda}{2}$$

.... *Equation 99*

Remember that m is any whole number. In this equation, we can include $m = 0$.

15.078 Measuring Fringes

Two glass slides are l m long and are touching at the left end and are separated by y m at the right-hand end. Consider a ray of monochromatic light striking the top slide x m from the left hand side. The separation of the slides is t m. This is shown in the diagram below (*Figure 67*):

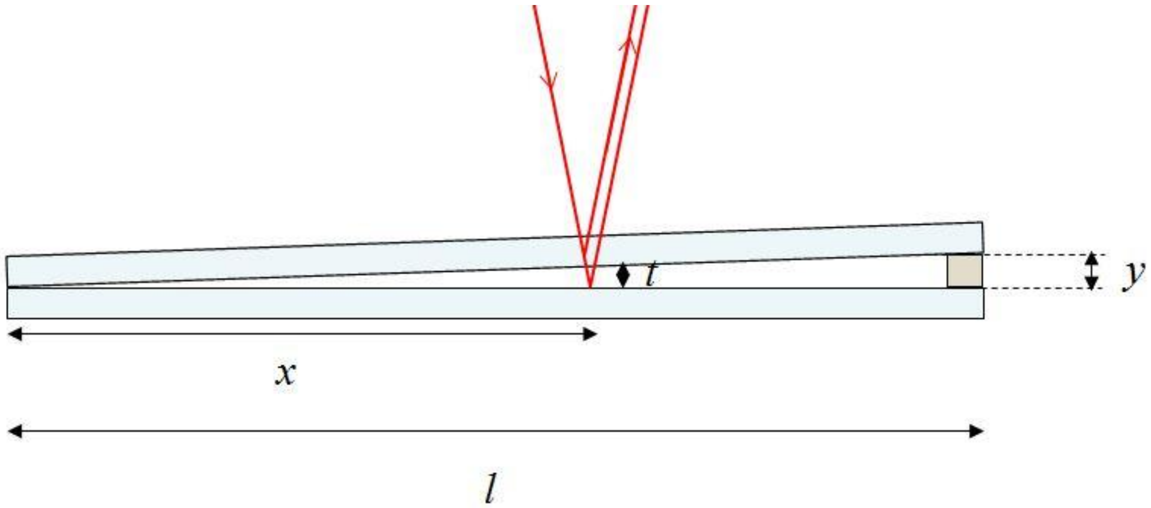


Figure 67 Measuring Fringes

There are m dark fringes in the distance x. We can see that there are two **similar triangles**. One has a base x and height t, while the other has a base l and height y. Therefore:

$$\frac{t}{y} = \frac{x}{l}$$

............*Equation 100*

Rearranging:

$$x = \frac{tl}{y}$$

.... *Equation 101*

Since:

$$t = \frac{m\lambda}{2}$$ .... *Equation 102*

We can substitute to give:

$$x = \frac{m\lambda l}{2y}$$ .... *Equation 103*

We measure the distance between the m^{th} and the $(m+1)^{\text{th}}$ dark fringe to give us a distance Δx. This needs to be done with a **travelling microscope** as the fringe spacing is in the order of about 1 mm. From this we can write:

$$\Delta x = \frac{(m+1)\lambda l}{2y} - \frac{m\lambda l}{2y}$$ .... *Equation 104*

Which gives us:

$$\Delta x = \frac{\lambda l}{2y}$$ .... *Equation 105*

Worked Example Two thin pieces of glass are 10.0 cm long. At one end, a piece of thin paper separates the ends of the glass slide. Monochromatic light of wavelength 612 nm is shone onto the glass. At a certain point, the separation between two adjacent fringes is found to be 1.51 mm. Work out the thickness of the paper.
Answer Use: $$y = \frac{\lambda l}{2\Delta x}$$ y = (612 × 10^{-9} m × 0.100 m) ÷ (2 × 1.51 × 10^{-3} m) y = **20.3 × 10^{-6} m** = 20 μm

The fringe separation decreases as the wavelength of the light decreases.

15.079 Interference by Division of Wavefront

In the process of interference by division of amplitude, a single wave was split and recombined. In interference by division of wavefront, we are combining two separate waves. The processes described in Topic 7 Tutorial 7 and Tutorial 8 arise due to the **division of wavefronts**.

For any interference effect to be observed, remember that the waves have to be **coherent** by:

- having the same wavelength.
- having a constant phase relationship.

The easiest way to achieve this is with a laser.

To study interference effects, we need to have a **point source**. A laser is close to a point source, which makes things much easier. Other light sources are **extended** sources, and getting coherence from the whole source is impossible. This is because photon production is random, and photons consist of short trains of waves.

To explain this, we will think about an example from sound waves. Consider the unpleasant experience of being close to an earth-strike in a thunderstorm. The lightning bolt is an extended source, with an irregular shape. The initial noise is an intensely loud high-pitched bang. All up the length of the stroke, the same would be heard. However, as the sound waves propagate away from the source, the higher frequency sounds are absorbed, leaving only the low frequency sounds, which we hear as a rumble. As sounds reach you from higher up the stroke, you hear them as a rumble of varying intensity, as the lightning bolt has (or had) an irregular shape.

To get coherence from an extended light source, we can screen off the extended source and allow the light to escape though a narrow hole, which makes the source act as a point source (*Figure 68*).

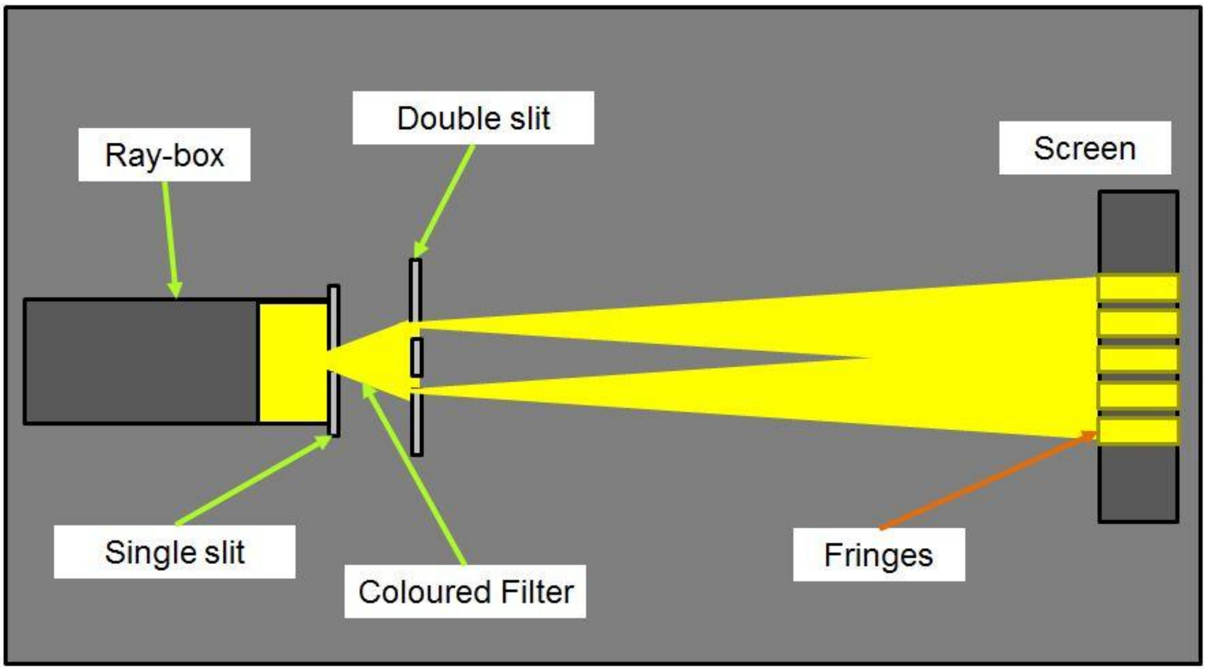


Figure 68 Young' double slit with a ray box

15.0710 Young's Double Slits

The basic treatment of this is to be found in Topic 7 Tutorial 7. The experiment can be carried out using apparatus as in the diagram above, but it is very difficult to see anything convincing. It is much more easily carried out using a laser (*Figure 69*).

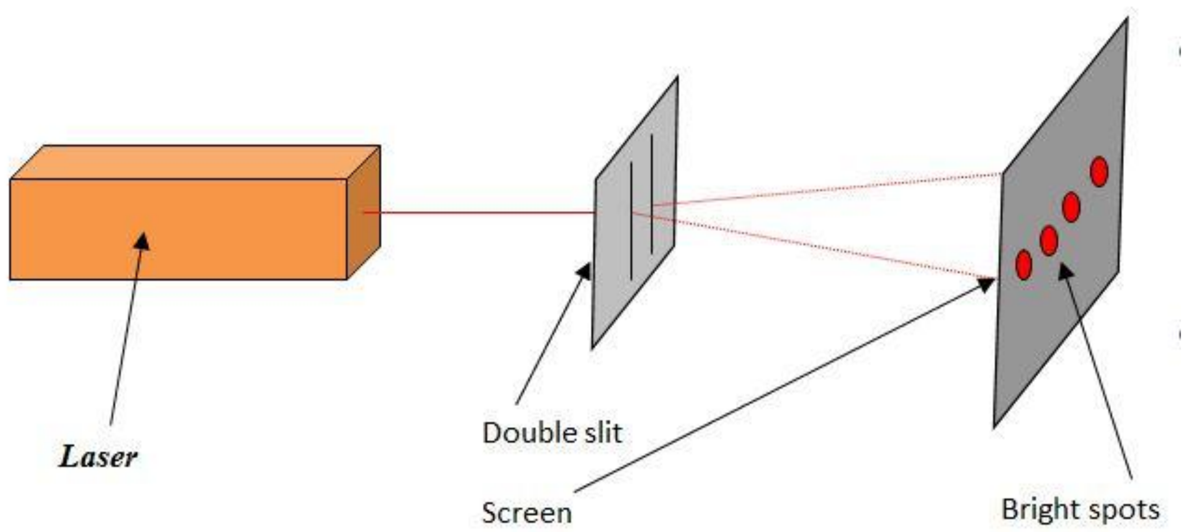


Figure 69 Young's double slit with a laser

On the screen we see the pattern in the diagram, along with the intensity of the bright spots (*Figure 70*):

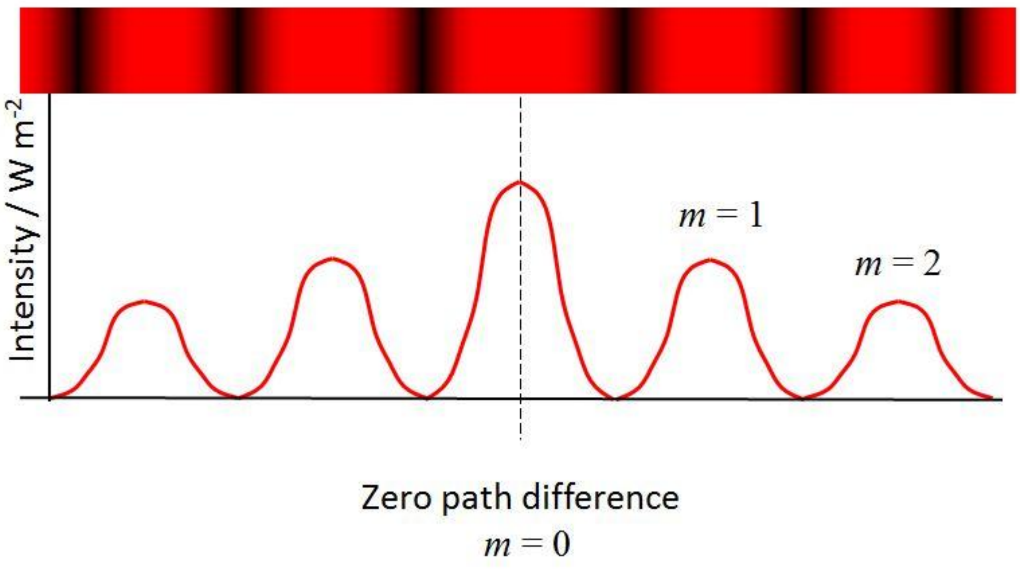


Figure 70 Pattern observed with a laser

The dark fringes occur when the path difference is an odd number of half wave lengths. The bright fringes happen when there is an even number of half wavelengths, or a whole number of wavelengths. The central bright spot occurs when the two waves have zero path difference. The first bright fringe occurs when the path difference is 1 whole wavelength, and so on. The pattern is symmetrical.

In this tutorial, we use the physics code m for number of wavelengths, rather than n. We will rerun the derivation from Topic 7, using the Physics codes used in this tutorial:

- **Wavelength** is the same (λ).
- **Distance** to the screen is the same (D).
- **Slit separation** is d here (instead of s).
- **Fringe separation** is Δx (instead of w).
- **Number of wavelengths** or **orders** is m (instead of n).

Consider a ray of monochromatic and coherent light of wavelength λ m falling onto a double slit. The spacing between the centres of the slits is d m. The distance from the slit to the screen is D m, where $D >> d$. The fringe separation is Δx m. This is shown below (*Figure 71*):

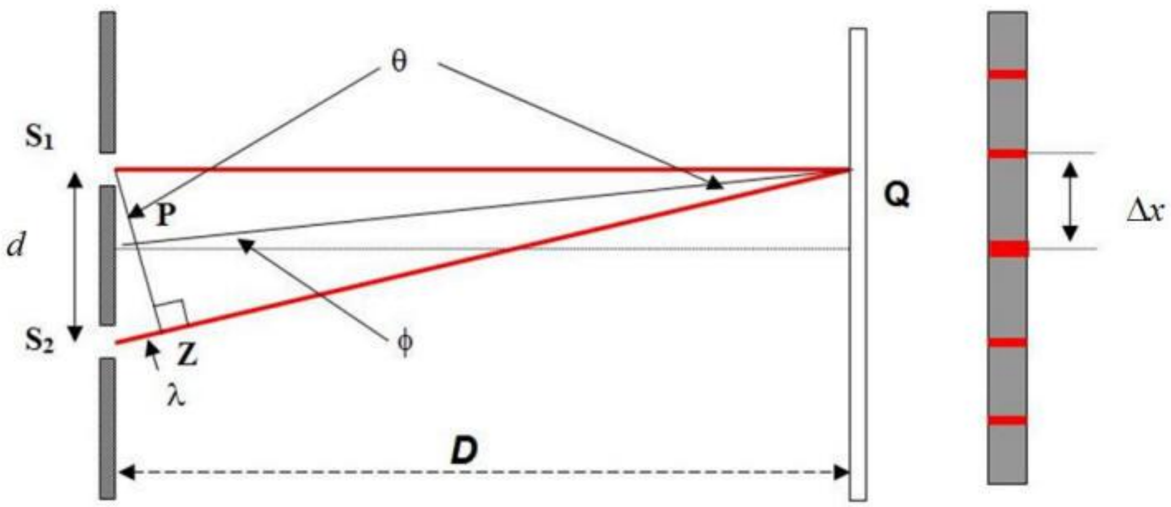


Figure 71Derivation of interference equation

For the **central** bright fringe, the path difference is 0.

$$S_2O - S_1O = 0 \quad \text{............. Equation 106}$$

For the **first** bright fringe

$$S_2Q - S_1Q = \lambda \quad \text{........... Equation 107}$$

Notice that in the triangle **S_1S_2Z**, **S_2Z** is λ.

We know that **S_1S_2** is d.

Therefore:

$$\sin\theta = \lambda/d \quad \text{........... Equation 108}$$

For the triangle **OPQ**,

$$\tan\phi = \Delta x/D \quad \text{............. Equation 109}$$

Although in this diagram, it is clear that $\theta \neq \phi$, in the real thing, we can assume that $\theta = \phi$, as the real set up is very much longer.

We know that for small angles in radians $\sin\theta = \tan\theta$.

Therefore:

$$\frac{\lambda}{d} = \frac{\Delta x}{D} \quad \text{............ Equation 110}$$

Rearranging:

$$\lambda = \frac{\Delta x d}{D}$$

.... *Equation 111*

Worked Example A laser is shone onto a double slit of which the slit separation is 0.050 mm. The distance from the screen to the slits is 4.5 m. If the wavelength is 615 nm, what is the separation of the fringes $m = 0$ and $m = 1$?
Answer Formula: $\Delta x = \frac{\lambda D}{d}$ Δx = (615 × 10^{-9} m × 4.5 m) ÷ 0.050 × 10^{-3} m = 0.055 m = 5.5 cm

Questions

Tutorial 15.07

15.07.1

What would happen to the resultant if the path difference was not quite an odd number of half wavelengths?

15.07.2

Look at the worked example on Page 89. What would be the path difference for the example above, if the detector reads zero?

15.07.3

Laser light of wavelength 620 nm is shone onto a screen as shown. A thin glass slide 0.1 mm thick is placed into the bottom ray. The glass has a refractive index of 1.51.

(a) Calculate the optical path difference.

(b) Discuss how the pattern on the screen might change, if at all.

15.07.4

Refer to *Figure 60*. The incident ray strikes the oil surface at an angle of incidence of 31 degrees.

(a) Show that the angle of refraction into the oil is about 20 degrees.

(b) Calculate the angle at which the ray is refracted into the water. Give your answer to an appropriate number of significant figures.

15.07.5

Refer to *Figure 60*. What would the energy of Ray 3 in units? What is the implication of this?

15.07.6

A thin layer of oil of refractive index 1.51 is illuminated with red light of wavelength 600 nm, which is normally incident on the surface. The surface appears dark. The path difference has been calculated as 10 wavelengths. What is the thickness of the film?

15/07.7

Refer to *Figure 64*. Why is there a second phase change of π radians?

15.07.8

Green light of wavelength 530 nm is normally incident on a lens coated with magnesium fluoride of refractive index 1.38. What is the thickness of magnesium fluoride that need to be applied to a camera lens to prevent the reflection of the green light from the lens?

15.07.9

Look at *Figure 66*. Why are the geometric path difference and the optical path difference both the same?

15.07.10

Consider the statement on Page 104: Remember that m is any whole number. In this equation, we can include $m = 0$.

State whether there is a light or dark fringe where the two slides touch. Explain your answer in terms of thickness and path difference.

15.07.11

In carrying out some measurements to study wedge fringes, a student is too hippy lazy to use the travelling microscope. Instead, he measures the length x and counts the number of dark fringes. He works out the Δx term by dividing x by the number of fringes. Explain whether or not this is wrong.

15.07.12

Two microscope slides are 8.0 cm long and touch on one end. At the other end, there is a piece of paper 0.030 mm thick. The set-up is exposed to monochromatic light of wavelength 633 nm. Calculate the fringe spacing.

15.07.13

A Young's slits experiment is set up with a slit separation of 0.400 mm. The fringes are viewed on a screen placed 1.00 m from the slits. The separation between the $m = 0$ and $m = 10$ bright fringes is 1.40 cm. What is the wavelength of the monochromatic light used? Give your answer to an appropriate number of significant figures.

Tutorial 15.08 Polarisation of Light

SQA Syllabus

Contents

This is quite a long and challenging tutorial. Work through it slowly and carefully. If you don't understand it, ask for help from your tutor.

Before you attempt this tutorial, you should review the section Topic 7 Tutorial 2 that covers polarisation. The important thing to remember is that only **transverse** waves can be polarised. Longitudinal waves cannot be polarised as they oscillate in the direction of propagation. We will consider polarisation of **electromagnetic waves**. The polarisation of mechanical waves can be used as a simple model for the polarised properties of electromagnetic waves, which consist of an electric component and a magnetic component in phase.

15.081 Polarising Filters

These are often called **polaroids** after the company that developed them. These are made of long-chain polymers that are oriented in a particular direction like this (*Figure 72*):

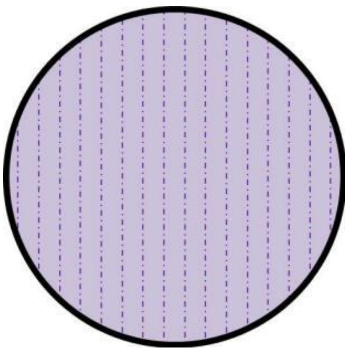


Figure 72 A polarising filter

Light from a light bulb is unpolarised, meaning that the photons have random orientations. The molecules transmit the component of the light parallel to their alignment. Therefore, if the filter is vertically aligned, the light is **vertically polarised**. More specifically, it's the **electric field** component that is vertically polarised. If the electric field component is transmitted, so also is the perpendicular **magnetic field** component (*Figure 73*).

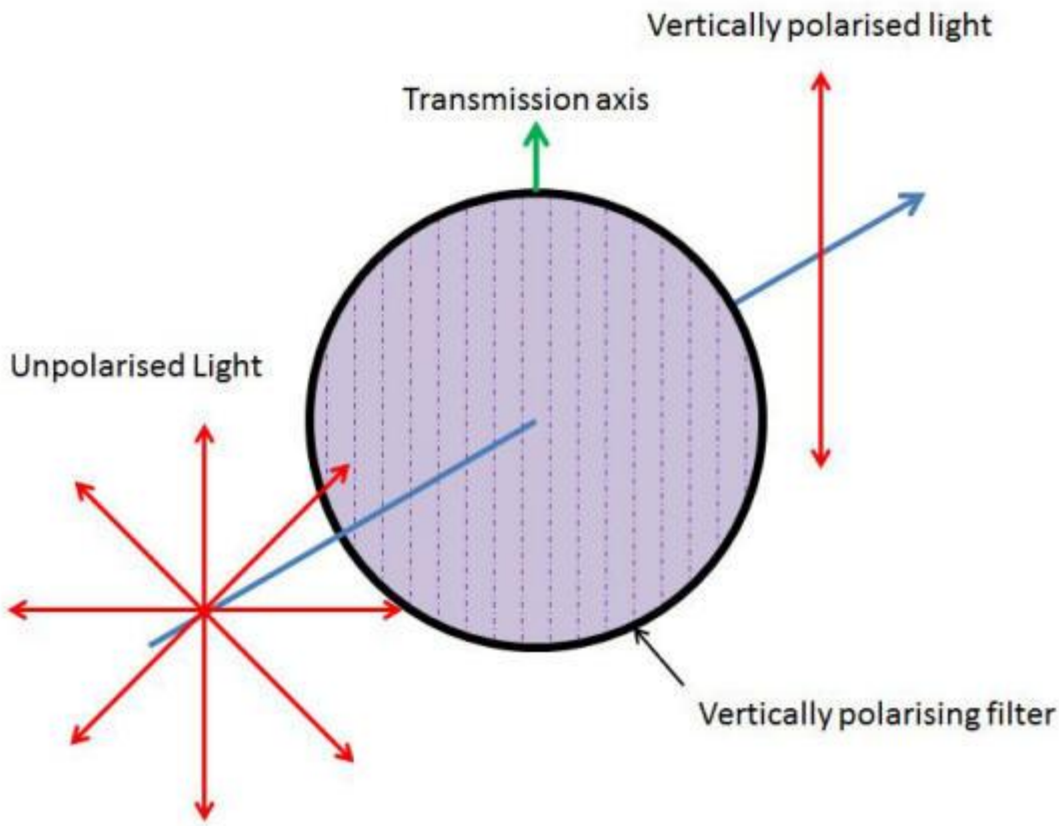


Figure 73 Polarisation of light

The orientation of the filter is sometimes called the **transmission axis**. If the orientation of light is at 90° to the transmission axis, no transmission occurs. If, however, the light is oriented at an angle to the transmission axis of a vertical polarising filter, the vertical component of the light is transmitted. The idea is shown below (*Figure 74*):

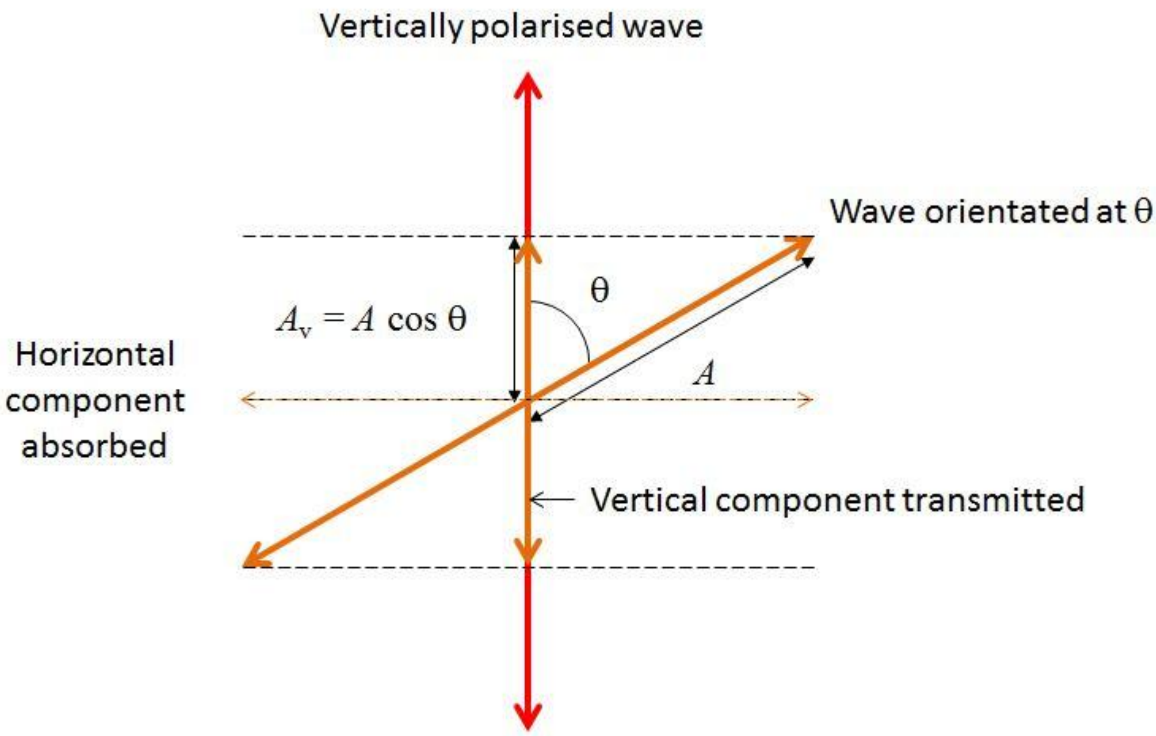


Figure 74 Polarisation at an angle

The wave at angle θ has the same amplitude, *A*, as the vertically polarised wave. Since the electric field component is a vector quantity, we can separate it into vertical and horizontal components:

$$A_v = A \cos \theta$$ .... *Equation 112*

and

$$A_h = A \sin \theta$$ .... *Equation 113*

Since the horizontal component is absorbed, we can ignore it.

A vertically polarised wave refers to the electric field component. The magnetic field component is horizontal, but it is transmitted. The polariser does NOT split the electric and magnetic components from each other, otherwise the whole physical nature of the electromagnetic wave would be altered.

The **amplitude** represents the **intensity** of the light, I. Intensity is the energy per unit area. As the production of photons is a random process, and their orientation is random, we can say that, on average, 50 % of the light intensity is transmitted through a vertically oriented polarising filter. 50 % of the intensity is absorbed.

Of course, the same argument can be applied to a horizontally oriented filter.

15.082 Two Polaroid Filters

We can put a second filter into the light path as shown below (*Figure 75*):

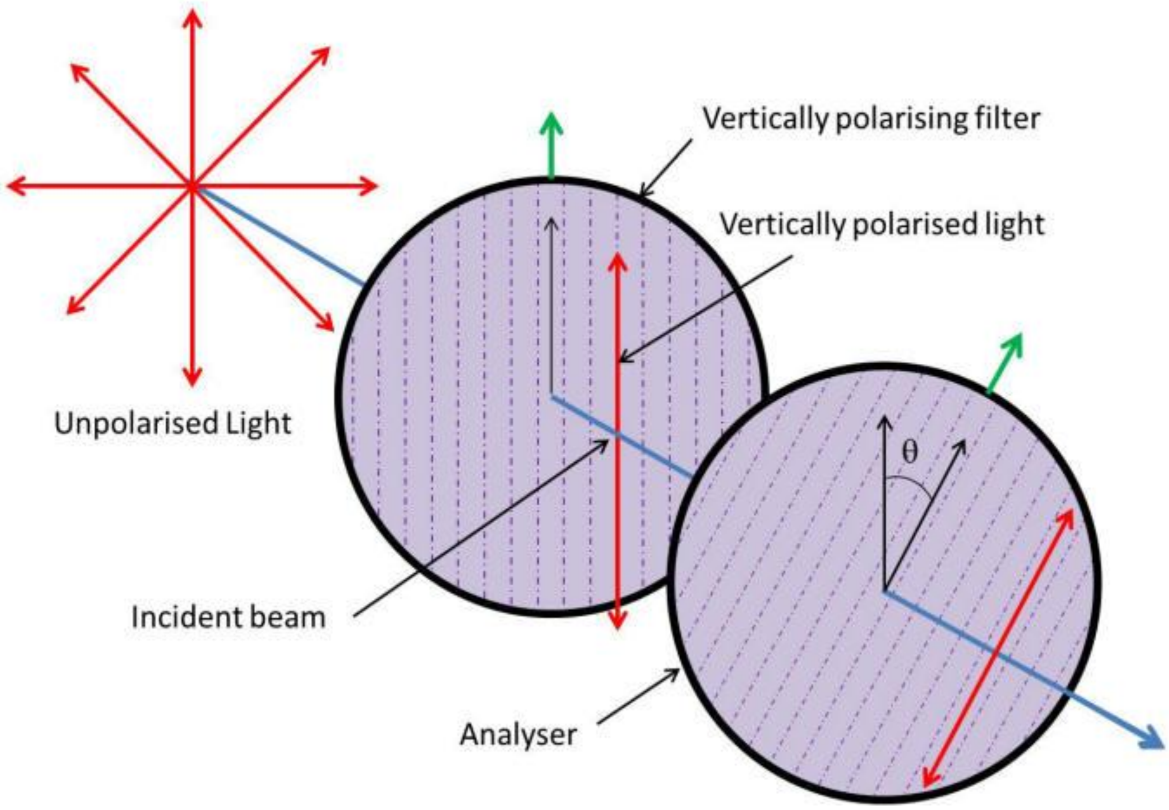


Figure 75 Adding a second polariser

The second filter is called the **analyser**. The analyser is turned at an angle of θ to the **incident beam** of vertically polarised light. Let's look straight on at the beam that has been twisted through the angle θ (*Figure 76*):

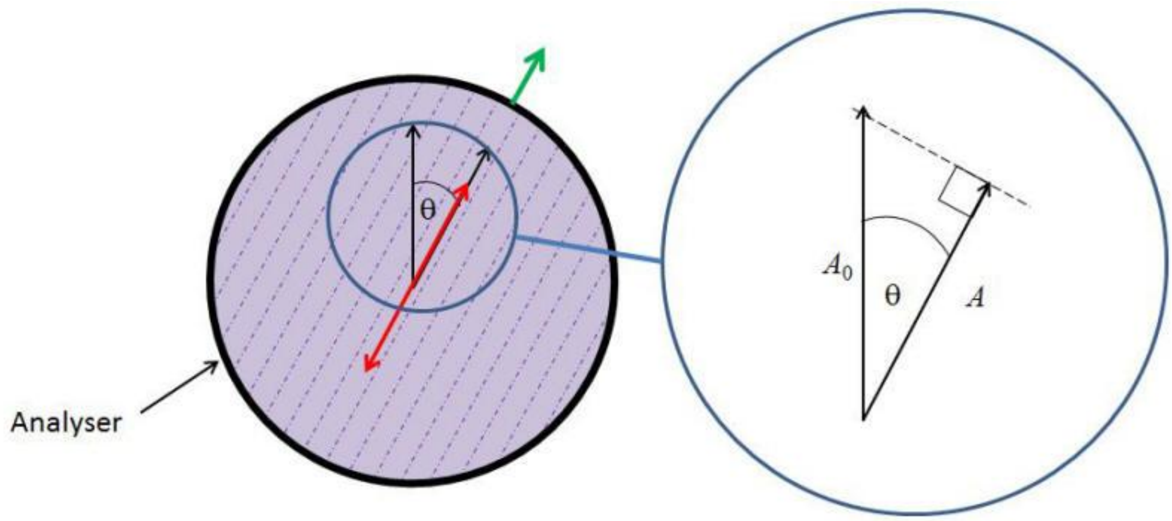


Figure 76 Moving the analyser by a certain angle

Suppose the amplitude of the incident beam is A_0. From the diagram we can see that the amplitude component parallel to the transmission axis is A, and:

$$A = A_0 \cos \theta$$ .... *Equation 114*

In Topic 7 Tutorial 1, we saw the relationship between energy and amplitude:

$$E \propto A^2$$

.... *Equation 115*

We developed the theme in Topic 7 Tutorial 4 to give the relationship:

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

.... *Equation 116*

15.083 Malus' Law

The point of this is to say that if **energy** varies as the **square of the amplitude**, the **power** must vary as the square of the amplitude. Hence the **intensity** (or **irradiance**), defined as **power per unit area**, will vary as the square of the amplitude. We give the intensity the physics code I, and the units are **Watts per square metre** (W m^{-2}).

So, we can write an expression that gives us the intensity:

$$A = A_0 \cos \theta$$ .... *Equation 117*

If we square the relationship we get:

$$A^2 = (A_0 \cos \theta)^2$$ .... *Equation 118*

In turn this becomes:

$$A^2 = A_0{}^2 \cos^2 \theta$$ .... *Equation 119*

So, we can rewrite this for the **intensity**:

$$I = I_0 \cos^2 \theta$$ .... *Equation 120*

This is called **Malus' Law**.

15.084 Experiment to Show Malus' Law

This experiment shows the change in intensity with angle. The apparatus is set up like this (*Figure 77*):

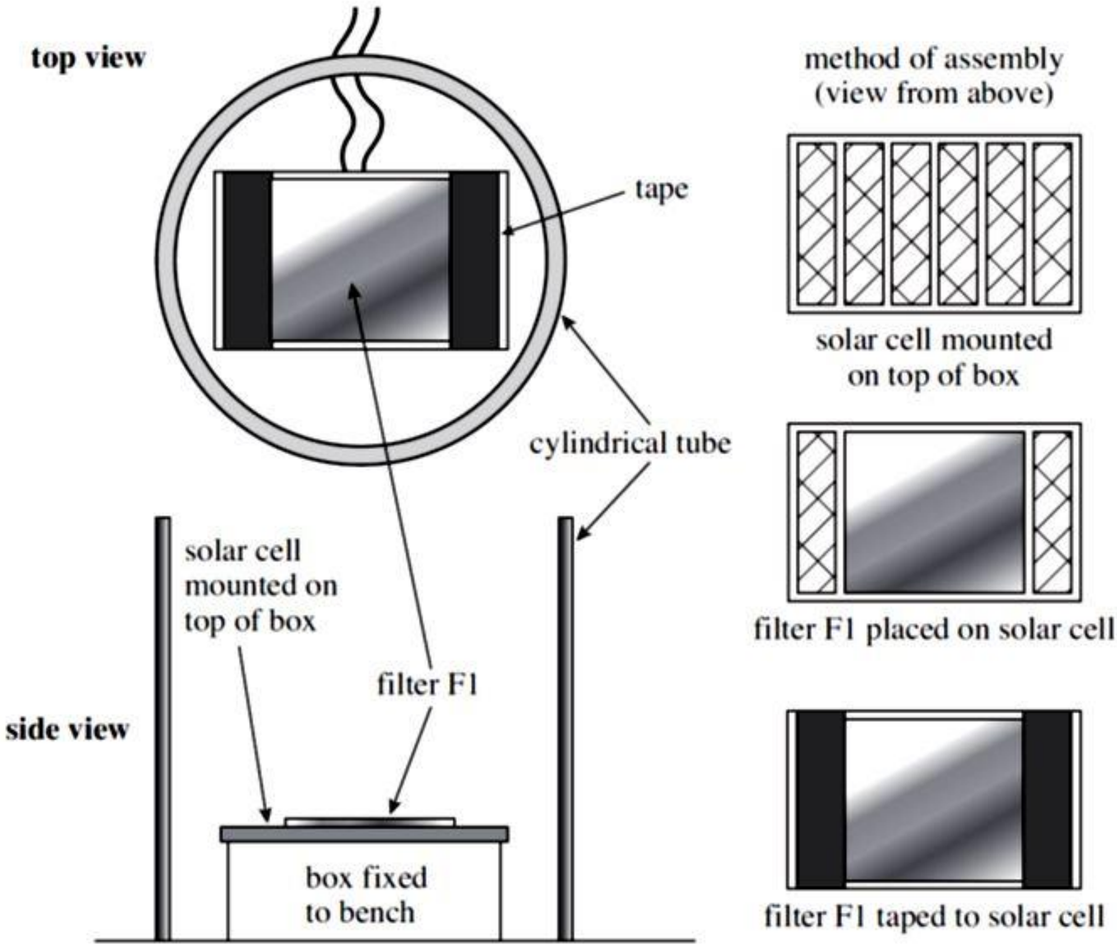


Figure 77 Experiment on Malus' Law (Source: AQA Specimen practical paper)

The light intensity is measured using a solar cell. The intensity is represented by the output voltage.

In this experiment the intensity of the light is reflected in the output of the solar cell. As you change the angle of the second polaroid (the **analyser**), the output of the solar cell changes. You take readings every 20 degrees.

Data modelling shows what the graph should look like, if Malus' Law is followed (*Figure 77*):

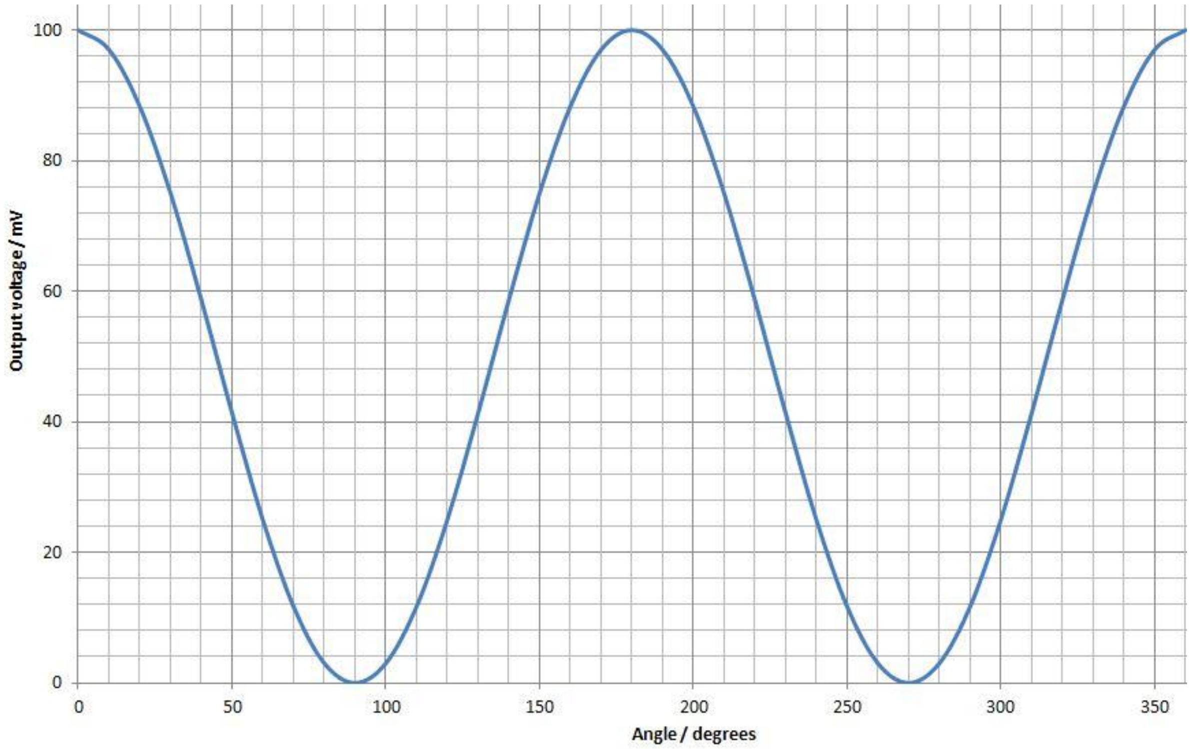


Figure 78 Data modelling of Malus' Law

It is quite challenging to get decent data from this experiment. At least two repeat readings should be made and an average taken. Also, the model above assumes that the crossed polaroid filters block out all the light. In reality they do not.

15.085 Uses for Polarisation

Polarising filters have many applications. The most obvious one is the polaroid sunglasses that cut down glare off wet surfaces. Many drivers use them.

This is a polarising filter for a camera (*Figure 79*):

Figure 79 A polarising filter for a camera

The polarising filter reduces glare that is reflected off shiny surfaces. This can be seen on these two pictures (*Figures 80* and *81*):

Figure 80 Note the reflections from the leaves

Figure 81 No reflections from the leaves

Polarising filters also can be used to emphasise clouds (*Figures 82* and *83*):

Figure 82 No filter

Figure 83 Clouds emphasised

Notice how the clouds stand out more in *Figure 83*. The polarising filter is cutting out the horizontally polarised light. You can see how the right hand picture is also brighter, with less of a blue tint. Both pictures were taken at the same time with the same camera.

Geologists use polarising filters in their **petrographic microscopes** to study minerals. Here is a picture of a thin slice of a rock (*Figure 84*):

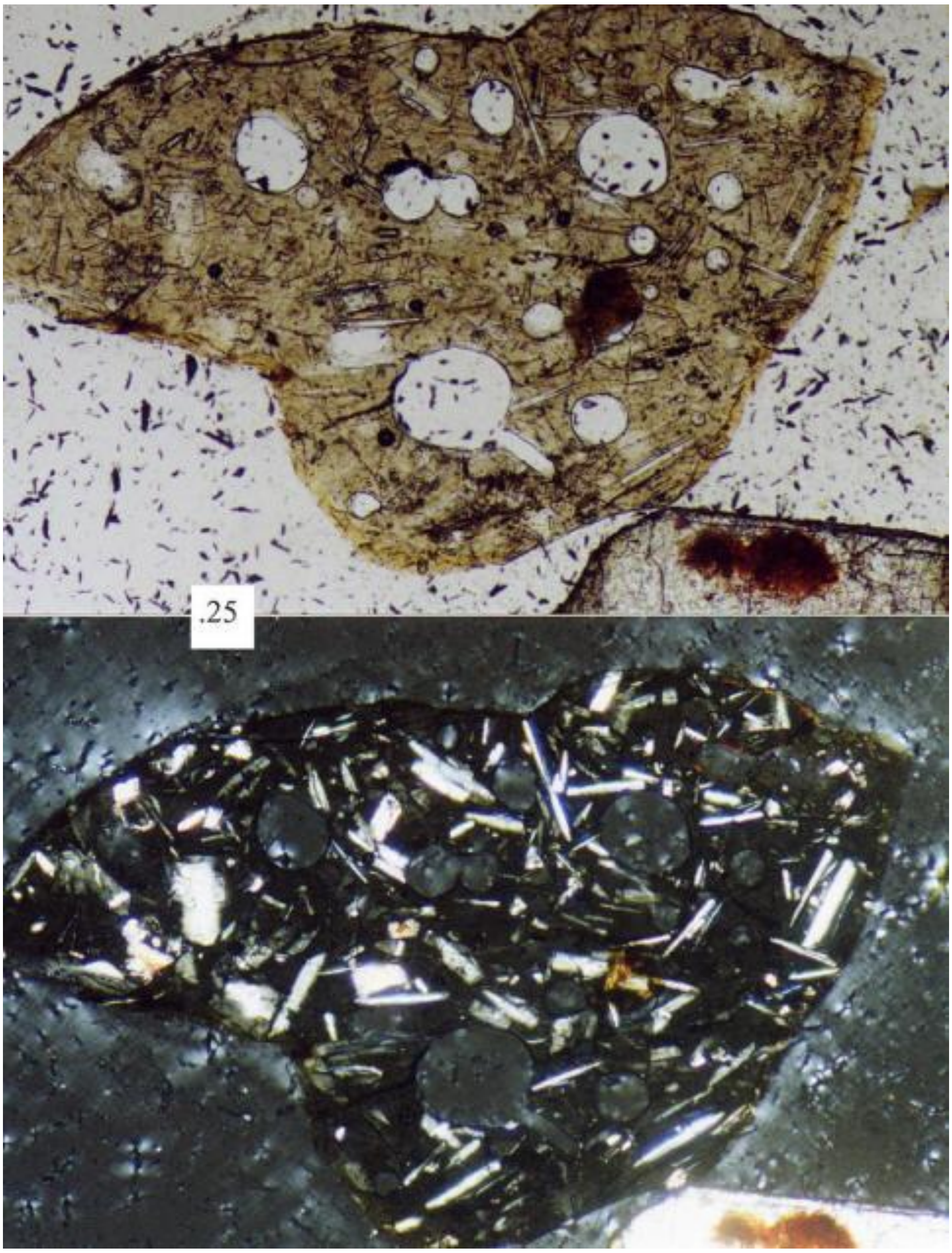


Figure 84 Polarising filter in a geological microscope (Image by Matt Affolter - Wikimedia Commons)

The top image is taken in plane polarised light, while the bottom image is taken in cross-polarised light. You can see how the individual crystals stand out more clearly in the bottom image.

15.086 Liquid Crystal Displays

This picture shows the liquid crystal display of the laptop on which I am preparing these notes. I am holding a photographic polarising filter to the screen so that the polarisers are crossed. You can see the result (*Figure 85*):

Figure 85 Holding a polarising filter to a computer screen

A light source is placed behind the screen. A polariser with a vertical transmission axis produces vertically polarised light. This is shown in the picture (*Figure 86*):

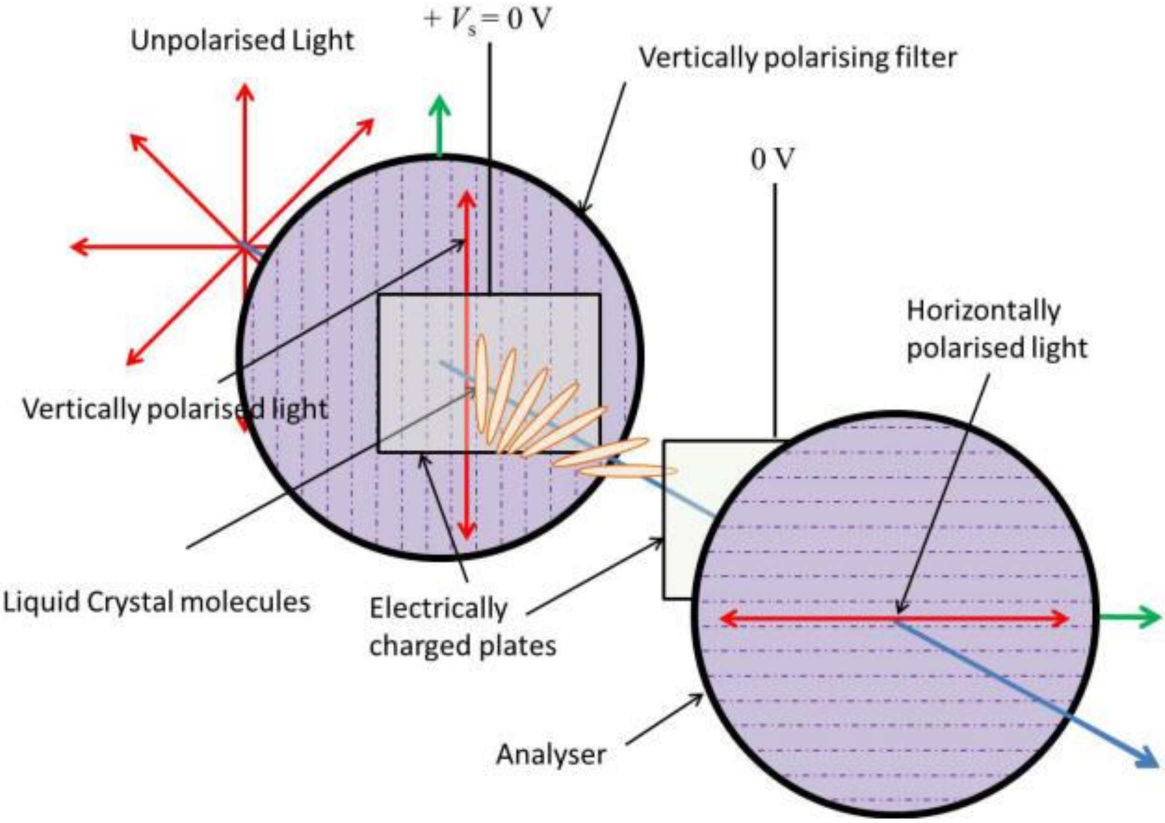


Figure 86 Working of a Liquid Crystal Display

When there is zero voltage across the plates, the liquid crystal molecules from into a spiral shape and twist the vertically polarised light to the horizontal. The horizontally polarised light can pass through the analyser which has a horizontal transmission axis.

When a voltage is applied across the plates, the liquid crystal molecules arrange themselves so that the vertically polarised light does not change its orientation.

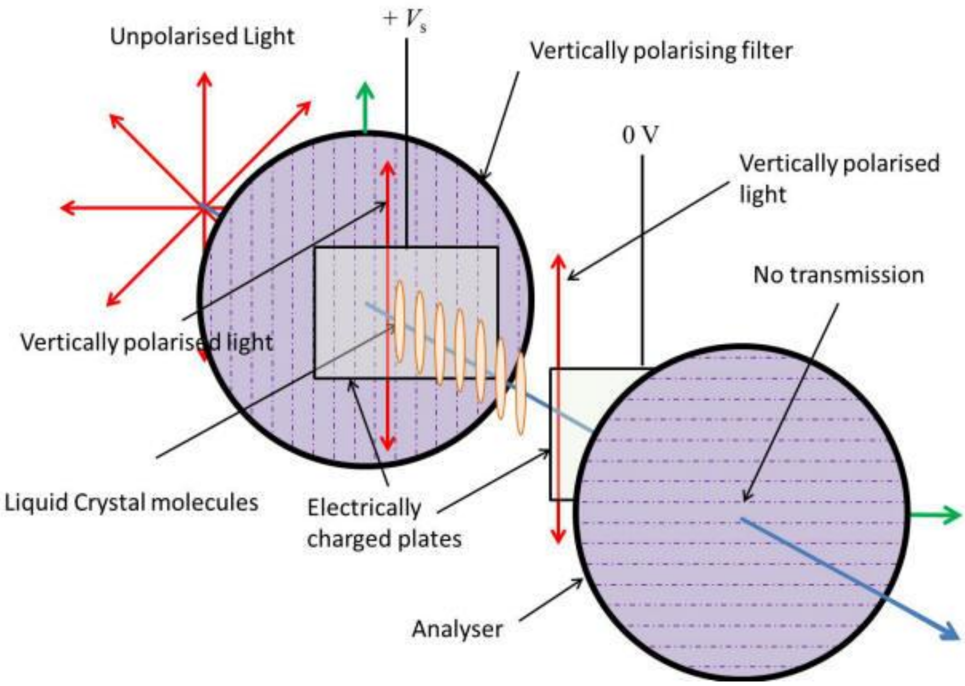


Figure 87 Dark LCD

Therefore, the vertically polarised light does not pass through the analyser. Therefore, the display is dark.

Figure 88 LCD on a calculator

LCD displays on computers are often referred to as **pixels** (picture elements). 1 pixel is about 0.26 mm. The pixels have red, green, and blue filters to make the different colours.

Some liquid crystal displays have the analyser with a vertical transmission axis.

Many solutions twist polarised light. Some molecules in chemistry have an **isomer** that twists the light **to the left**, while the other isomer twists the light **to the right**. The molecules have the same formula but are mirror images of each other. They are called **optical isomers**. Biological molecules are left-hand optical isomers. Right-hand isomers cannot be processed in many biological systems.

The extent and **sense** (whether the light is turned to the left or right) to which a material twists polarised light can be measured with a **polarimeter**.

15.087 Polarisation by Reflection

In the picture below (*Figure 89*), you can see leaves reflecting light.

Figure 89 Light reflected from the surface of a leaf

With a polarising filter, the reflection is much reduced. (Yes, they are the same.)

Figure 90 Reflection much reduced by a polarising filter

We can conclude that the reflected light is (horizontally) **polarised**. When light reflects of a **non-metallic** (or insulating) surface, it is partially polarised. At a certain angle, it is fully polarised. Metallic surfaces reflect rays in random orientations, so the reflected light is **unpolarised**. So, let's look what happens when unpolarised light strikes a non-metallic surface (*Figure 91*):

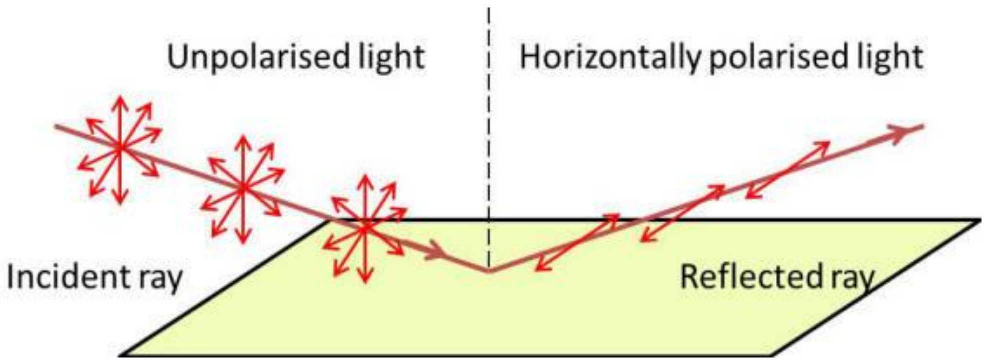


Figure 91 Light reflected at a certain angle from a non-metallic surface is polarised

The reflected ray is usually not completely polarised. However, there is a case where the light is completely polarised:

- When monochromatic light is used.
- The angle of incidence (and reflection) is a particular value.

15.088 Brewster Angle

This was worked out by a Scottish physicist, David Brewster (1781 - 1868). In the diagram above, we considered the reflected ray but ignored the transmitted ray. Let's look at both rays from the side (*Figure 92*).

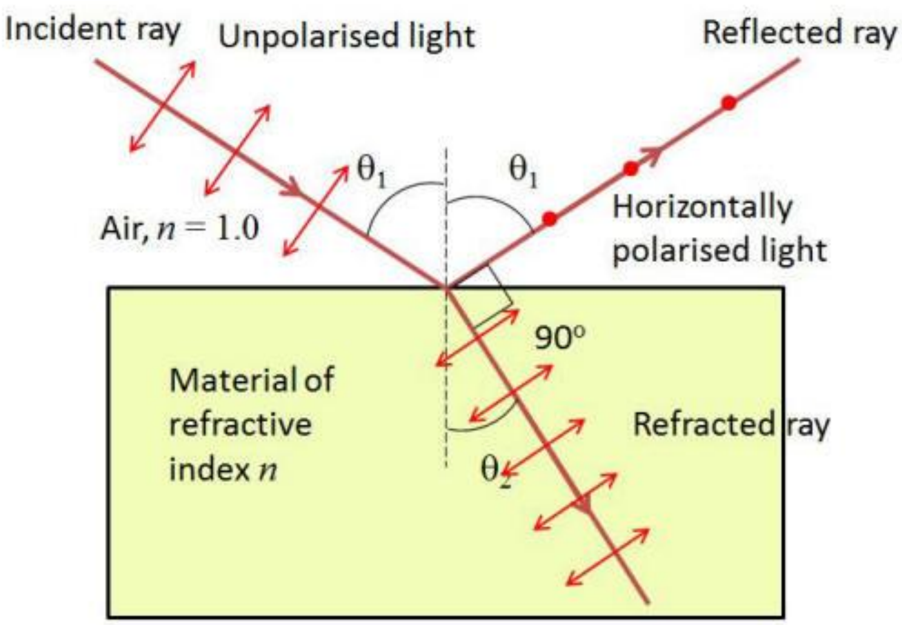


Figure 92 Reflected ray and refracted ray

The incident ray is **unpolarised**. The refracted ray is polarised **vertically**. The reflected ray at this angle is **fully polarised** in the **horizontal** orientation. The key point that is critical for this is that the **angle between the refracted ray and the reflected ray is 90°**. This is called **Brewster's Law**, or the **Brewster Angle**.

The angle of incidence is θ_1, and the angle of refraction is θ_2. The angle of reflection is, of course, θ_1. We can see that:

$$\theta_1 + 90^o + \theta_2 = 180^o$$ .... *Equation 121*

It doesn't take a genius to see that:

$$\theta_1 + \theta_2 = 90^o$$ .... *Equation 122*

So:

$$\theta_2 = 90^o - \theta_1$$ .... *Equation 123*

Since we have a refraction here, we can apply Snell's Law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$ .... *Equation 124*

Substituting for θ_2 from *Equation 123*:

$$n_1 \sin \theta_1 = n_2 \sin (90 - \theta_1)$$ .... *Equation 125*

Since:

$$\sin (90 - \theta_1) = \cos \theta_1$$ .... *Equation 126*

We can write:

$$n_1 \sin \theta_1 = n_2 \cos \theta_1$$ .... *Equation 127*

Rearranging:

$$\frac{\sin\theta_1}{\cos\theta_1} = \frac{n_2}{n_1}$$ .... *Equation 128*

Since sin ÷ cos = tan, we can write:

$$\tan\theta_1 = \frac{n_2}{n_1}$$ .... *Equation 129*

Since we normally observe the horizontal polarisation in air, the usual value for n_1 is 1.00. So, we can write the equation as:

$$\tan\theta_1 = n_2$$ .... *Equation 130*

This equation describes **Brewster's Law**, and the angle θ_1 is called the **Brewster Angle**.

Note that in the SQA syllabus, the angle θ_1 is given the code i_p (the **incident polarising** angle).

Worked example What is the Brewster angle for glass, of which the refractive index is 1.51, when observed in air?
Answer Formula: $$\tan\theta_1 = n_2$$ $\tan\theta = 1.51$ $\theta = \tan^{-1} 1.51 =$ **56.5°**

15.089 Birefringence

Some crystals have different refractive indices (plural of *index*), depending on how the light strikes the boundary of the crystal and its orientation. Calcite ($CaCO_3$) is one such example (*Figure 93*).

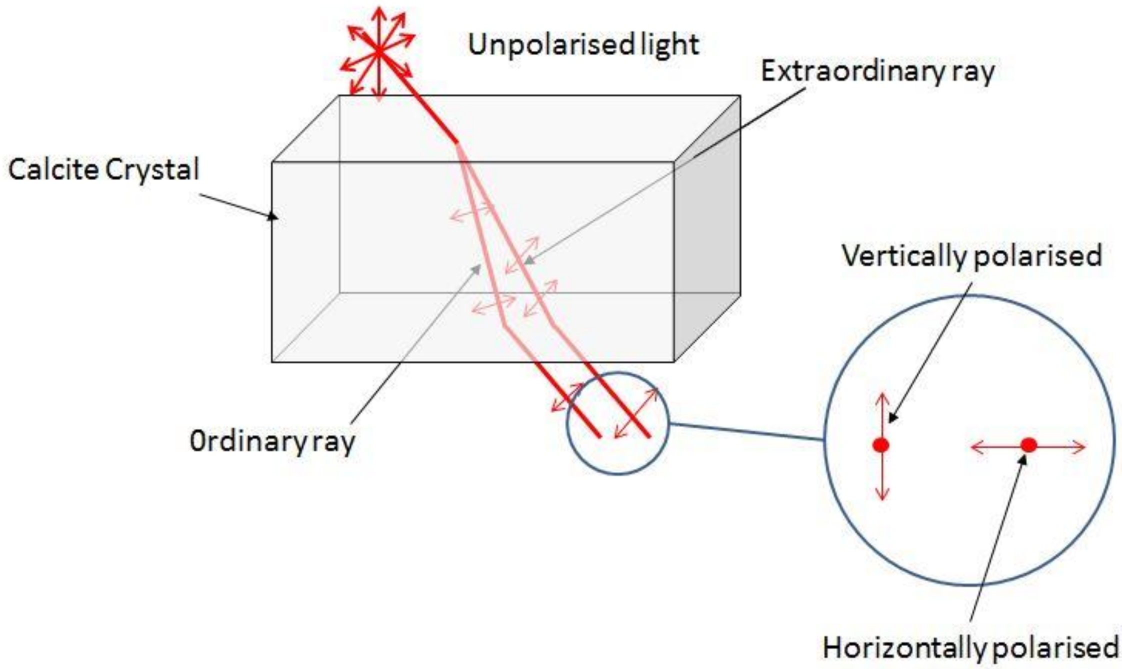


Figure 93 Birefringence

In the diagram, unpolarised light strikes the crystal, the crystal structure polarises the light into both vertical and horizontal direction. The vertically polarised ray refracts with a refractive index of 1.6854. This is called the **ordinary ray**. The ray that is polarised in the horizontal direction is called the **extraordinary ray** and has a refractive index of 1.4864.

If you observe the two rays with a **vertical polarising filter**, you will observe only the **vertically polarised ray** (the **ordinary** ray). Similarly, **a horizontal polarising filter** will reveal only the **horizontally polarised** ray (the **extraordinary** ray).

Other crystals show the same phenomenon, including quartz, water ice, sodium nitrate, and titanium dioxide. In some crystals, the extraordinary ray has a higher refractive index. Calcite has the most marked birefringence.

15.0810 Photoelasticity

The property of birefringence is seen in many transparent polymers such as Perspex. The mechanism is similar to the that described above. The study of the way such materials behave under polarised light is called photoelasticity. The material can alter the orientation of polarised light. When a sample of such a material is placed between crossed polaroids, the shape can be seen like this (*Figure 94*):

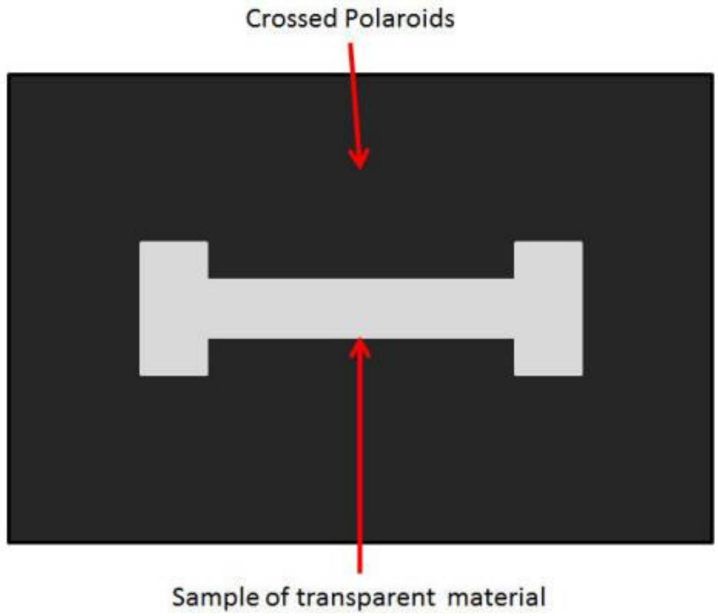


Figure 94 Sample of Perspex placed between crossed polaroids

When the object is stressed, the orientation of the polarised light is changed further by the **stress lines**, to give **interference patterns**. In monochromatic light this can be seen (in a very stylised way) as in *Figure 95*:

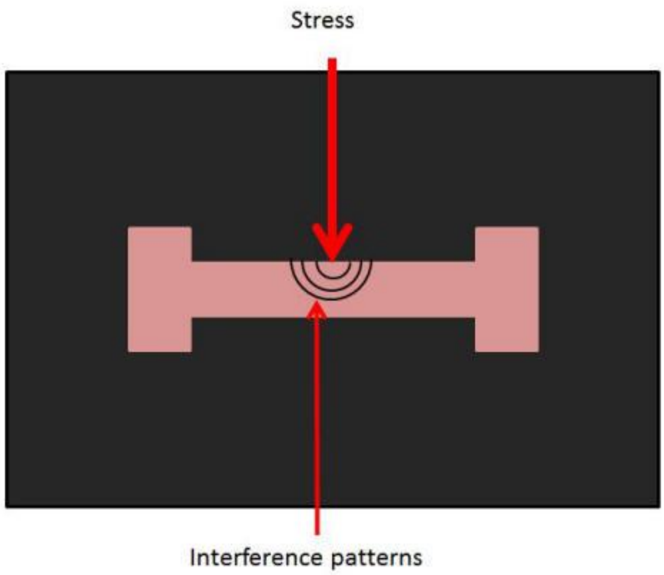


Figure 95Stress lines in a sample shown between crossed polaroids

If the light is white light, the interference patterns are highly coloured as white light is a mixture of all sorts of different wavelengths.

The stress patterns are used by engineers to predict how stress lines will act in real structures. This will help to avoid weak points on the structure, the results of which could be catastrophic.

Questions

Tutorial 15.08

15.08.1

A ray of light of intensity 3 W m^{-2} is oriented at 50° to the transmission axis of a polarising filter 10 cm × 10 cm.

(a) What is the intensity of the light is transmitted?

(b) Assuming that the absorbed light is converted to heat, what is the rate of heating of the filter? Give the correct unit.

15.08.2

The intensity of a vertically polarised ray of light is reduced to 50 % of its original value. Show that the analyser has to be turned through 45° to achieve this.

15.08.3

In an experiment like the one above on Page 122, the voltage given out by the sensor is 60 mV when the transmission axis of the analyser is lined up with the transmission axis of the first polariser. When the analyser is turned through a certain angle, the sensor voltage drops to 20 mV. What is the angle that has been turned?

15.08.4

Explain how the figures on this calculator (*Figure* 88) are visible.

15.08.5

Some liquid crystal displays have the analyser with a vertical transmission axis.

How would you know if the display was of this type?

15.08.6

The same block as in the example on Page 134 is now immersed in water of refractive index is 1.33. What is the Brewster angle now?

15.08.7

A ray of light strikes a crystal of calcite at an incident angle of 35° from the air. Calculate the angle of refraction of the ordinary ray and the extraordinary ray. The refractive indices for the ordinary and extraordinary rays are 1.6854 and 1.4864 respectively.

15.08.8

Why can the sample in *Figure 94* be seen?

4. Sports Science (Option C)

Tutorial 15.09 Coefficient of Restitution

Welsh Board and Eduqas Syllabus

Contents

These notes are for students studying Option C in the Welsh Board and EDUQAS syllabus.

15.091 Coefficient of Restitution

When you drop a ball onto the ground, it bounces back, but the height to which it bounces is ALWAYS less than the height it fell from. (If you throw the ball at the ground in a temper, it will bounce higher and you may be disqualified... We are not considering this here.) You may well do a simple experiment in which you measure the bounce height of a ball.

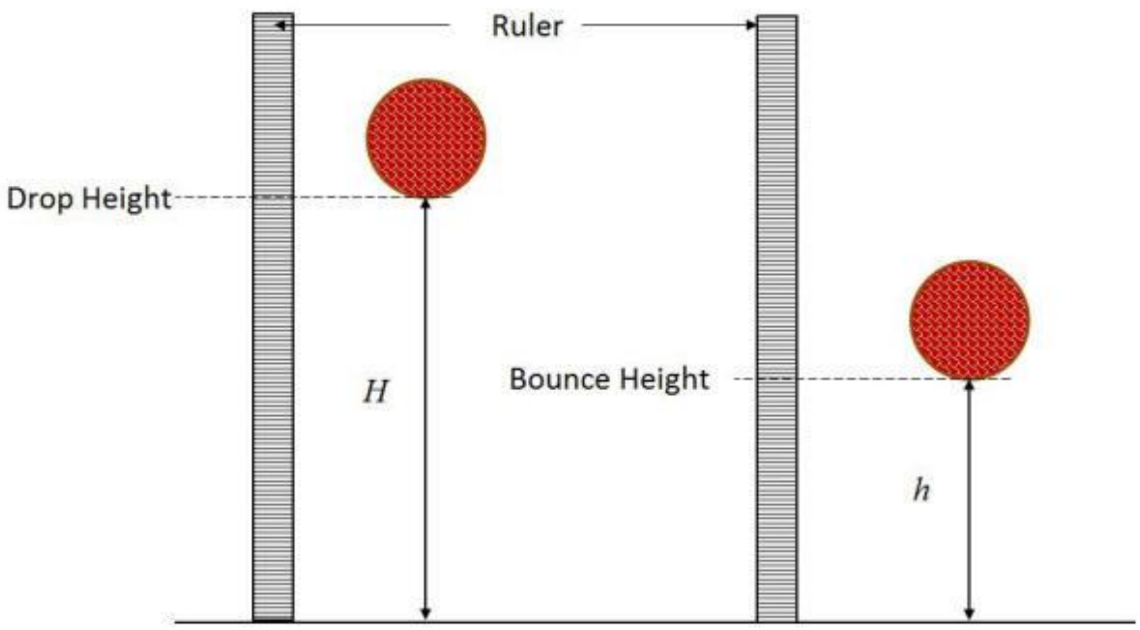


Figure 96 Measuring the bounce of a ball

If we measure the drop height, and the bounce height, we find that they are in a constant ratio, which we will look at later.

We measure the speed as the ball hits the floor and then as it rebounds. A good quality data-logger can be used for this. You need to place it at a point that is about the diameter of the ball above the floor.

We find that the speed just before the ball hits the floor and the speed after the ball bounces off the floor are related in this ratio:

$$\frac{\text{Speed of separation}}{\text{Speed of approach}} = \text{constant}$$

.... *Equation 131*

The **speed of separation** is the difference between the velocities of objects A and B (*Figure 97*):

Figure 97 Speed of separation

So, we can say that:

Speed of separation = velocity of A - velocity of B

In this case, the relative velocity is the same as the measured speed. The velocity of the floor is 0 before and 0 after. We are ignoring the signs of direction, and the movement of the floor is considered to be negligible.

The constant is called the **coefficient of restitution** and is given the Physics code e. It has no units.

$$e = \frac{v_{\text{after}}}{v_{\text{before}}}$$ .... *Equation 132*

The coefficient of restitution is less than 1. This is because energy is always lost as a ball hits a surface:

- Sound is given off (you can hear it bounce).
- Some is converted to heat.
- The ball is deformed as it bounces, and work is done in deforming the ball.
- When the ball bounces, elastic energy is converted back to kinetic. Some is lost due to hysteresis.

There are many factors that affect the coefficient of restitution. A cold squash ball has a low coefficient of restitution. It hardly bounces at all. When it is warmed up, the coefficient of restitution is much higher. You could investigate that by putting squash balls in water baths at different temperatures.

Some materials have a low coefficient of reinstitution. Others have a high value. Different surfaces can affect the coefficient of restitution. Balls bouncing on a carpet do not bounce as high as those bouncing from a hard floor.

A perfectly inelastic collision results in a coefficient of restitution of 0.

15.092 When 2 Balls Collide

Sports such as bowls involve one ball hitting another. Usually, one ball is stationary. In this next argument, we will think of two balls that strike each other in **one dimension**, i.e. the centres of mass are in line. We will ignore friction.

Consider two balls, A and B of masses m_1 and m_2 respectively. They come together and collide (*Figure 98*):

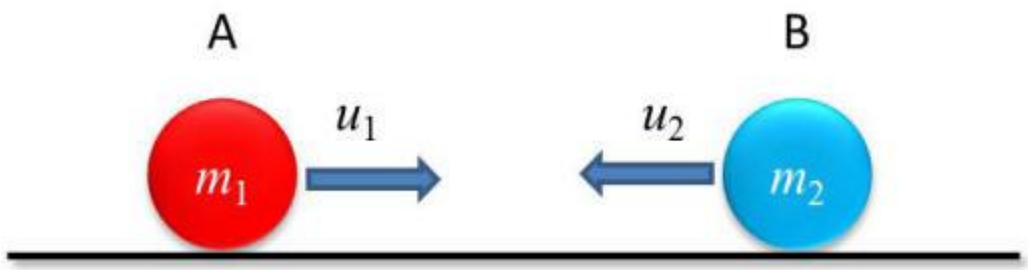


Figure 98 Two balls colliding

We can add up the momenta:

$$p_{\text{before}} = m_1 u_1 + m_2 u_2 \quad \text{.......... Equation 133}$$

After the collision we see:

Figure 99 After the collision

We again add up the momenta:

$$p_{\text{after}} = m_1 v_1 + m_2 v_2 \quad \text{........... Equation 134}$$

We know that momentum is conserved, so we can write:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$ .... *Equation 135*

When we work on momentum calculations like this, we must use the **velocities**, as direction is critical. Left to right is considered positive.

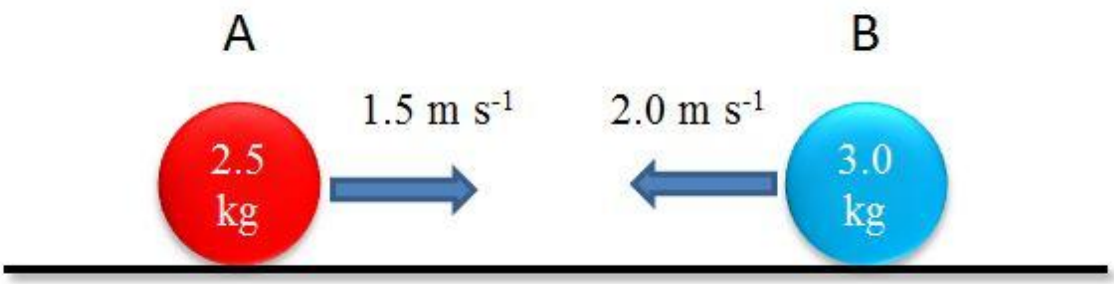
The coefficient of restitution for the two balls is given by the expression:

$$e = \frac{v_A - v_B}{u_A - u_B}$$ .... *Equation 136*

When using the coefficient of restitution equation, we must use the **speed** (magnitude of velocity) without the direction.

Worked example

Ball A has a mass of 2.5 kg, and its initial velocity is 1.5 m s^{-1} to the right. Ball B has a mass of 3.0 kg, and its initial velocity is 2.0 m s^{-1} to the left. They collide head on.



If the coefficient of restitution is 0.75, what are the final velocities of balls A and B?

Answer

Momentum before = (2.5 kg × +1.5 m s^{-1}) + (3.0 kg × -2.0 m s^{-1})
= 3.75 kg m s^{-1} + -6.0 kg m s^{-1} = -2.25 kg m s^{-1}

Momentum after = (2.5 kg × v_1 m s^{-1}) + (3.0 kg × v_2 m s^{-1}) = -2.25 kg m s^{-1}

$2.5\,v_1 + 3.0\,v_2 = -2.25$ kg m s^{-1}

Now we use the coefficient of restitution:

$$0.75 = (v_2 - v_1) \div (2.0 - 1.5)$$

$$v_2 - v_1 = 0.75 \times (2.0 - 1.5) = 0.375$$

Only use the <u>speeds</u> (i.e. the magnitudes of the velocities) for the coefficient of restitution. Do not use the signs, or it will mess the calculation up.

Watch the signs; it is very easy to get a wrong sign somewhere.

So, we have two simultaneous equations:

$$v_2 - v_1 = 0.375$$
$$2.5\,v_1 + 3.0\,v_2 = -2.25$$

Substitute for v_2:

$$v_2 = 0.375 + v_1$$

$$2.5\,v_1 + 3.0\,(0.375 + v_1) = -2.25$$
$$2.5\,v_1 + 3.0\,v_1 = -2.25 - 1.125$$
$$5.5\,v_1 = -3.375$$

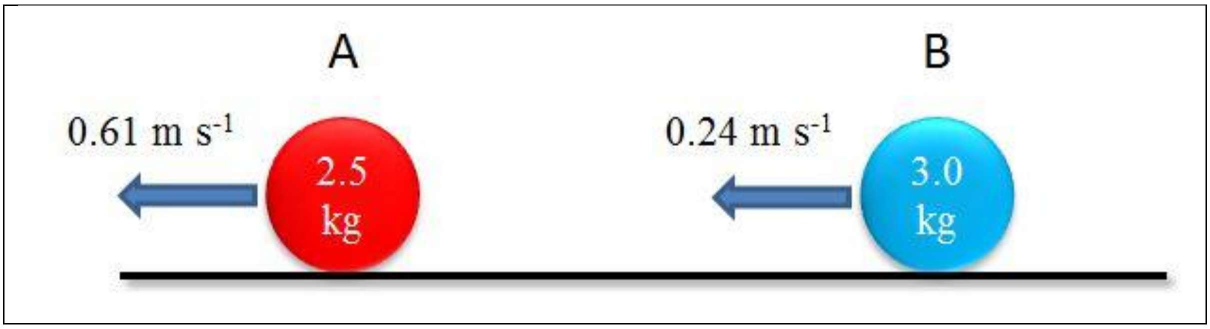
v_1 = **<u>-0.614 m s^{-1}</u>**

Now substitute for v_1:

$$2.5 \times -0.614 + 3.0\,v_2 = -2.25$$
$$3.0\,v_2 = -2.25 + 1.535 = -0.715$$

v_2 = -**<u>0.238 m s^{-1}</u>**

Ball A travels at 0.61 m s^{-1} to the left, while ball B travels at 0.24 m s-1 to the left, like this:



If the balls stick together after the collision, the coefficient of restitution is 0.

In the example above, kinetic energy has been lost.

15.093 Bounce of a Ball

Consider a ball being dropped from a height H. It bounces off the floor, back to a height h.

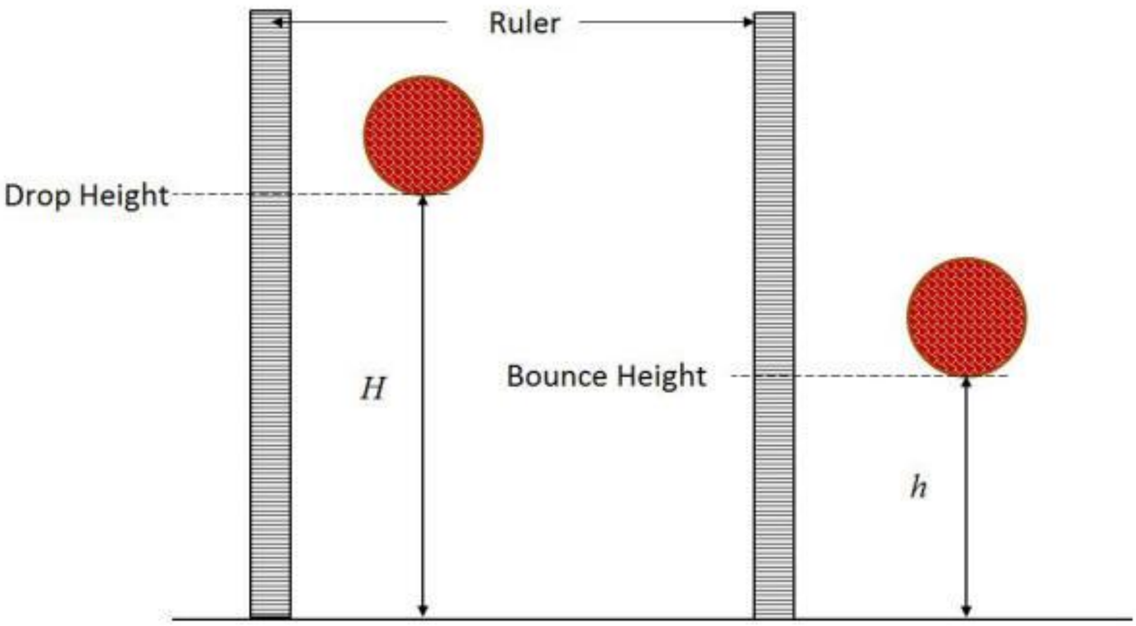


Figure 100 Bounce of a ball

When the ball is released, it will accelerate towards the ground at an acceleration g (= 9.81 m s^{-2}). When the ball hits the ground, there is a change in momentum (impulse) leading to a force that accelerates the ball upwards (Newton II). Its final velocity is 0, when the height is h.

We know that a suitable equation of motion would be:

$$v^2 = u^2 + 2as$$ .... *Equation 137*

We can adapt that for our situation. We will use downwards as negative. The acceleration is $-g$ and the displacement downwards is $-H$. We will call the speed before v_{drop}. So, we can write:

$$(v_{\text{drop}})^2 = 0 + 2 \times -g \times -H$$ .... *Equation 138*

Therefore, the two minus signs cancel out:

$$(v_{\text{drop}})^2 = 2gH$$ .... *Equation 139*

This is important because you cannot get a square of a negative that is a real number.

When the ball bounces back, the final speed is zero, and the height reached is h. The displacement of h is upwards, so it's positive. We will call the speed of the bounce v_{bounce}. So, we write:

$$0^2 = (v_{\text{bounce}})^2 + 2 \times -g \times h$$ .... *Equation 140*

Therefore:

$$(v_{\text{bounce}})^2 = 2gh$$ .... *Equation 141*

The minus signs helpfully cancel out.

We know that the **coefficient of restitution**, e, for a bouncing ball is:

$$e = \frac{v_{\text{after}}}{v_{\text{before}}}$$ *........... Equation 142*

Therefore:

$$e^2 = \frac{{v_{\text{bounce}}}^2}{{v_{\text{drop}}}^2}$$ *.......... Equation 143*

And we can write:

$$e^2 = \frac{2gh}{2gH}$$ *........... Equation 144*

And it doesn't take a genius to see that:

$$e^2 = \frac{h}{H}$$ *............. Equation 145*

Therefore, we can write a final expression for coefficient of restitution:

$$e = \sqrt{\left(\frac{h}{H}\right)}$$ *........... Equation 146*

A lively ball has a high coefficient of restitution. A football bounces well on a hard surface, and reasonably well on a grass surface. A cricket ball on grass is as lively as a lump of lead.

Questions

Tutorial 15.09

15.09.1

When measuring the bounce of a ball, you need to place it at a point that is about the diameter of the ball above the floor. Why do you think the ball should be placed at this point?

15.09.2

The coefficient of restitution of a ball is 0.675. What is the speed of the ball as it rebounds having hit the floor at 5.50 m s^{-1}?

15.09.3

If the balls stick together after the collision, the coefficient of restitution is 0. Explain why the statement above is true. What kind of collision is this?

15.09.4

Using the values for masses and speeds in the example above on Page 145, calculate the loss of kinetic energy. What proportion of the kinetic energy remains as kinetic energy?

15.09.5

A ball has a coefficient of restitution of 0.75. It is dropped from a height of 1.0 m.

(a) Show that the speed of the ball is about 4.4 m s^{-1} as it is about to hit the floor.

(b) Work out the speed of the bounce as the ball just leaves the floor. Give your answer to an appropriate number of significant figures.

(c) Work out the height of the bounce.

Tutorial 15.10 The Bernoulli Effect

Welsh Board, Eduqas and IB Syllabus

Contents

This tutorial is for students of the Welsh Board and Eduqas who are doing Physics in Sport Option.

The last section on Laminar and Turbulent Flow discusses the Reynolds Number which is in the IB Option B only

This is quite a long tutorial.

15.101 Aerofoils

In 1732, the Swiss physicist and mathematician, Daniel Bernoulli (1700 - 1783), noticed that an increase in speed of a fluid resulted in a decrease in pressure. The equation we will look at is called the **Bernoulli Equation** but was actually the work of Leonhard Euler (1707 - 1783). At the time, it was a physical observation. Now we know that it is important in the way that the wing of an aeroplane lifts the machine off the ground and enables it to fly. This is summed up in the picture below (*Figure 101*):

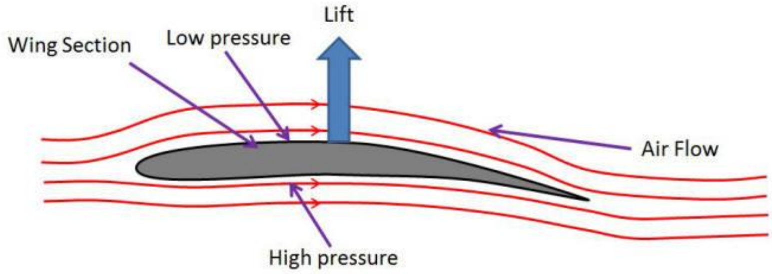


Figure 101 Aerofoil

The wing section is often called an **aerofoil**. The air is split and travels **faster** around the top of the wing than the bottom. There is a region of **high pressure** underneath the wing as a result of the slower moving air. There is a region of **low pressure** above the wing due to the faster passage of the air. The **pressure difference** (multiplied by the area) results in an upwards force called **lift**. If the lift is greater than the weight, the aeroplane will take off. The effect can be increased by lowering **flaps**, and the pilot lowers the flaps to take off at a lower speed. Flaps a lowered more as the aeroplane comes in to land.

The downside for this is that the aerofoil causes **drag**. Some aeroplanes have a laminar flow aerofoil that is symmetrical. This will support the aeroplane with little drag but needs a high speed for it to work. Flaps are particularly important for flying at low speed, taking off and landing. If the flaps fail, a flapless landing can be decidedly scary!

In motor sport, racing cars have an **inverted** aerofoil that works in the opposite sense to the wing on an aeroplane. As the car goes faster, the aerofoil puts an extra pressure on the rear tyres to increase grip as the car travels at speeds of between 50 m s^{-1} and 100 m s^{-1} (*Figure 102*).

Figure 102 Inverted aerofoil on a racing car (Image by Brian Snelson - Wikimedia Commons)

You can see the aerofoils on the front and back of this (rather dated) racing car. They should not be confused with the "go-faster" bits of plastic that some manufacturers add to their sporty models. They were popular in the 1980s and 1990s, as shown in this picture (*Figure 103*):

Figure 103 Go-faster bits of plastic on a sporty car (Image by Rupert Kent - Wikimedia Commons)

At road legal speeds (30 m s^{-1}) a **spoiler** like this is almost ineffective, impairs the view from the rear-view mirror, causes more drag, and could present a serious hazard to a cyclist or biker who runs into the back of the car. (OK, they shouldn't, but they do.)

15.102 The Continuity Equation (IB students only)

Consider an ideal fluid passing through a pipe. It has two cross-sections like this (*Figure 104*):

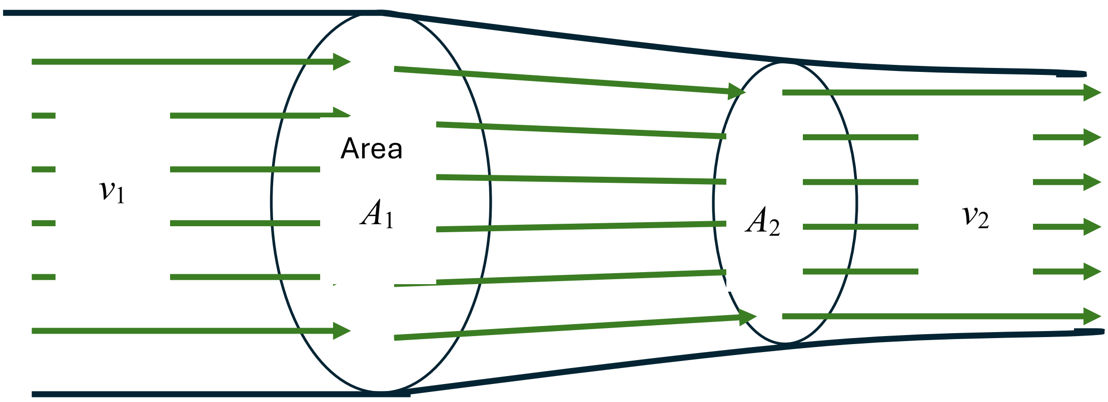


Figure 104 Ideal fluid flowing through a pipe narrowing cross section

Consider a length of the pipe which is l long and A in cross sectional area. The first thing we can say is that it has volume V.

$$V = Al$$

,,,,,,,,,,, *Equation 147*

It is filled with an ideal fluid which we will consider as a liquid, which is **incompressible**. A pipe full of static fluid is not very useful. We want it to flow at a certain rate. To get this rate, we divide both sides by time:

$$\frac{V}{t} = \frac{Al}{t}$$

.... *Equation 148*

The liquid has density ρ. We know that:

$$m = \rho V$$

...........*Equation 149*

From *Equations 148* and *149* we can write an expression for the mass per unit time:

$$\frac{m}{t} = \frac{\rho V}{t} = \frac{\rho A l}{t}$$

.... *Equation 150*

Since the term l/t gives us velocity v, we can write:

$$\frac{m}{t} = \rho A v$$

.... *Equation 151*

Let us refer back to *Figure 104*. The fluid velocity through area A_1 is v_1, while the fluid velocity through area A_2 is v_2. So, we can put these into *Equation 151*.

$$\frac{m}{t} = \rho A_1 v_1$$

.... Equation 152

Similarly:

$$\frac{m}{t} = \rho A_2 v_2$$

.... Equation 153

The same amount of fluid must come out as goes in. Since the m/t term is the same, we can combine *Equations 152* and *153* to give:

$$\rho A_1 v_1 = \rho A_2 v_2$$

.... Equation 154

The density of the fluid cannot change either, so the ρ terms cancel out:

$$A_1 v_1 = A_2 v_2$$

.... Equation 155

This relationship is called the **continuity equation**. If the diameter of the pipe is reduced, the speed of the fluid must increase. This what happens if you put your thumb over the end of a garden hose.

Worked Example

A pipe of 12 mm diameter carries water that moves at 0.60 m s^{-1}. At the end of the pipe, there is a nozzle of diameter 5 mm. What is the speed of the water as it comes through the nozzle?

Answer

Work out the areas:

$$A = \pi D^2/4$$

$A_1 = \pi \times (1.2 \times 10^{-2}\text{ m})^2 \div 4$
$A_1 = 1.13 \times 10^{-4}\text{ m}^2$

$A_2 = \pi \times (5.0 \times 10^{-3}\text{ m})^2 \div 4$
$A_2 = 1.96 \times 10^{-5}\text{ m}^2$

Use:

$$A_1 v_1 = A_2 v_2$$

$v_2 = (1.13 \times 10\text{-}4\text{ m}^2 \times 0.60\text{ m s}^{-1}) \div 1.96 \times 10^{-5}\text{ m}^2$
$v_2 =$ **3.46 m s^{-1}**

When working out the area, make sure that you read the question carefully. If the diameter is mentioned use:

$$A = \pi D^2/4$$

If the radius is mentioned, use:

$$A = \pi r^2$$

Make sure you know what you are dealing with. Many students come a cropper with this one.

15.103 Pressure as Energy Density

Energy density is defined as **energy per unit volume**. The units are joules per cubic metre (J m^{-3}). An obvious example is the energy from a cubic metre of fuel. In Topic 6 Tutorial 3, we saw that the area under the stress-strain graph was energy per unit volume. A stretched longbow in an archery competition has an energy per unit volume. Another example would be the stretched spring in a clay-pigeon trap, which is a device that hurls out clay pigeons (ceramic discs) for shooters to fire at in shooting competitions.

We know that pressure is defined as **force per unit area**.

$$p = \frac{F}{A}$$

.... *Equation 156*

We know that if we push against a force for a distance, s, in the direction of the force, we do a **job of work**:

$$W = Fs$$ .... *Equation 157*

So, we can multiply the pressure equation on both sides by s:

$$ps = \frac{Fs}{A}$$

.... *Equation 158*

We can rearrange this to give:

$$p = \frac{Fs}{As}$$

.... *Equation 159*

We know that area × distance = volume, and that force × distance moved (in the direction of the force) = work, so we can say:

$$p = \frac{W}{V}$$

.... *Equation 160*

This is **energy per unit volume**, in other words, **energy density**.

Energy density from other sources can be described as pressure. Consider a star. There is a lot of energy in a large volume. Therefore, the star has an energy density, better known as radiation pressure, which prevents the star from collapsing under the influence of gravity.

15.104 Kinetic Energy as Energy Density

Any moving body has kinetic energy, regardless of whether it's solid, liquid, or gas. Of course, we know the equation:

$$E_\mathrm{k} = \frac{1}{2}mv^2$$

.... *Equation 161*

We are in the game of turning things into energy density, so we divide the kinetic energy of the fluid by the volume:

$$\frac{E_\mathrm{k}}{V} = \frac{1}{2} \times \frac{mv^2}{V}$$

.... *Equation 162*

We have the term mass ÷ volume in the equation. That is **density**, ρ. So, we can write:

$$p = \frac{1}{2}\rho v^2$$

.... Equation 163

Remember that energy density is **pressure**. This is the kinetic energy per unit volume, often called the **kinetic pressure**, or the **dynamic pressure**.

15.105 Potential Energy and Energy Density

We know that the formula for potential energy is:

$$E_{\mathrm{p}} = \mathrm{mg}\Delta h$$

.... Equation 164

We can convert the above equation into energy per unit volume:

$$\frac{E_{\mathrm{p}}}{\mathrm{V}} = \frac{mg\Delta h}{V}$$

.... Equation 165

Just as we did above, we can say that the mass ÷ volume term is the density:

$$p = \rho g \Delta h$$

.... Equation 166

This equation is used as the equation that describes how pressure increases with depth. It is more commonly written as:

$$p = h\rho g$$ *.......... Equation 167*

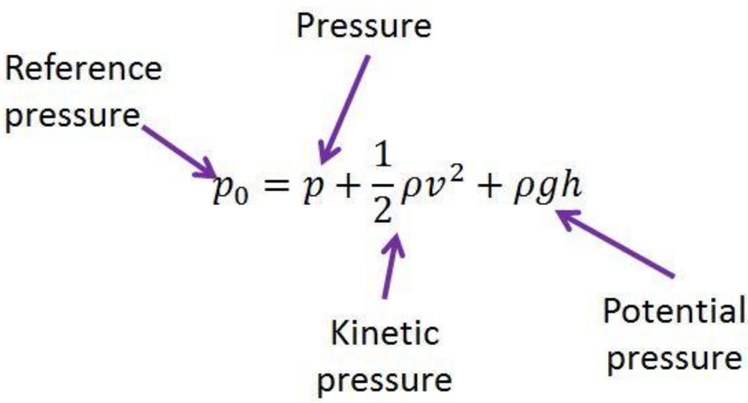
15.106 The Bernoulli Equation

We saw above that pressure is equivalent to energy density. So, the Bernoulli Equation can be considered in terms of the Law of Conservation of Energy:

Total energy density = Energy density + kinetic energy density + potential energy density

So:

Reference pressure = pressure + kinetic pressure + potential pressure



.... *Equation 168*

The reference pressure is often taken as atmospheric pressure, 1.035×10^5 Pa.

In the air, the potential pressure (energy per unit volume) is very small compared to the atmospheric pressure. So, we can simplify the equation to:

$$p_0 = p + \frac{1}{2}\rho v^2$$

.... *Equation 169*

In the syllabus, the equation has been rearranged to:

$$p = p_0 - \frac{1}{2}\rho v^2$$

.... *Equation 170*

We are interested in the **pressure differential**, rather than the absolute value for the pressure. So, we can write:

$$\Delta p = p_0 - p = \frac{1}{2}\rho v^2$$

.... *Equation 171*

15.107 Force on a spinning ball

If a ball is given spin, we get the following (*Figure 105*):

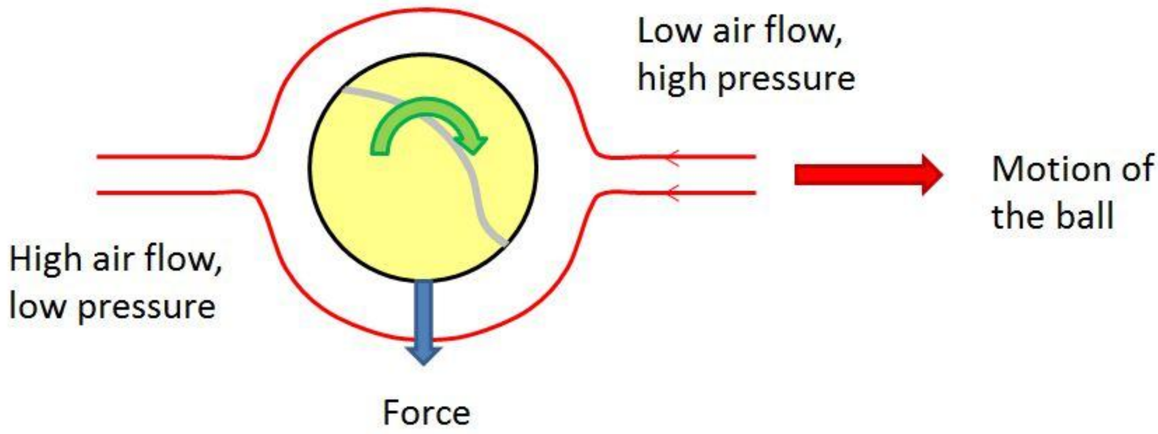


Figure 105 Force on a spinning ball

The force makes the ball deviate from a straight line.

The force acting on the ball is sometimes called the **Magnus Effect**.

Footballers do a similar trick to make the ball curve through the air (hence *bend it like Beckham*). It requires considerable intuitive skill, which I never had. Open confession is good for the soul. I am hopeless at all ball games and cannot kick, catch, hit, or throw a ball to save my life. I became a rower and a runner.

15.108 Extension - Fluids in a constriction

Consider a fluid of density ρ passing at a speed v through a pipe that has a constriction in it, like this (*Figure 106*):

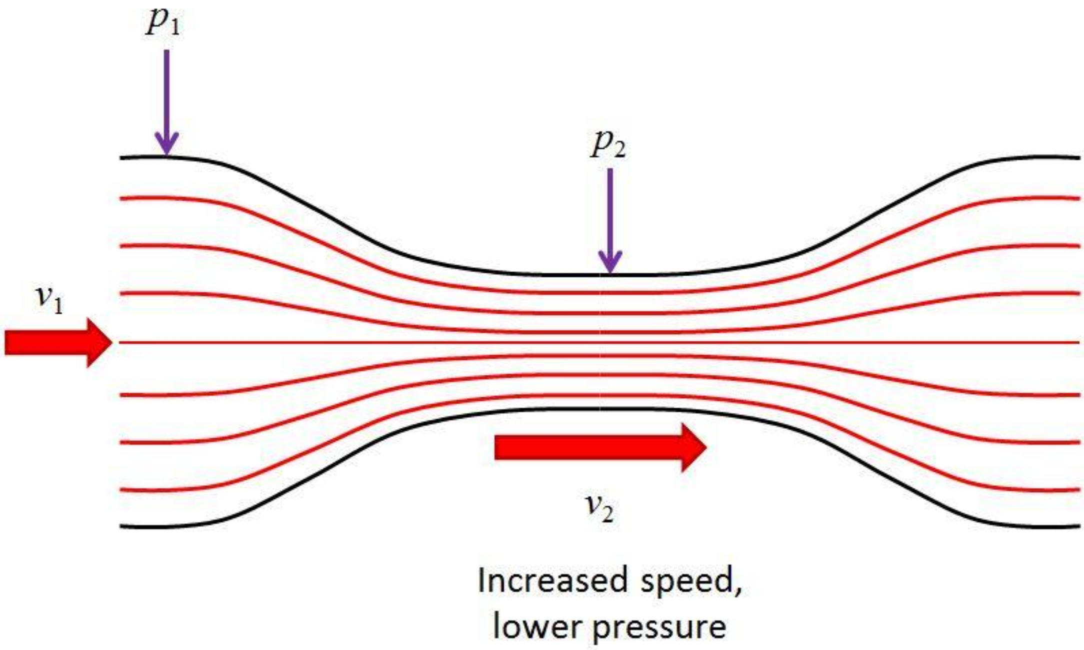


Figure 106 Fluid in a constriction

The potential energy per unit volume should be considered as well as the kinetic energy per unit volume. So, we write:

$$p_0 = p + \frac{1}{2}\rho v^2 + \rho g h$$

.... *Equation 172*

At the point p_1, we can therefore write:

$$p_0 = p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1$$

.... *Equation 173*

Similarly at point p_2:

$$p_0 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2$$

.... *Equation 174*

So, we can equate the two expressions (*Equations 173* and *174*) through p_0:

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2$$

.... *Equation 175*

This assumes that there is **laminar** (or smooth) flow that is in **steady state**, meaning that the flow is constant. It is a simple model. At university level you will find turbulence and other changes factored in. The mathematics is more complex.

Examples of this include:

- The **Venturi** in a carburettor.
- The **water injectors** on a high pressure steam boiler.

15.109 Drag

Drag is a result of collisions of air molecules on the body of a cyclist. The faster cyclist, the more frequent the collisions are, and the greater the change of momentum. We know that **change in momentum** results in **force**. You can feel drag for yourself by pedalling fast on a bicycle. When you are travelling slowly, there is little air resistance. The faster you go, the greater the air resistance becomes. And that means you have to work harder.

Drag on a racing bicycle can be reduced by having a carefully designed frame with streamlined tube cross-sections rather than round. The wheels have a narrow profile and fewer spokes (*Figure 107*). The rider wears close-fitting clothing made of Lycra ® and streamlined helmets. Some male cyclists shave the body hair on their legs and arms.

Figure 107 Racing bicycle (Image from Wikimedia Commons)

The best of these machines, built with hi-tech materials, can cost more than a car (and are ridden by MAMILs and OMILs - Middle-aged Men In Lycra and Old Men In Lycra).

Drag depends on these factors:

- The cross-sectional area facing the direction of movement (A).
- The density of the fluid (ρ).
- The square of the speed (v^2).
- The drag coefficient (C_D).

The formula for the **drag force** is this:

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

.... *Equation 176*

The term $\frac{1}{2}\rho v^2$ is the **kinetic energy per unit volume**, or **dynamic pressure**. This can be given the physics code q, so we can write:

$$F_D = qC_DA$$

.... Equation 177

We can rearrange the equation to write:

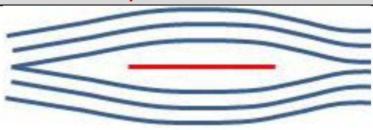
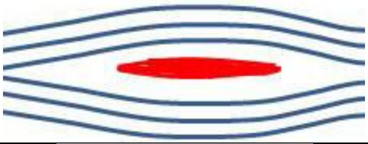
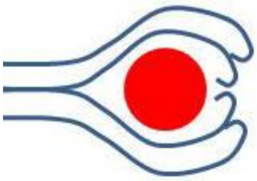
$$C_D = \frac{F_D}{qA}$$

.... Equation 178

Therefore, the drag coefficient is the:

ratio of the drag force to the product of the dynamic pressure and area.

The drag coefficient is determined experimentally and is affected by a large number of different factors, some of which can be quite complex. In a sporting context the drag coefficient explains why fast moving projectiles like a javelin are long and thin. However, there is a fly in the ointment - skin friction. The idea is shown in the table below:

Shape and Form	Form Drag / %	Skin Friction / %
	0	100
	10	90
	90	10
	100	0

You can see that a long thin shape has very low drag, but high **skin friction**. However, the skin friction is rather less significant in causing drag than the cross-sectional area.

The flight characteristics of a javelin are often studied by third year undergraduate students for their dissertations. The Question 15.10.8 is a simplification.

15.1010 Skin Friction (Extension only)

This eight-oared racing shell (*Figure 108*) is long and thin. It has a low form drag, but a high skin friction.

Figure 108 An eight-oared racing shell

The skin friction on this eight-oared racing shell arises from turbulence between the surface of the shell and the water. The skin friction is the ratio of the skin shear stress (from the turbulence) and the dynamic pressure.

In our example of the javelin, the skin friction was very small. In the case of this racing shell, it is significant. The equation is:

$$C_f = \frac{\tau_w}{q}$$
.... *Equation 179*

where:

- C_f - skin friction (no units).
- τ_w - skin shear stress (N m^{-2}).
- q - dynamic pressure (N m^{-2}).

The **dynamic pressure** is given by:

$$q = \frac{1}{2}\rho v^2$$
.... *Equation 180*

where:

- ρ - density (kg m^{-3}).
- v - speed (m s^{-1}).

Relating skin friction to drag force involves several different factors, which are beyond the scope of our discussion.

15.1011 Laminar and Turbulent Flow in Fluids (IB only)

Before you do this section, you may find it helpful to review Stokes' Law (Topic 6 Tutorial 4)

Consider a body of fluid flowing through a wide pipe (*Figure 109*). In the middle of the pipe, the flow is **uniform**. This means that the particles of fluid are all moving with the same speed. We say that the flow is **laminar**.

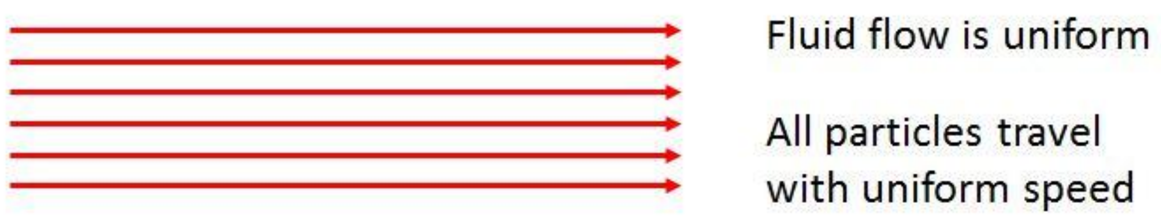


Figure 109 Uniform fluid flow

Around the inside surfaces of the pipe, there is **skin friction**. The flow around the edges is **turbulent**. The flow is uneven. The **churning** of the fluid takes energy (*Figure 110*). The effect can be reduced by ensuring that the surfaces of the pipe are as smooth as possible, but it is impossible to make the pipe perfectly smooth. Even making the pipe relatively smooth is very expensive.

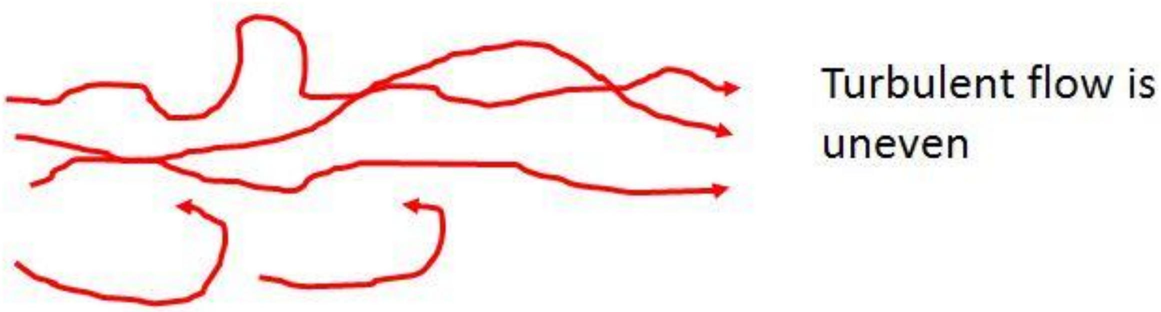


Figure 110 Turbulent flow

The picture below shows a transition point where a fluid (a rising column of hot air) changes from laminar flow to turbulent flow (*Figure 111*).

Figure 111 Changing from laminar to turbulent flow

The concept of laminar and turbulent flow is important in a wide range of applications including:

- Design of aeroplanes and other fast moving vehicles.
- Safe flight of aeroplanes through different atmospheric conditions.
- Efficient bulk movement of liquids through pipes.
- Efficient mixing of liquids.

The speed can be measured at different points across the pipe. In this case we are looking at just half the pipe, from its centre to its edge (*Figure 112*):

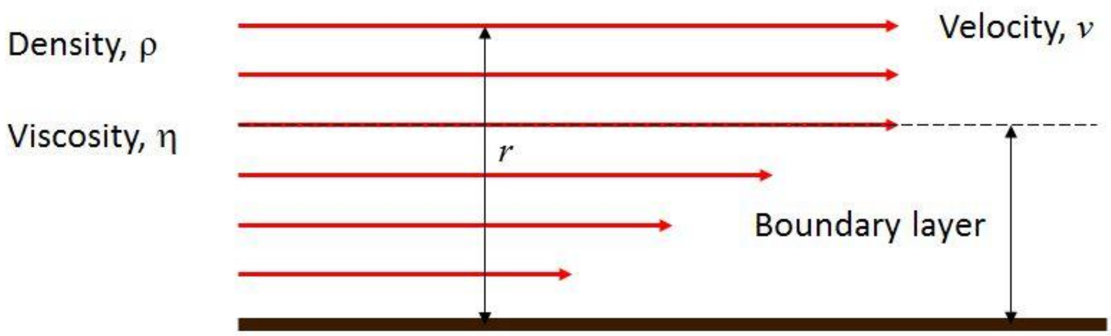


Figure 112 Variation of fluid flow from the centre of a pipe to its edge.

In the middle of the pipe, there is little resistance from the inside surface of the pipe. However, irregularities on the inside surface of the pipe will cause turbulence. This will result in a reduced speed. A **boundary layer** forms in which the flow becomes more turbulent the closer to the edge that we get. If the pipe is very smooth, the boundary layer is narrow. If the pipe is rough, (like most pipes), the boundary layer is significant. Also, if the pipe is narrow, the boundary layer takes up a greater proportion of the bore of the pipe.

We can use a relationship to determine whether the flow will laminar or turbulent. This is called the **Reynolds Number** (named after Osbourne Reynolds (1842 - 1912)). The Reynolds number is formally defined as the:

Ratio of the inertial forces to the viscous forces

We can write this as:

$$R_e = \frac{\text{inertial force}}{\text{viscous force}}$$

.... *Equation 181*

The physics code for the Reynolds Number is R_e. As a ratio, the Reynolds number has **no units**.

The **inertial** effects are observed within the boundary layer, as turbulent flow from the sides of the pipe affect the laminar flow towards the middle of the pipe. Counteracting this is the **viscosity** of the fluid, which reduces the turbulence. As a general rule:

- If the Reynolds Number is **low**, it means that **viscous** forces predominate, and the flow is **laminar**.
- If the Reynolds Number is **high**, the **inertial** forces are predominant, therefore the flow is **turbulent**.

Consider a fluid of density ρ flowing at a velocity v through a pipe of radius r. The viscosity is η (the strange symbol, η, is "eta", a Greek letter long 'ē').

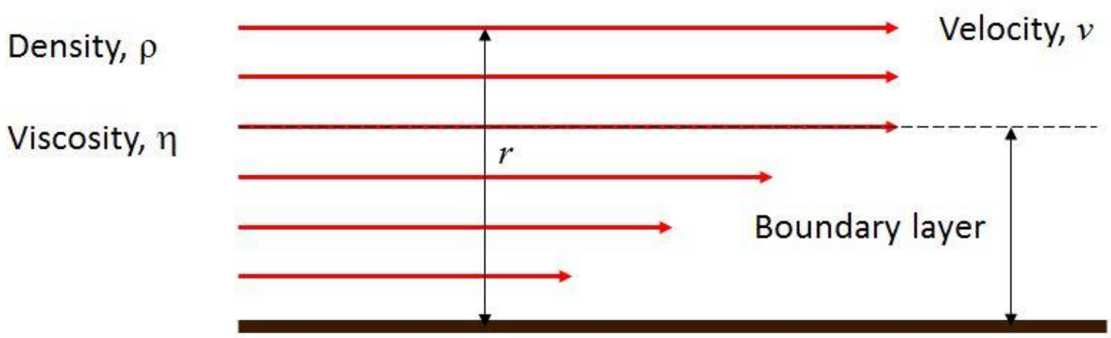


Figure 113 Variation of fluid flow from the centre of a pipe to its edge.

The equation for the Reynolds Number is:

$$R_e = \frac{vr\rho}{\eta}$$

.... *Equation 182*

Where:

- R_e is the Reynolds Number (no units).
- v is the fluid velocity (m s^{-1}).
- r is the radius of the pipe (m).
- ρ is density of the fluid (kg m^{-3}).
- η is the viscosity of the fluid (kg m^{-1} s^{-1} or Pa s or N s m^{-2})

As a rule, a Reynolds number of less than 1000 suggests the flow is laminar. Between 1000 and 2000, the flow becomes turbulent. Above 2000, the flow is turbulent.

Worked Example Water is flowing through a pipe of 50 mm radius at a speed of 0.010 m s^{-1}. The density of water is 1000 kg m^{-3} and the viscosity is 8.90 × 10^{-4} Pa s. Calculate the Reynolds Number. Is the flow laminar or turbulent?
Answer $$R_e = \frac{vr\rho}{\eta}$$ R_e = (0.010 m s^{-1} × 0.050 m × 1000 kg m^{-3}) ÷ 8.90 × 10^{-4} Pa s = **560** This would suggest the flow was laminar.

We can do a similar calculation to show that as the speed increases, the flow becomes more turbulent.

Although we have considered the Reynolds Number for flow within a pipe, the same relationship can be applied the width of a wing, or the length of a hull.

In the sources used for this discussion, the **diameter** of the pipe is used.

Questions

Tutorial 15.10

15.10.1

By converting to base units, show that the units for pressure are consistent with those for energy density.

15.10.2

A stream of water has a density of 1.00×10^3 kg m^{-3}. It is moving at 3.5 m s^{-2}.

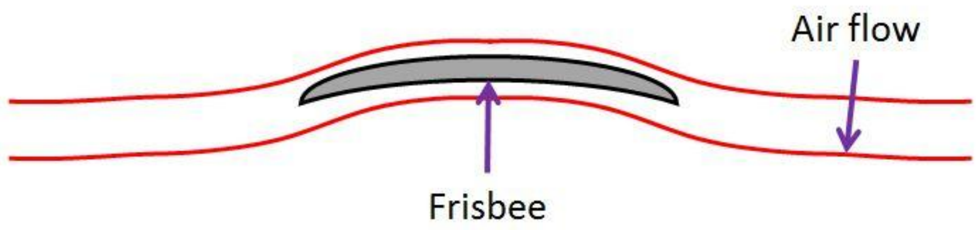
What is the pressure it exerts due to its movement?

15.10.3

A diver dives to 20 m off the coast to explore the seabed. The density of seawater is 1025 kg m^{-3}. What is the pressure acting on her?

15.10.4

A Frisbee is a toy that looks like the lid on a biscuit tin. It is thrown by one person to another, and it floats on the air. A cross-section of the toy is like this:



The Frisbee is 30 cm across and is flying at 5.5 m s^{-1}.

The atmospheric pressure is 1.01×10^5 Pa.

The density of air is 1.23 kg m^{-3}

(a) Assuming that the floating is a result of the Bernoulli Effect, work out the magnitude of the drop in air pressure acting on the top surface of the toy.

(b) Work out the magnitude of the force acting on the toy.

(c) State and explain the direction of the force acting on the toy.

(d) Which data item did you not use?

15.10.5

State what shape the path will be. Explain your answer.

What assumption have you made?

15.10.6 (*Challenge*)

A tennis ball has a diameter of 6.75 cm and a mass of 58.0 g. It is spinning at 500 rpm and has a forward speed of 20 m s^{-1}.

The atmospheric pressure is 1.01 × 10^5 Pa.

The density of air is 1.23 kg m^{-3}

(a) Show that the rotational speed of the circumference of the ball is about 1.8 m s^{-1}.

(b) Calculate the value of the total difference in pressure acting on the ball.

(c) Calculate the value of the force acting on the ball.

(d) Work out the radius of the path of the ball, assuming its forward speed remains constant.

15.10.7

What are the units for the drag coefficient? Explain your answer.

15.10.8

A javelin of mass 600 g has a diameter of 30 mm. It is thrown through the air at a speed of 20 m s^{-1}. The athlete accelerates the javelin to its flight speed in 0.10 s.

Calculate:

(a) The acceleration of the javelin.

(b) The force required to accelerate the javelin.

(c) The drag force acting on the javelin if the drag coefficient is 0.01.

(d) The range of the javelin if it's thrown at an angle of 45°; (Look at Topic 5 Tutorial 9)

(e) The kinetic energy of the javelin.

(f) The flight path is about 56 m. Work out the lost energy due to the drag force.

Density of air = 1.23 kg m^{-3}

Acceleration due to gravity = -9.81 m s^{-2} (i.e. downwards).

15.10.9 (*Challenge*)

A racing shell has a total mass of 750 kg. It is travelling at 5.7 m s^{-1}. The crew stop rowing and allow the boat to run. It comes to a stop in a distance of 40 m.

Assume that all the kinetic energy is lost as a result of the drag force and skin force from the water.

(a) Show that the kinetic energy of the racing shell is about 12 kJ.

(b) Calculate the magnitude of the acceleration of the racing shell using an appropriate equation of motion.

(c) Calculate the value of the total skin friction and drag force.

(d) The area **perpendicular** to the direction the movement of the boat is about 0.047 m^2. What is the drag force if the drag coefficient is 10 %?

(e) Calculate the skin friction. Give your answer to an appropriate number of significant figures.

15.10.10

Glycerine flows through a pipe of 1.0 cm diameter. The flow is laminar if the Reynolds Number is 1000. Calculate the maximum speed that the glycerine can flow for the flow in the pipe to remain laminar.

For glycerine, $\eta = 1.93 \times 10^{-3}$ Pa s.

For glycerine, $\rho = 832$ kg m^{-3}.

5. Energy and the Environment

Tutorial 15.11 Global Temperatures

Welsh Board and Eduqas Syllabus

Contents

This tutorial is for students of the Welsh Board and Eduqas.

It is quite a long tutorial, so you might want to work through it in stages.

15.111 Thermal Equilibrium

The Earth has the right environment for carbon-based life as we know it. The characteristics of our planet are that:

- The temperatures are suitable for life.
- There is water which has chemical properties that make it suitable as a medium for life processes.
- There are molecules that are the building blocks of life.
- There is an oxygen-rich atmosphere.

The lowest temperature recorded on the Earth has been -88 °C, while the highest has been +58 °C. However, the average temperature is a benign and comfortable 14 °C, even though there are climatic variation. As I write these notes (July 2018), there have been several weeks of hot and dry weather in the United Kingdom. However, the weather in March and April 2010 was decidedly colder than average with a strong easterly wind (the "Beast from the East").

The **Zeroth Law of Thermodynamics** states that when two objects are in thermal contact, heat energy flows from the hotter object to the cooler object until the two reach the same temperature. When both are at the same temperature, no energy is transferred, a state that is called **thermal equilibrium**.

The Earth is in Thermal Equilibrium with the Sun. The Sun transfers heat by radiation to the Earth by day. The Earth reradiates the radiation, mostly at night (*Figure 114*).

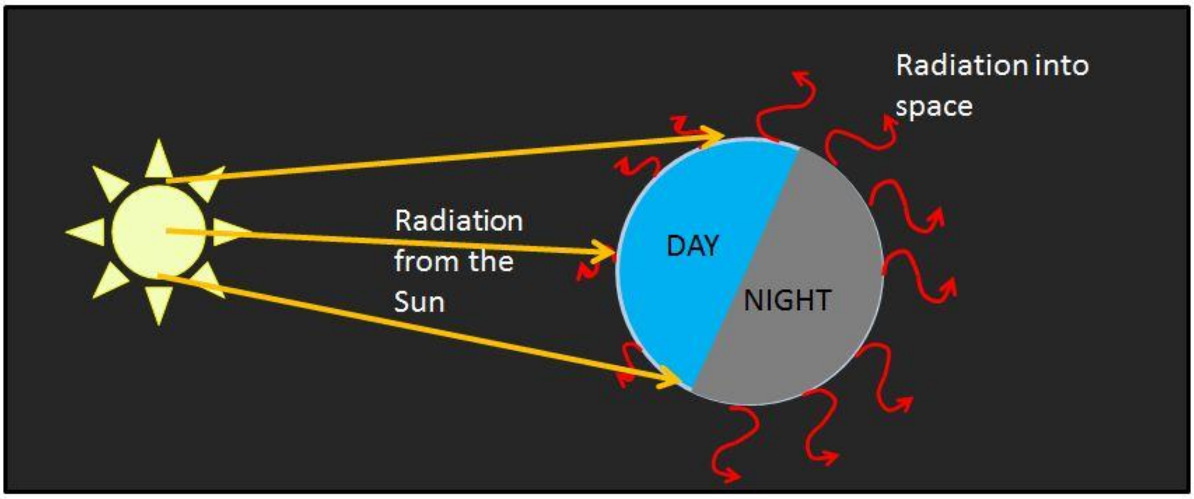


Figure 114 Thermal energy reradiated by the Earth.

There is some **re-radiation** during the day, but most is at night. This diagram shows a simple version of events, assuming that the radiation from the Sun is evenly distributed during the day, and that the radiation from the Earth is even at night.

The heating of the Earth's surface is not even, leading to areas around the tropics being much warmer than areas near the pole. Additionally, the Earth rotates on an axis that is about 23° to the vertical, as a result of which there are the seasons, which are more marked the closer to the poles you are. These is two of the many reasons that the Earth has climatic and weather variations. The atmosphere is very **energetic** as well as **chaotic**, which is why accurate weather forecasting is so very difficult. Some of the world's most powerful computers are employed to process the fiendishly complex equations and models that describe the global weather.

15.112 Albedo

Not all the radiation that heads towards the Earth is absorbed. Some is scattered off the atmosphere and the surface back into space. The reflected radiation is called the **albedo** (coming from a Latin word meaning whiteness.) Albedo is defined as the ratio of the scattered power (back into space) to the total incident power (from the Sun):

Albedo = scattered power (W) ÷ total incident power (W)

Albedo has the physics code α (alpha) and has no units. It is a fraction between 0 and 1. Zero albedo means that all the power from the sun's radiation is absorbed. The surface is a perfect absorber. It is black. An albedo of 1 means that all the power is reflected. In reality, neither of these extremes is achieved.

Snow has an albedo 0.9, which means that 90 % of the incident solar radiation is reflected, and 10 % is absorbed. Sea ice has an albedo of about 0.5 to 0.7 meaning that the ice absorbs between 30 % and 50 % of the solar radiation. Once melt-water pools start to form, the albedo drops to about 0.15, and the rate of radiation absorption increases. Albedo explains why snow melt might not be completed until late spring. Open oceans have an albedo of 0.06 (6 %).

The ground also has an albedo. The albedo of soil is 0.17 (17 %), which freshly laid tarmac has an albedo of 0.04 (4 %).

Emissivity is defined as the ratio between the energy emitted from a surface, and the energy that would be emitted if that surface was a black body.

A black body is regarded as a perfect emitter of radiation.

The **emissivity** is a quantity with no units with a value between 0 and 1. It is related to the albedo by:

$$\text{emissivity} + \text{albedo} = 1$$ .... *Equation 183*

Therefore, if we have a newly laid tarmac surface (albedo 0.04), the emissivity is:

$$\text{emissivity} = 1 - 0.04 = 0.96$$

Therefore, the emissivity of a tarmac surface is 96 % of a perfect black body

15.113 The Greenhouse Effect

There is much concern about the **greenhouse effect**. Let's explain this using a traditional model of a greenhouse (*Figure 115*);

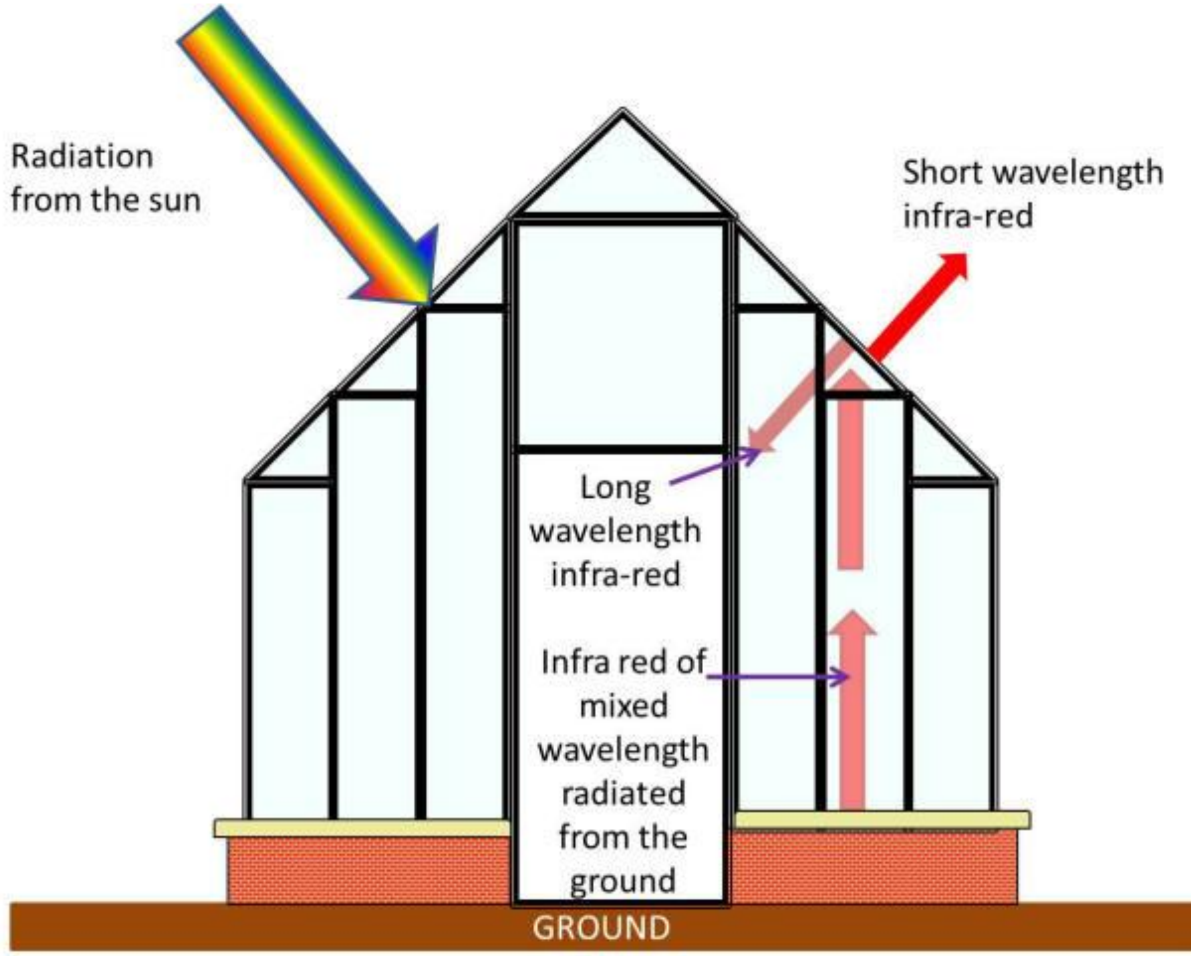


Figure 115 The greenhouse effect

Radiation of all wavelengths (white light) lands on the greenhouse. Some of it is **reflected**, but most is **transmitted** by through the glass, to be absorbed by the soil. The soil gets warm, and emits a mixture of **infra-red** radiations, shown by the red arrow with the dark red border. The glass can transmit the **short wavelength** infra-red, but reflects the **long wavelength** infra-red. The glass reduces **radiative transfer**. Therefore, the greenhouse retains much of the **internal** energy of its interior, so its temperature is higher than the external temperature.

The model referred to above is rather simplistic. The **convection** of air heated by radiation from the ground has not been considered in the explanation. Hot air rises (because it's less dense) and collects in the roof. Opening a roof vent reduces the temperature considerably, by letting out the hot air.

In the Earth's atmosphere, radiative transfer is reduced by **greenhouse gases**. Most gases consisting of molecules of two atoms (diatomic gases) and all gases consisting of three or more atoms can absorb and emit infra-red radiation. The three main gases in the atmosphere, nitrogen (N_2), Oxygen (O_2), and argon (Ar) do not absorb or emit infra-red. They are described as being **IR-transparent**. Infra-red radiation may be emitted as a result of collisions between molecules.

In the atmosphere, the main gases that can absorb and emit infra-red radiation are:

- Water vapour (H_2O).
- Carbon dioxide (CO_2).
- Methane (CH_4).
- Ozone (O_3).

These are called the **greenhouse gases**.

The picture here (*Figure 116*) shows the idea of carbon dioxide molecules interacting with rays of infra-red radiation which are a mixture of short and long wavelengths. Carbon dioxide is the most common greenhouse gas.

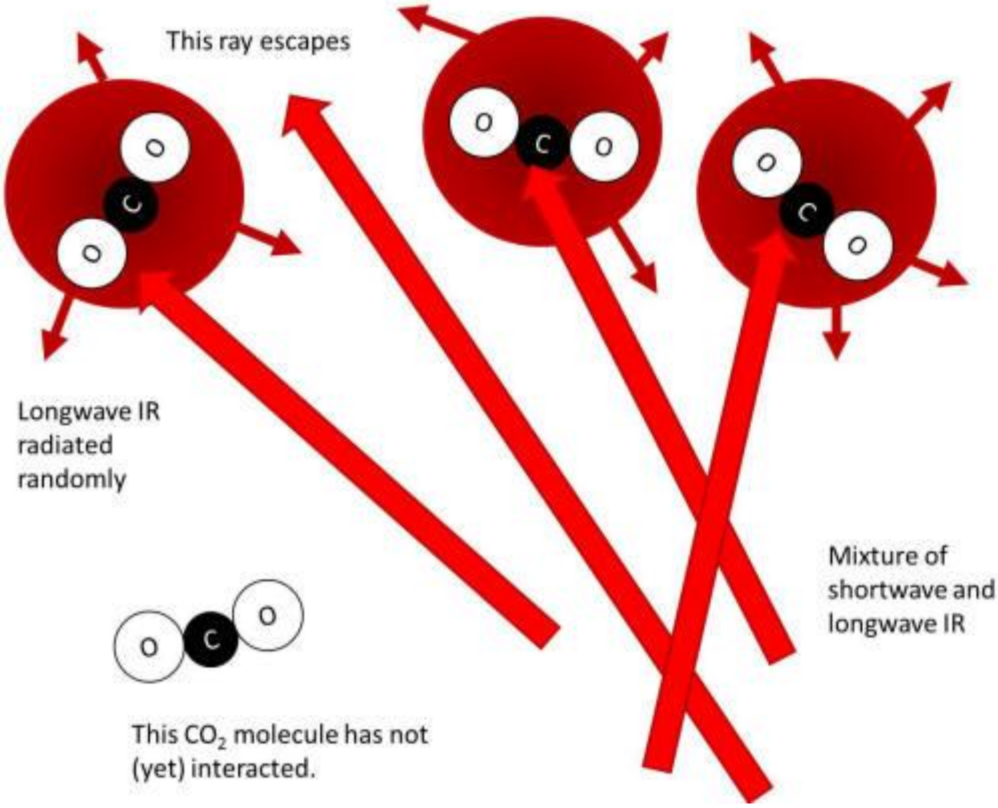


Figure 116 Interaction of carbon dioxide molecules with IR radiation

The short wavelength IR photons are not absorbed as much as the long wavelength photons. These are re-radiated randomly (just like the photons from an excited atom). Some photons will interact with other greenhouse gas molecules and be re-radiated. Many will increase the internal energy of the molecules in the atmosphere, resulting in an increase in temperature. The more carbon dioxide there is, the more likely that such events will happen.

The internal energy of carbon dioxide can result in the molecules:

- Stretching asymmetrically.
- Stretching symmetrically.
- Twisting on their axes.
- Bending.

This is shown in *Figure 117*.

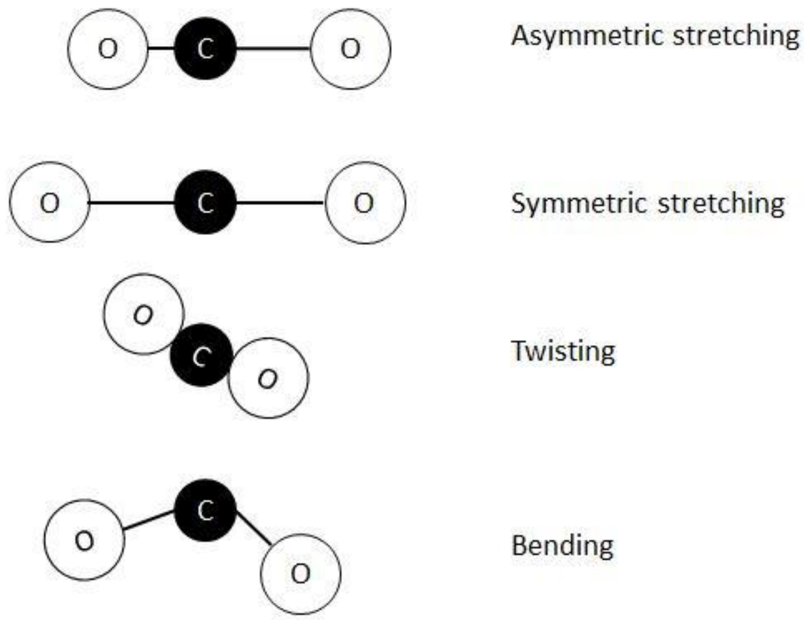


Figure 117 Behaviour of carbon dioxide molecules

The peak infra-red emission is at about 15000 nm. This is at the boundary of **infra-red** and **microwaves** in the electromagnetic spectrum. There are other peaks at 2000 nm, 3000 nm, and 4000 nm as shown on the graph:

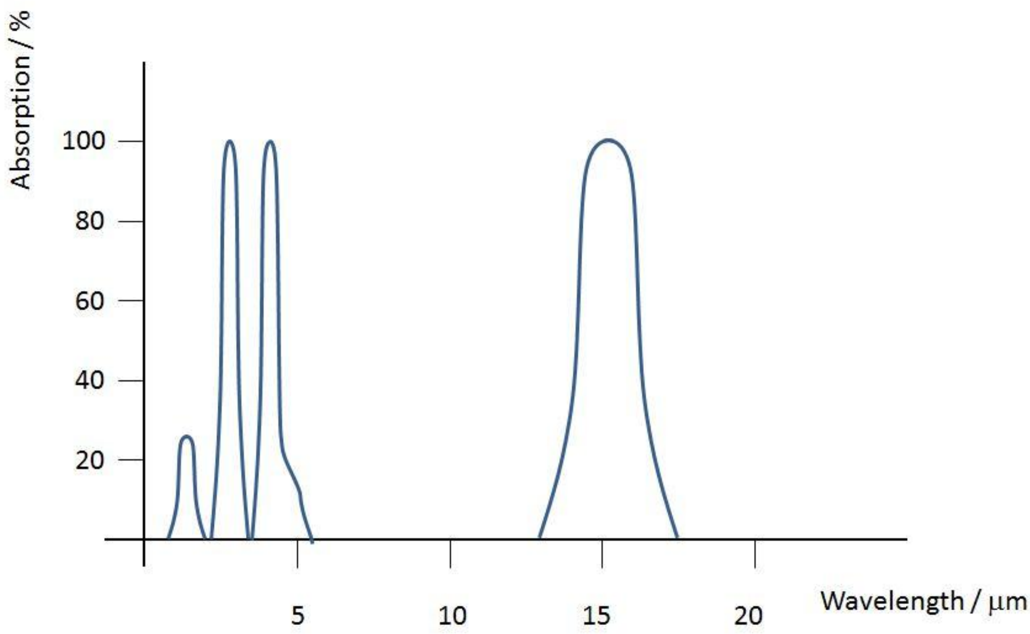


Figure 118 Emission peaks of carbon dioxide

Other molecules have different patterns unique to them.

Molecules can absorb energy at only specific wavelengths. The interaction is through the **electric field component** of the **electromagnetic wave** with the **charge distribution** of the molecule. The excited state lasts from several microseconds to tenths of a second. This is many times longer than the excitation lifetime of an electron.

Do not confuse the excitation of an electron with the excitation of molecules.

The **concentration of carbon dioxide** has risen markedly since 1960 as shown in the diagram (*Figure 119*):

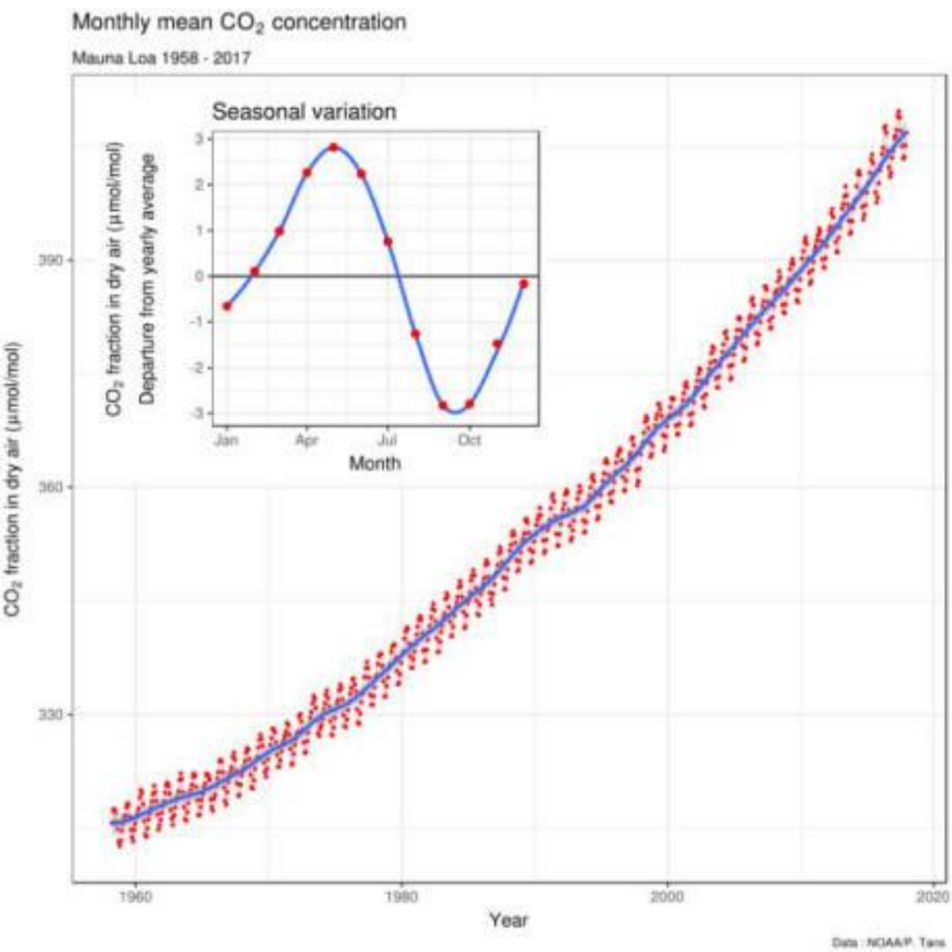


Figure 119 Rise of atmospheric carbon dioxide concentration

Most scientists believe that the rise in CO_2 levels is due to **human activities**. These include:

- Use of fossil fuels in internal combustion engines (e.g. cars, lorries, diesel trains, aeroplanes, and ships).
- Generating electricity using fossil fuels.
- Cutting down and clearing trees.
- Heating homes.
- Agriculture.

The level of carbon dioxide has been increasing since the industrial revolution in the eighteenth and nineteenth centuries. It was only in the latter part of the twentieth century that people realised that the increase in atmospheric CO_2 is not sustainable.

Among the effects of increased CO_2 emission are:

- An increase in global average temperatures.
- Extinction events of temperature-sensitive organisms, for example, coral reefs.
- Seabirds are finding food more difficult to find as rising sea temperatures are making their food move into higher latitudes.
- Thawing of permafrost regions - this will release methane, another greenhouse gas.
- More extreme weather events.

A single abnormally hot summer (or abnormally cold winter) is NOT sufficient evidence for global warming.

The trends need to be over decades.

Nations are coming together to address this issue. In these tutorials, we will look at some ways that the problems can be overcome, but it will take a long time. One example is the abolition of petrol and diesel cars by 2040. However, there are problems with **electric cars** (nothing new - the electric car was around before the petrol car):

- Battery range is limited.
- Home battery chargers are useful only if your house has a driveway to park the car - cars parked in the street won't have easy access to these.
- Rapid chargers still take a long time.
- Batteries are very expensive - many are hired at over £100 a month (more than many spend on petrol).
- The infra-structure for electricity will not cope if everyone were charging up their electric cars.
- Many electric cars have their batteries as part of the structure of the car. Minor damage to the battery case can result in the scrapping of the car.
- Battery faults can lead to thermal runaway, leading to catastrophic fires.

Some right-wing politicians, mostly in the USA, say that the whole idea of human-made global warming is nonsense.

15.114 Solar Energy Transfer

The only way that energy is transferred from the Sun across 150 million km of space to the Earth is by **electromagnetic radiation**.

Stars like the Sun glow in the same way as other glowing objects. If we turn the voltage up across a light bulb from zero volts up to its normal voltage, we see the filament glow a dull red, then to orange, to yellow to white. If we look at a spectrum as this happens, we see that (*Figure 120*):

- there is a continuous range of colours,
- but the relative intensity changes.

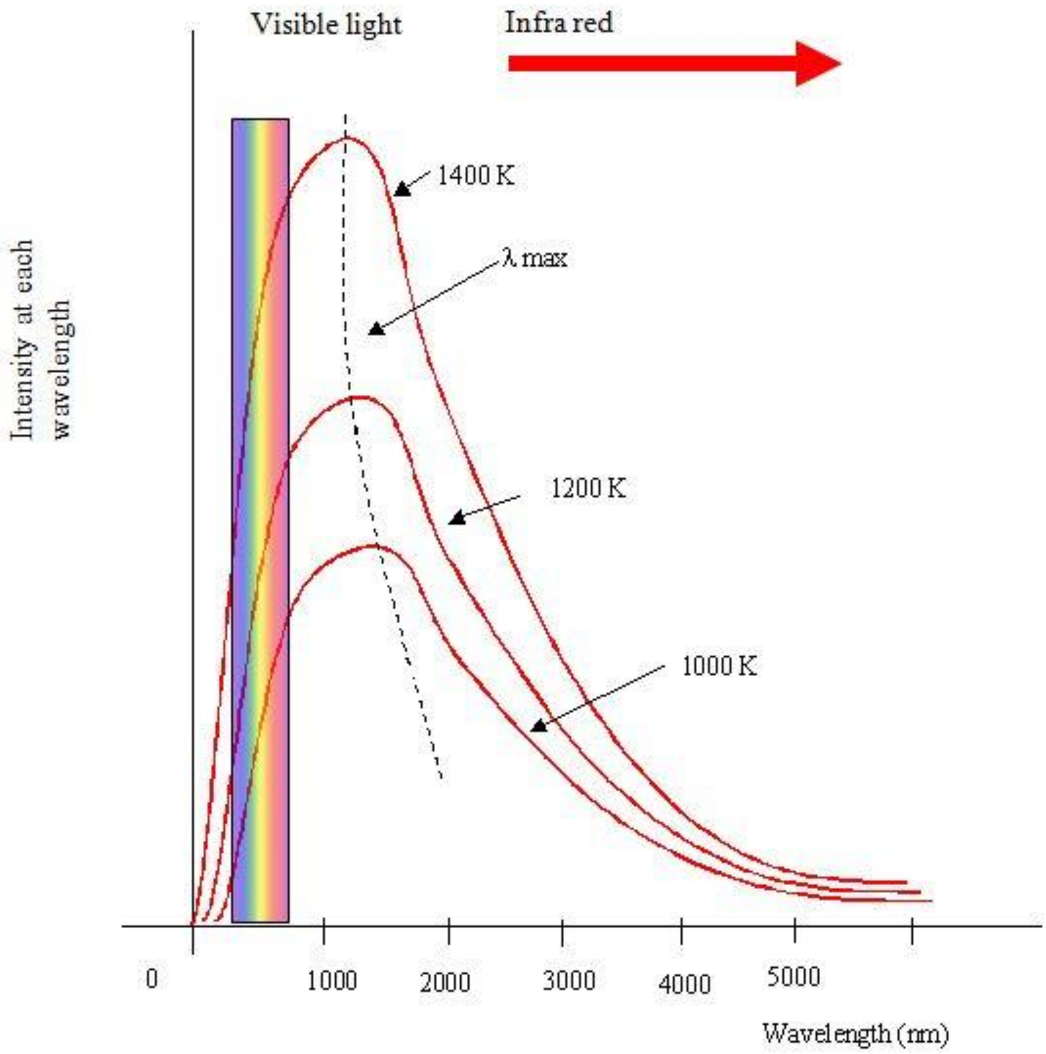


Figure 120 Spectrum of objects glowing at high temperatures

The light that we see is the resultant of that mixture of colours and other wavelengths. On this graph, the visible spectrum is to the left, between 300 and 600 nm. To the right are wavelengths of infra-red radiation.

We do not see that objects do not green because even if the peak wavelength were in the green region, 500 nm, there are also red and blue components as well. Therefore, the star appears white, because red, green, and blue make white.

15.115 Black Body Radiation

We look at the temperature of stars by looking at their colours. A lot of energy is given off as **thermal radiation.** Objects that are red hot have a temperature of about 1200 K. To understand how the colour of an object depends on its temperature, we need to understand the concept of a **black body**. A black body is a perfect absorber so that all radiation that falls on it is absorbed.

A black body is NOT the same as a black hole

A perfect absorber is a perfect emitter. Therefore, if we heat it up it will emit radiation including visible light (*Figure 121*). This is true (to a first approximation) for stars. Note the following for black bodies:

- a hot object emits radiation across a wide range of wavelength;
- there is a peak in intensity at a given wavelength;
- the hotter the object the higher the peak;
- the hotter the object the shorter the peak wavelength.
- the **area under the graph** is the total energy radiated per unit time per unit surface area.

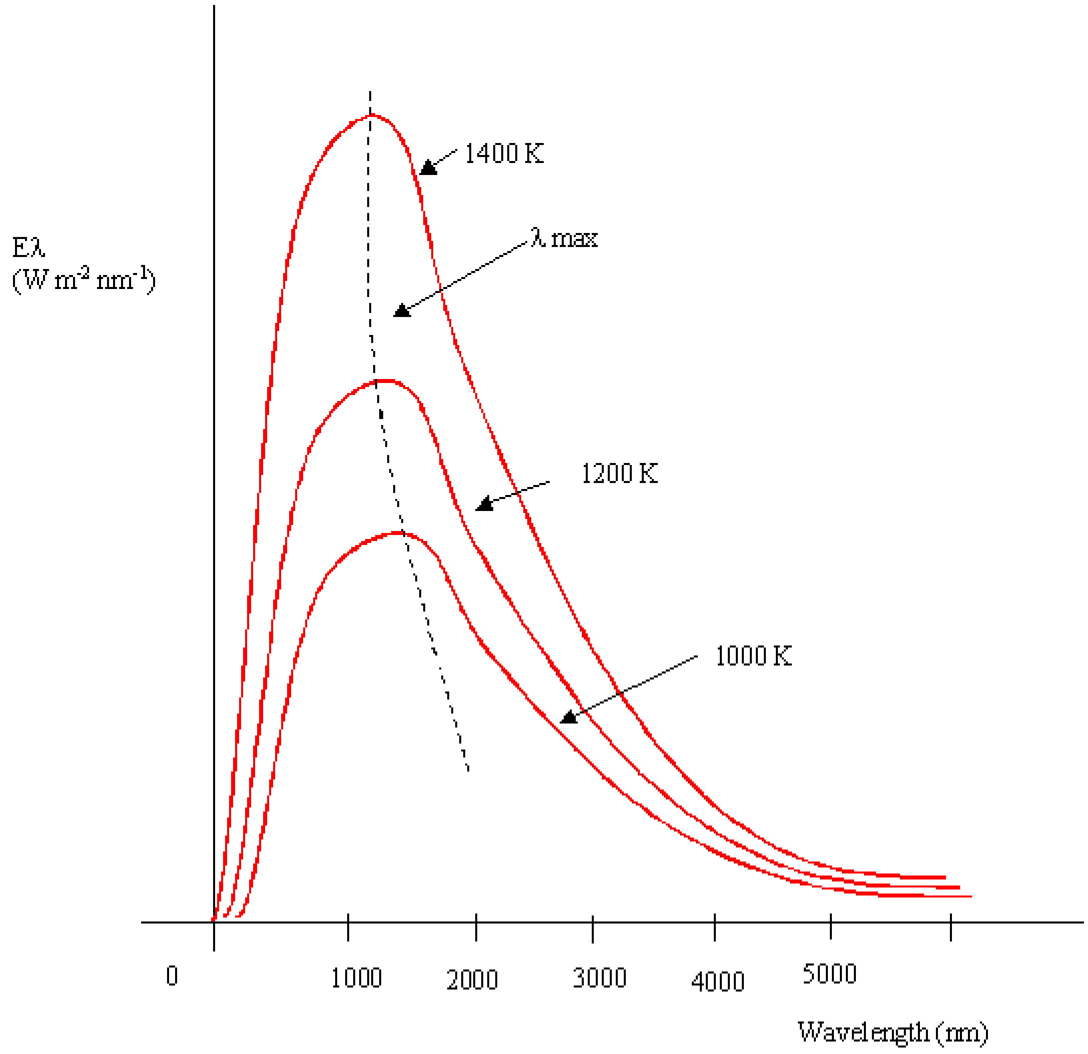


Figure 121 Emission from a black body

15.116 Absorption of Solar Radiation by the Atmosphere

Most radiation that reaches the Earth's surface is made up of visible and infra-red radiation. The atmosphere filters out the more energetic short-wavelength radiations like ultra-violet radiation. Much of the radiation is scattered by the atmosphere (which is why the sky appears blue). Also, the energy will be reflected by clouds and haze. The solar radiation that reaches the ground directly from the Sun is called **direct solar radiation**. **Indirect radiation** is radiation that has been scattered by the sky and the clouds or reflected from objects.

The graph (*Figure 122*) show the energy absorption.

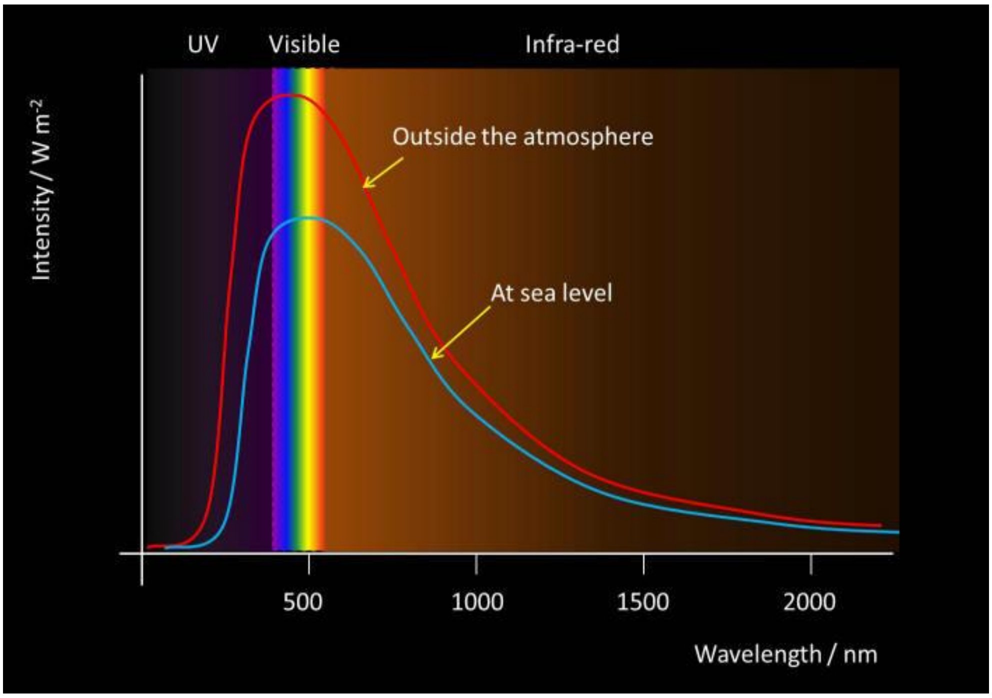


Figure 122 Intensity of absorbed radiation against wavelength

The graph has been simplified to remove peaks at different wavelengths that are transmitted from the Sun. Also, the atmosphere absorbs some frequencies more than others.

Some ultra-violet radiation does reach the surface of the Earth. There are three kinds:

Ultra-violet Radiation	**Wavelength Range / nm**	**Notes**
UV-A	320 - 400	Not absorbed by ozone. Can penetrate water depending on turbidity. Causes sunburn. Clouds absorb it. It can inhibit photosynthesis by reducing the efficiency of electron transport. It can cause fluorescence. See Topic 3 Tutorial 5.
UV-B	280 - 320	It can penetrate up to 20 m in seawater, depending on turbidity and chemistry. Penetrates less far in fresh water. Photons are more energetic than UV-A, and cause mutations by making Thymine dimers in the DNA. This can lead to cancer. It appears to impair photosynthesis in plants.
UV-C	200 - 290	Although UV-C makes up 0.5 % of the radiation, it is the most damaging to living organisms. It is absorbed readily by ozone in the stratosphere.

17.117 Wien's Law

The **peak wavelength** is λ_{max} which is the **wavelength at which maximum energy is radiated**. This is **inversely proportional** to the **Kelvin** temperature. It is called **Wien's Displacement Law** (as the peak is displaced towards shorter wavelengths). We write it as:

$$\lambda_{max}\,T = \textbf{constant} = \textbf{0.00289 m K}$$ .... *Equation 184*

Worked example
What is the peak wavelength of a black body emitting radiation at 2000 K? In what part of the electromagnetic spectrum does this lie?
Answer λ_{max} = 0.00289 m K ÷ 2000 K λ_{max} = 1.45 x 10^{-6} m = **1450 nm** This is in the infra-red region.

17.118 Luminosity of the Sun

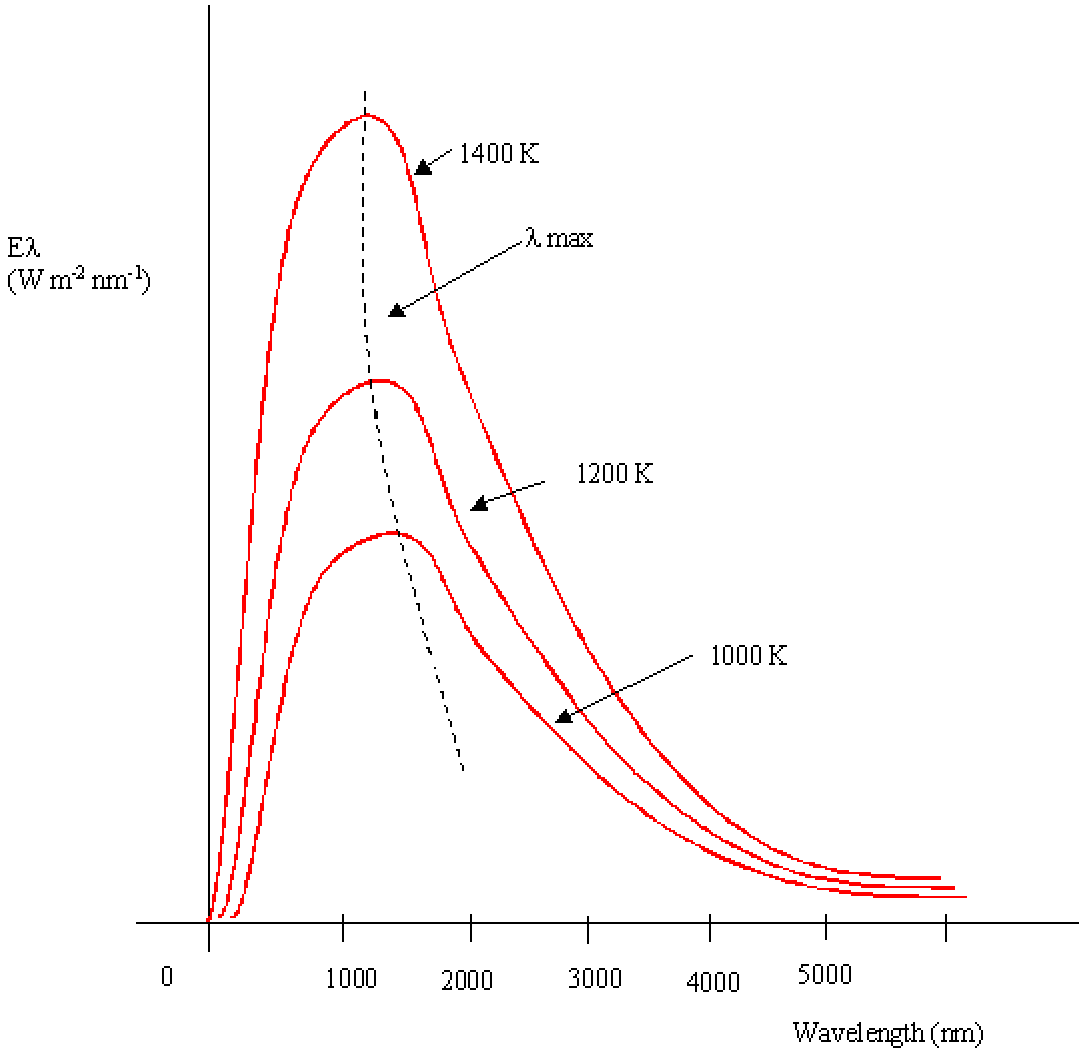


Figure 123 Emission from a black body

The area under the graph above is related to the rate at which a black body radiates energy. The **luminosity** of a star is the total energy given out per second, so it's the **power**. From the graph the luminosity increases rapidly with temperature, which gives rise to **Stefan's Law**. Formally this is stated as:

The total energy per unit time radiated by a black body is proportional to the fourth power of its absolute temperature.

In other words, double the temperature and the power goes up sixteen times. In physics code we write:

$$P = \sigma A T^4$$ .... *Equation 185*

[P- Power (W); σ - Stefan's constant; A - area (m^2); T - temperature (K)]

The strange looking symbol σ is "sigma", a Greek letter lower case 's'. It **Stefan's Constant**.

σ = 5.67 x 10^{-8} W m^{-2} K^{-4}

We can treat a star as a perfect sphere ($A = 4\pi r^2$) and a perfect black body. So, for any star, radius r, we can write:

$$P = 4\pi r^2 \sigma T^4$$ .... *Equation 186*

(Note: in some text books the power may be represented as **luminosity** with the physics code L)

Stars with the same **absolute magnitude** have the same **power output**. We can justify this statement by considering stars P and Q:

- Power of P = $P_P = A_P \sigma T_P^4$ where A_P is the area of P and T_P is the surface temperature of P.
- Power of Q = $P_Q = A_Q \sigma T_Q^4$ where A_Q is the area of Q and T_Q is the surface temperature of Q.

We can equate the two expressions to give:

$$A_P \sigma T_P^4 = A_Q \sigma T_Q^4$$ .... *Equation 187*

So, we can write:

$$\frac{A_{\mathrm{P}}}{A_{\mathrm{Q}}} = \frac{{T_{\mathrm{Q}}}^4}{{T_{\mathrm{P}}}^4}$$

.... Equation 188

So, if the temperatures are the same, the areas will be the same. Therefore, the radii will be the same.

15.119 The Power of the Sun

On the equator, the average intensity of the Sun's rays is about 1400 W m^{-2}. In practice, some is absorbed by the atmosphere, and some is reflected as heat, but we will use this in a calculation to work out the power given out by the Sun.

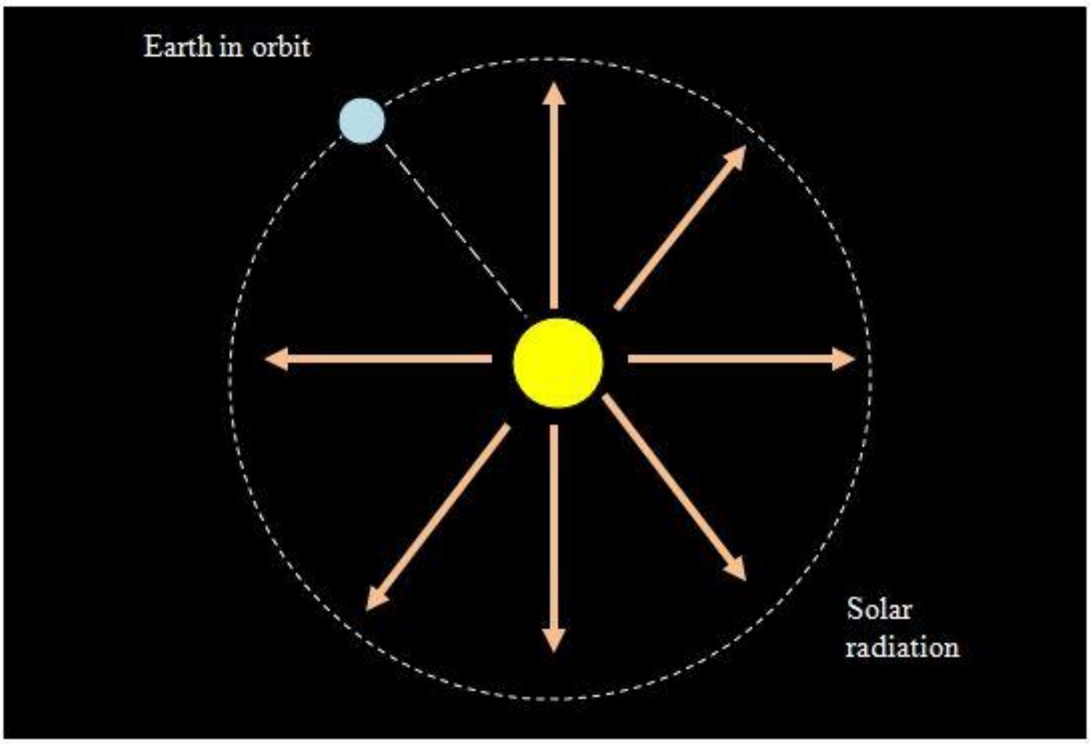


Figure 124 The Earth's orbit

We can work out the power of the Sun by working out the total area of a sphere that has the radius of the Earth's orbit.

$$A = 4\pi r^2 = 4 \times \pi \times (1.50 \times 10^{11})^2 = 2.83 \times 10^{23}\ \text{m}^2$$

Since each square metre receives 1400 W, the total power of the Sun is:

2.83×10^{23} m^2 × 1400 W m^{-2} **= 3.96 × 10^{26} W** = 4.0 × 10^{26} W (to 2 s.f.)

You may have noticed that this figure is slightly lower than the answer worked out in Question 15.11.7, but some energy is absorbed and reflected, so 1400 W m^{-2} is slightly too low.

15.1110 Rising Sea Levels

Twenty thousand years ago, the British Isles was a peninsula on the north-west coast of Europe. It was between two large rivers, the Seine and the Rhine. Doggerland, off the current North Sea coast, was a range of low hills, home to herds of animals and the first human inhabitants of this region. Between the south coast of England and France, a range of chalk hills separated the two great rivers. The soil in Kent is the same as that in Champagne, and Kentish wine producers make a sparkling wine that is very similar to Champagne.

During the ice age, there were very cold winters, but the summers could be benign. The global mean temperature then was 10 °C, compared to 14 °C today. The icesheets that covered much of Northern Europe started to melt. About seven thousand years ago (yesterday in geological terms), a massive lake of meltwater built up in what is now the southern North Sea, hemmed in by ice dams. One of these burst resulting in a massive flood that breached the range of hills between the Rhine and the Seine to form what is now the English Channel. It also swept away the hill that formed one side of the Solent Valley to separate the Isle of Wight from the mainland.

The sea-level has risen by about 120 m during the melting process which started 19000 years ago and finished 6000 years ago. The sea-level rose by about 1 metre every year. Rising sea-levels are therefore nothing new. The sea is actually retreating on the west coast of Wales. Harlech Castle was on sea-cliffs when it was built in the Fourteenth Century. It is now about 3 km inland. The UK mainland is rising more in the west and sinking to the east. This is because the land is rising more in the west as it has been relieved of the weight of the ice.

Global warming is a major concern as it will cause sea-levels to rise, leading to low-lying lands being flooded. London could well fall victim in a couple of hundred years. Let us have a look at why the sea levels are rising:

- As the oceans get warmer, the water in the top layers **expands**. This is because the kinetic energy of the molecules increases with temperature. Density is mass per unit volume. The mass is due to the molecules, so if there are fewer molecules per unit volume, the other molecules have to go somewhere else. So, the whole body of the water gets bigger. Thus, water levels rise.
- Water from melting land-based (**grounded**) ice is pouring into the oceans. The evidence for this is that many glaciers have retreated a long way in previous decades.

These are intuitive. Let's consider what happens to a floating iceberg as it melts. How much does floating ice contribute to the rise in sea-levels? To answer this, we need to think about **Archimedes' Principle** and **density**. See Topic 5 Tutorial 1 to revise density.

15.111 Archimedes' Principle

Any object that floats in a liquid has an **upthrust that is equal to its weight**. Since the weight acts vertically downwards, the upthrust acts vertically upwards. The idea is shown below (*Figure 125*):

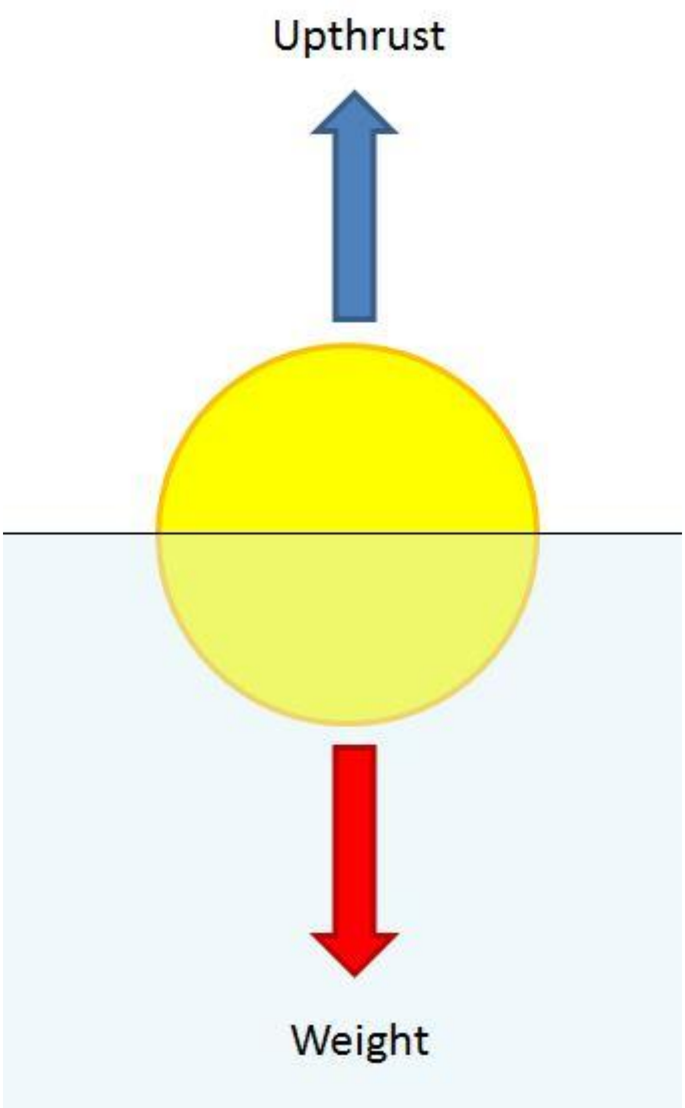


Figure 125 Archimedes' Principle

The weight of water displaced is the same as the weight of the object. If the upthrust is less than the weight, the object will sink.

When an object is totally immersed in the water, the volume of water displaced is the volume of the object. You will have used this idea to find out the density of an irregular object. See Topic 5 Tutorial 1.

Archimedes' principle states:

Any body wholly or partly immersed in a fluid experiences an upthrust equal to the weight of the fluid displaced

The equation associated with Archimedes Principle is:

$$F_r = t\rho g A$$

.... *Equation 189*

Please see Topic 5 Tutorial 4 for the derivation.

The volume of the object:

$$V = tA$$ .... *Equation 190*

The term t is the thickness of the object. This volume is the same as the volume of liquid that is displaced. The mass of liquid displaced is volume × density, so the mass of liquid displaced is:

$$m = tA\rho$$ .... *Equation 191*

Therefore, the weight displaced is:

$$F_r = tA\rho g$$ .... *Equation 192.*

We can write this in terms of the volume:

$$F_r = V\rho g$$ .... *Equation 193*

So, let's model a melting iceberg using a cube of ice from the freezer placed in 200 cm^3 water of temperature of 0 °C (*Figure 126*).

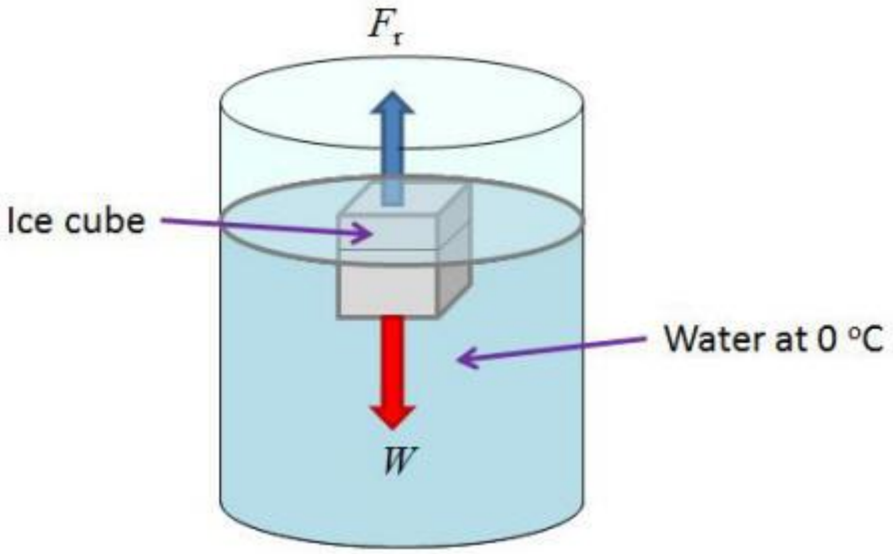


Figure 126 Ice floating in water

The ice cube has sides that are 0.030 m. Therefore, the volume is 27×10^{-6} m^3. The density of water ice is 919 kg m^{-3}. The density of water is 1000 kg m^{-3}.

The weight of water displaced will be 0.243 N by Archimedes' Principle.

As the ice melts, less water would be displaced, until eventually the 25 cm^3 of water in the cube is incorporated into the water in the beaker. The number of water molecules in the 27 cm^3 of water ice at a density of 919 kg m^{-3} is the same as the number of water molecules in 25 cm^3 of liquid water at a density of 1000 kg m^{-3}. Therefore, the water level remains the same.

From this, we can conclude that melting sea ice would not cause sea-levels to rise. The ice at the North Pole is sea-ice, so the melting of the sea-ice there would not have a significant effect on sea-levels.

The melting of the Antarctic icecap would make the sea-level rise by 61 metres.

Questions

Tutorial 15.11

15.11.1

Give two reasons why the model shown in the diagram (*Figure 114*) is too simplistic.

15.11.2

Why is internal energy, not heat, referred to in the paragraph on the Greenhouse Effect referred to on Pages 178 - 179?

15.11.3

Calculate the photon energy in J and eV for an infra-red photon of wavelength 15000 nm.

15.11.4

Refer to *Figure 122*. Explain how the UV of the sunlight outside the Earth's atmosphere differs from the UV detected at sea-level

15.11.5

Some years ago, it was discovered that chlorofluorocarbons (very un-reactive molecules used as aerosol propellants and refrigerants) were interacting with ozone in the stratosphere, leading to "the hole in the ozone layer". Explain why this caused a great deal of alarm, leading to a world-wide ban on these substances.

15.11.6

The Earth emits radiation with a peak of 10.5 mm. What temperature does this correspond to?

15.11.7

If the Sun has a radius of 6.96×10^8 m and a surface temperature of about 6000 K, what is its total power output? What is the power per unit area? What is the peak wavelength?

15.11.8

A diver is salvaging a spherical cannon ball from the wreck of an ancient ship. The cannon ball has a diameter of 10 cm and is made of iron of density 7900 kg m^{-3}. The density of seawater is 1030 kg m^{-3}.

What is the force needed to lift the cannon ball:

(a) on land?

(b) under the water?

Does the depth of the wreck matter?

Acceleration due to gravity = 9.81 m s^{-2}.

15.11.9

Refer to *Figure 126*. Show that the mass of the water in the ice cube is about 0.025 kg.

What is the weight?

15.11.10

Refer to *Figure 126*. What is the volume of water displaced by the ice cube?

What is the level to which the water would rise, assuming there was a volume scale on the beaker.

15.11.11

A glacier from a mountain range feeds ice into the sea. Every so often it "calves", meaning that a chunk breaks off and floats away as an iceberg. Would it contribute to the rise sea-levels?

Tutorial 15.12 Renewable and Non-Renewable Energy Sources

Welsh Board, Eduqas and IB Syllabus

Contents

This tutorial is for students of the Welsh Board, Eduqas, and IB Syllabuses.

The last two topics are for students studying the CEA syllabus.

This is a long tutorial. You may wish to go through it slowly in several sessions.

15.121 Non-Renewable Energy Sources

Non-renewable energy sources captured their energy from the Sun millions of years ago. They have given us the lifestyle that we enjoy today. Imagine life without electricity (think back to the last time your house lost its electricity). Our modern way of life depends on fossil fuels:

- **Oil** - remains of tiny sea-creatures that lived and died millions of years ago and were trapped in rock formations.
- **Coal** - trees that died millions of years ago, and their remains were compressed to from a sedimentary rock that can be burned.
- **Natural gas** - methane that is associated with both coal and oil.

All of these give off carbon dioxide, which is a greenhouse gas, when they are burned. Greenhouse gases (which we discussed in the previous tutorial) are associated with global warming with all the problems that climate change is bringing.

Another problem that the use of fossil fuels bring is that of **pollution**. Not only do fossil fuels give out carbon dioxide but also sulphur dioxide. All organisms have sulphur in them, coming from an **amino acid** called **cysteine**, which is an essential part of the proteins that make up living cells. Sulphur dioxide reacts with rain water to make **acid rain** that has caused long term damage to trees.

In the Twentieth Century, coal was burned in large amounts in open fires to heat homes, as well as for firing boilers for locomotives, and steam engines to power machinery. Mains gas was derived from coal, rather than methane that is used today. **Town gas** was principally **carbon monoxide**, which is highly toxic. The smoke from such activities led to **smog** which made for breathing difficulties for many people, sometimes fatal. It needs to be remembered also that in those days, most adults smoked tobacco, so a large proportion of the population had smoking related health issues. The number of fatalities led to **clean air legislation** which demanded **smokeless** fuels.

While smog has become less of a problem, pollution from **road traffic** has taken its place. The internal combustion engine in cars, lorries, and buses produces not only carbon dioxide, but also **carbon monoxide**, and **nitrogen dioxide**. **Catalytic converters** are now compulsory to reduce such pollutants. The diesel engine also produces tiny particles of soot. In recent years, motorists were encouraged to buy diesel cars because they use less fuel, hence less carbon dioxide is produced. However, the cars produced more **pollutants**, so manufacturers were forced to add **diesel particulate filtration** systems in to reduce these to an acceptable level. A number of manufacturers, not only Volkswagen, added **cheat software** into the engine management systems of the cars, so that when they were tested in stationary test facilities, the cars produced much less pollution than when out on the road. Volkswagen was caught in the **emissions scandal**, but other manufacturers have since owned up. The re-programming to cut down the emissions has had adverse effects on performance and reliability of affected cars.

Government has demanded that fossil fuel be banned as a sole fuel for cars by 2040. **Electric** and **hybrid** cars will be the rule. This brings another issue, how to charge the batteries of the cars. Millions of cars will need charging points. This brings up other questions:

- What about the **electricity infrastructure** needed to provide for the extra demand when everyone charges their car?
- What happens if a motorist does not have a drive on which to charge his/her car? In most towns, motorists know that it's impossible to have the same parking space every time.
- What about **charging points** if you are on a long journey? It takes high power chargers 40 minutes to charge up an electric car to 80 %. A facility may have 20 charging points, but there could well be 100 cars waiting… (It takes no more than 5 minutes to call in for petrol.)
- In some parts of the country, there are hardly any **public charging points**. Where I live, near the Yorkshire Dales, the nearest charging point is 40 km away.
- A government that chickens out at completing railway electrification projects on the basis that the erection of a few thousand electrification masts and several hundred kilometres of overhead wire costs too much is surely going to be completely "frit" about the cost of providing the infrastructure for charging points for every house.

Many countries that in the Twentieth Century had totalitarian regimes that kept their peoples poor have now embraced the technological age. Why should their people not have all the benefits that it brings? Why can't they drive around in the decent cars that we take for granted? Why should they not have good computers?

Another pressing problem has arisen in the international public consciousness, that of **plastic pollution**. Plastics are made from oil. They don't degrade naturally. While an increasing amount of such materials is recycled, a lot is dumped and some ends up in the sea. Marine plastic pollution is becoming an increasing threat to marine environments and the species that live there. The clean-ups that are being carried out are a small fraction of what is there.

All of these increase the demand for fossil fuels with all the attendant issues. Some wealthy and influential individuals and corporations say that pollution and global warming is not an issue, despite the evidence to the contrary. So there needs to be different ways of providing the energy to power the lifestyle that we take for granted.

And this is what we are going on to consider now.

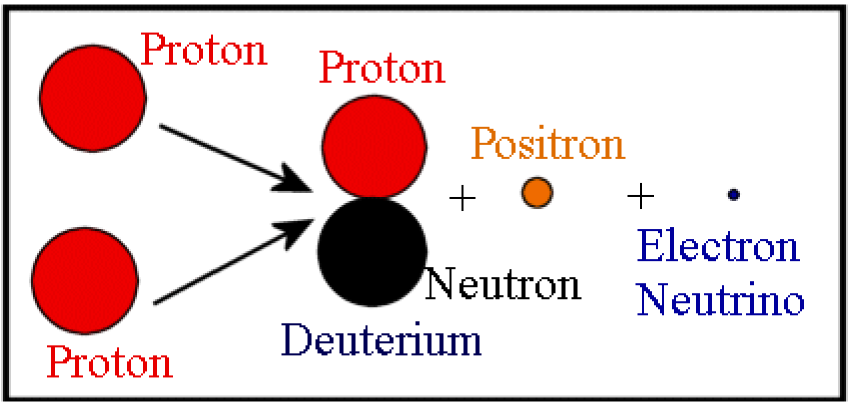
15.122 Power from the Sun

The Sun, like all stars, gets its power from **fusion**.

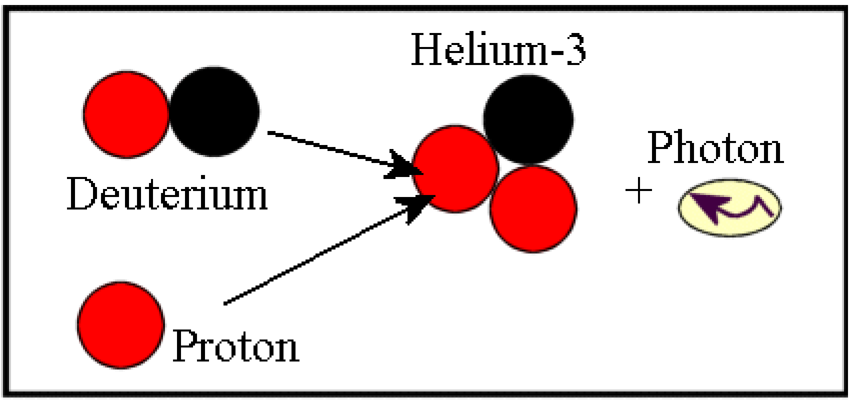
Fusion means joining nuclei together, every alchemist's dream. It is easier said than done. The idea is that **light nuclei are joined together**, increasing the binding energy per nucleon. This will result in lots of energy being given out.

The process in **stars** has three stages (*Figure 127*):

1. Proton + Proton ® Deuterium + positron + electron neutrino



2. Deuterium + proton ® Helium 3 + photon



3. Helium 3 + Helium 3 ® Helium 4 + proton + proton

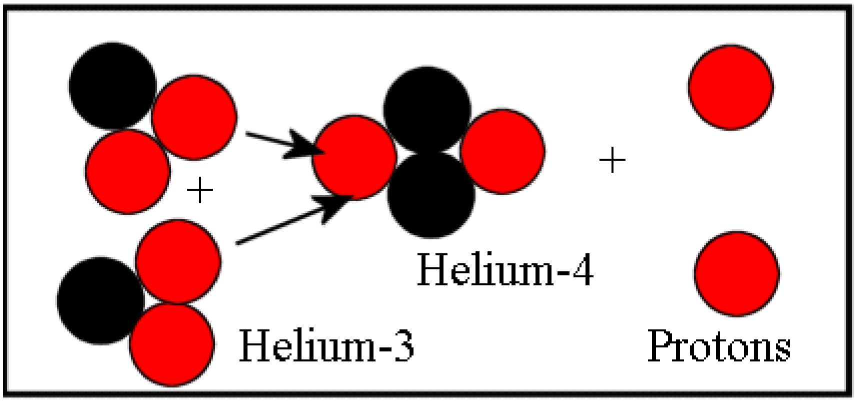


Figure 127 Stages in fusion

Since two protons are left over, the reaction is self-sustaining.

15.123 Intensity of Sunlight

Intensity of radiation is defined as:

Power per unit area

The physics code for intensity is I and the units are watt per square metre (W m^{-2}). The equation is:

$$I = \frac{P}{A}$$ ....*Equation 194*

The energy has a maximum value, often written I_0, at the source. We treat the source as a **point source**. As the light waves **propagate**, they spread out. For each doubling of radius, the intensity goes down by 4 times (*Figure 128*):

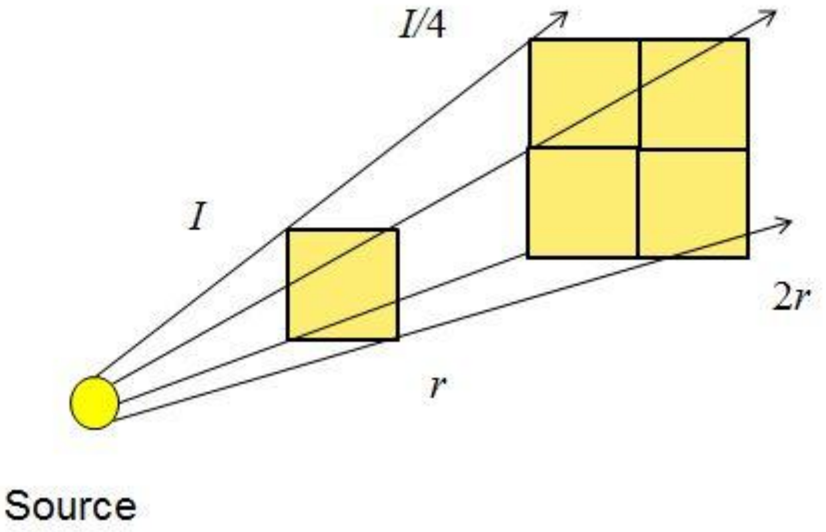


Figure 128 Inverse square law

This is called the **Inverse Square Law**. The equation for the inverse square law is:

$$\frac{I}{I_0} = \frac{k}{r^2}$$

.... *Equation 195*

where k is some constant, and r is the radius from the point source.

This is true for all waves. Note that intensity is sometimes called **irradiance**.

15.124 Intensity Calculations

We often don't know the values of I_0 or k, the constant. So, an equation like *Equation 195* is not very useful.

However, if we have a second equation like this:

$$I_2 = \frac{kI_0}{{r_2}^2}$$

.... *Equation 196*

we can do something with the two. We can combine these in a rearranged form to give us:

$$I_1(r_1)^2 = kI_0 = I_2(r_2)^2$$ .... *Equation 197*

So, we can write:

$$I_1(r_1)^2 = I_2(r_2)^2$$ .... *Equation 198*

We can rearrange this into a ratio:

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$ .... *Equation 199*

Using Stefan's Law and Wien's Law, the intensity of light on the Sun's surface is 7.3×10^7 W m^{-2}. Your answer to Question 15.12.3 is lower than this because not all the radiation is absorbed by the Earth's surface. Some is reflected by clouds and water.

15.125 Photovoltaic Cells

This garden lamp has a rechargeable battery in it that is charged up by sunlight during the day. At night the battery is discharged through the LED lamp (*Figure 129*).

Figure 129 A photovoltaic cell

The theory of photovoltaic effect is described briefly in Topic 14E Tutorial 3. The circuit symbol for a photovoltaic cell is:

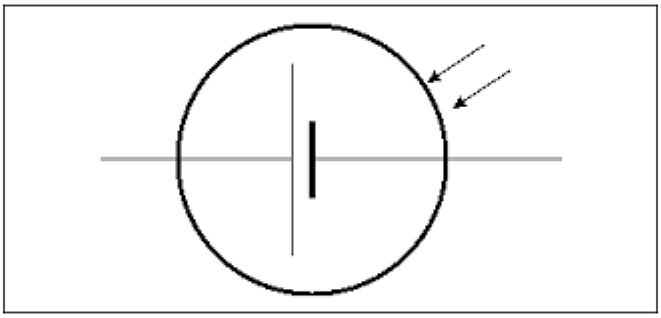


Figure 130 Symbol for a photovoltaic cell

Solar cells are not very efficient, about 10 % at the most. To get a useful amount of power, we need to have a large **array**. You can see large arrays of solar cells on the roofs of houses. They can feed batteries to store energy for later use. More commonly they are connected to the mains through an **inverter**, a device that converts the **direct**

current output of the solar cell into **alternating current** of the mains. The idea is shown in the picture below (*Figure 131*):

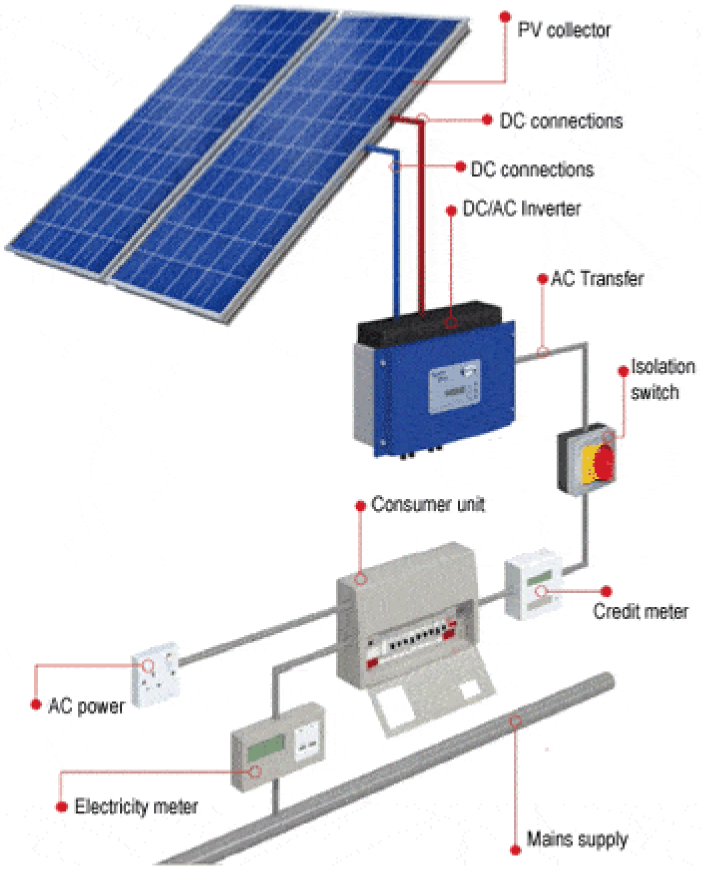


Figure 131 A photovoltaic array in a home installation (Image from www.powermyhome.co.uk)

The isolation switch allows the inverter to be isolated from the mains, in case something goes wrong. The current then passes through an electricity meter to work out how much electricity has been generated by the photovoltaic array. Then the credit meter is connected to the consumer unit.

If the output of the photovoltaic array is greater than the current being used in the house, the surplus is sold to the electricity board.

The output voltage of a typical photovoltaic cell is about 0.5 V. To get a useful voltage, the cells need to be arranged **in series**, like this (*Figure 132*):

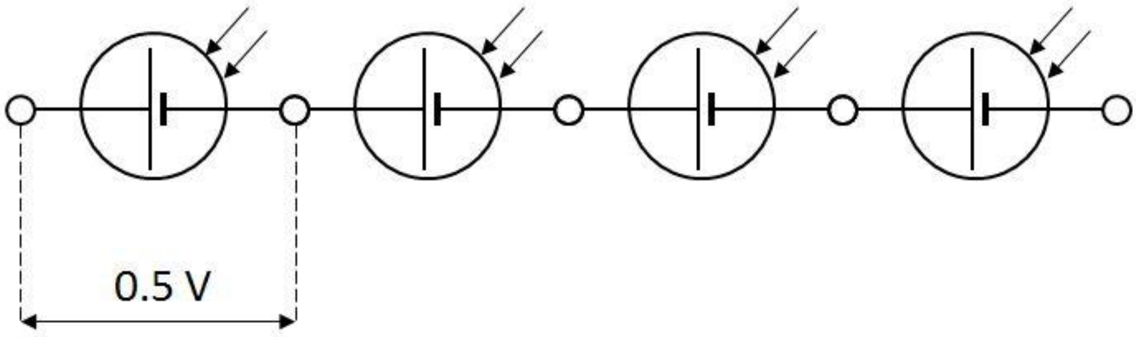


Figure 132 Photovoltaic cells in series

A typical photovoltaic cell will give a current of about 2.5 A. If we want a bigger current, we can place the solar cells in **parallel** like this (*Figure 133*):

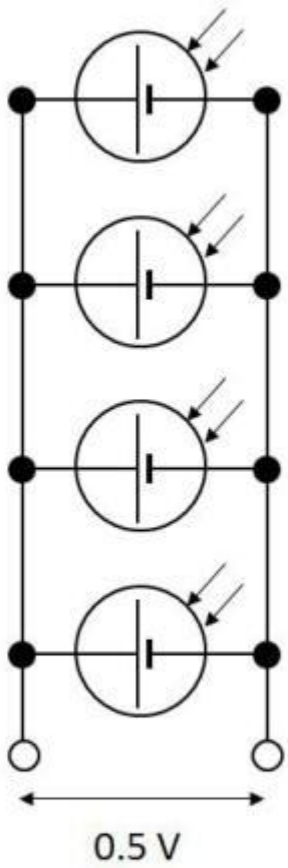


Figure 133 Photovoltaic cells in parallel

A single series array would give out 240 V at 2.5 A.

From your answer to Question 15.12.6, you can see that a very large number of solar cells is needed. Therefore, solar panels are expensive.

Solar cells are about 10 % efficient. Therefore, they can give out just one tenth of the solar radiation that falls on them.

In an array of photovoltaic cells, there are also parallel **diodes** (*Figure 134*):

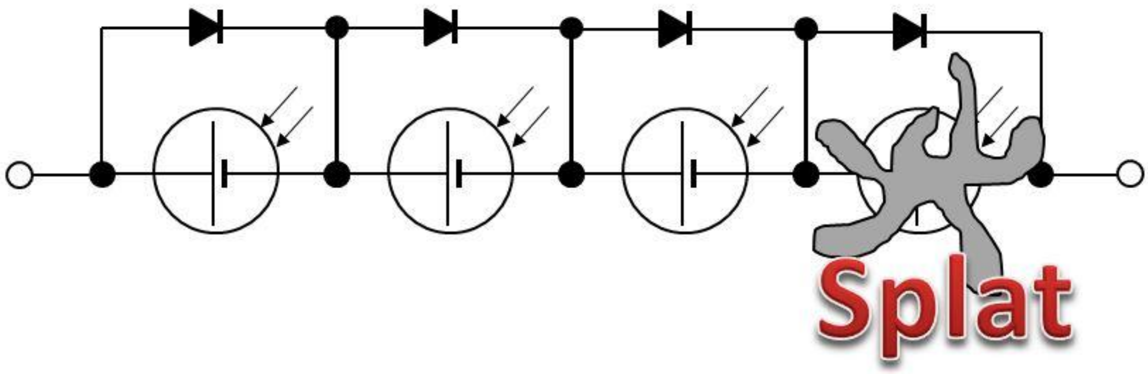


Figure 134 Diodes are placed in parallel across photovoltaic cells

Suppose a mess appeared on one of the cells. Without a parallel diode, the current would flow though the cell, which would act as a (rather poor) diode, and could get hot. The diodes by-pass the affected cells.

15.126 Wind Turbines

Wind turbines are a familiar feature on our landscape. They convert the kinetic energy from the wind into electrical energy to feed into the local grid. Large amounts of power can be generated by large arrays of turbines called wind farms. Some of these are off shore, such as the examples shown below that are just off the Belgian coast (*Figure 135*).

Figure 135 Off shore wind turbines (Image by Hans Hillewaert - Wikimedia Commons)

Although wind turbines are regarded as modern technology, the use of wind to power devices is many hundreds of years old. In the UK the technology was used to grind corn for bread flour. Wind was also used to power pumps in the East of England and in the Netherlands. Wind turbines to power electrical generators were built as early as 1887.

Wind turbines made in two different types:

- VAWT - vertical axis wind turbine (*Figure 136*).

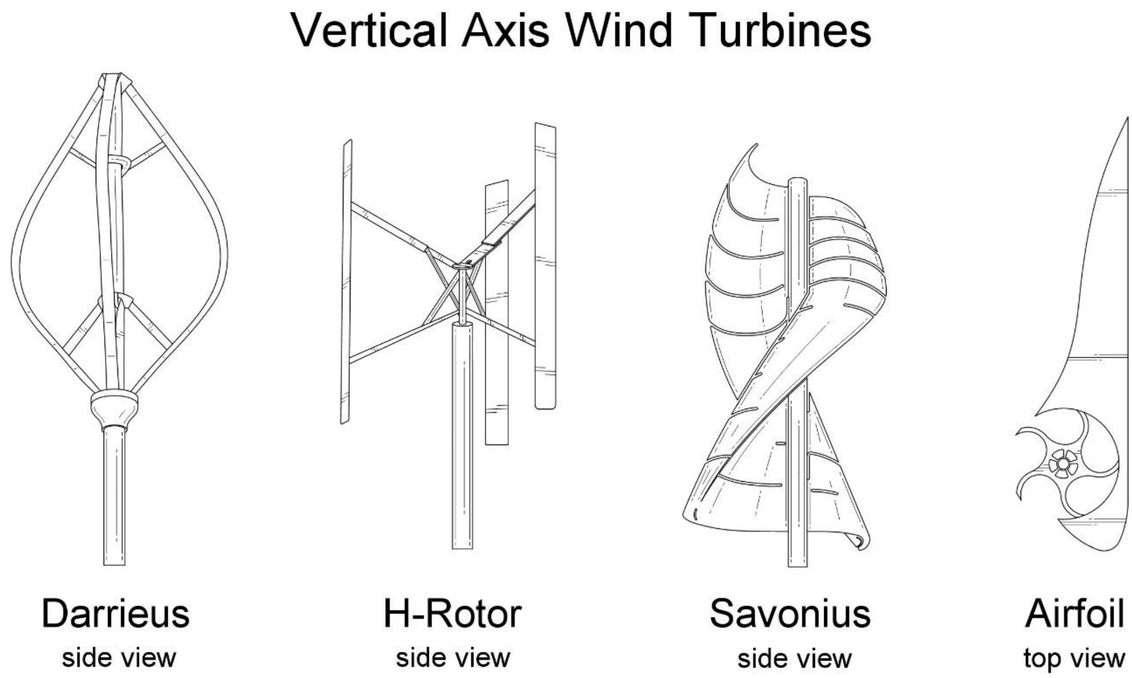


Figure 136 Vertical axis wind turbines (Littleengine0928 - Own work)

- HAWT - horizontal axis wind turbine.

The most common is the modern HAWT.

The picture below shows how a large wind turbine is made (*Figure 137*):

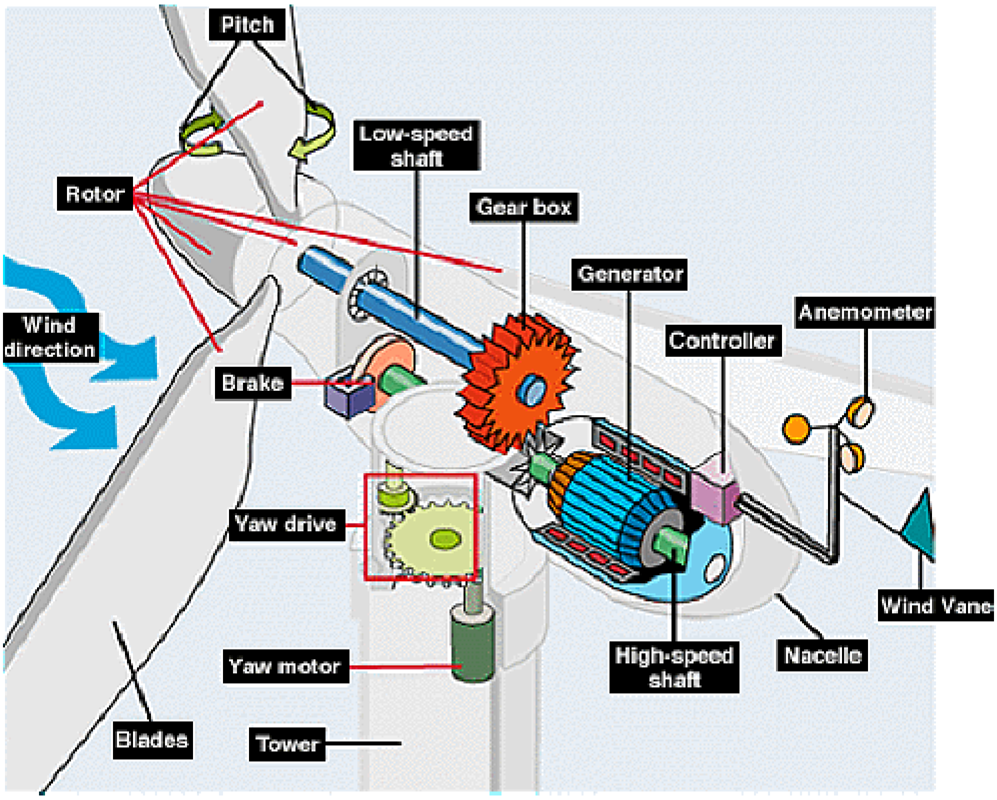


Figure 137 A horizontal axis wind turbine

The wind spins the **rotor** at a rate of about 1 revolution per second (1 Hz). The rotational speed is increased considerably by a **gearbox**. The **high speed shaft** turns the **generator** that generates the electricity. The **yaw drive motor** points the assembly into the wind, as it changes direction.

The whole assembly is contained in a housing called a **nacelle**. The nacelle is mounted on a tower that can be over 100 m high.

The **anemometer** detects wind speed. If the wind speed gets too high, damage can be done to the machine. In this case, the blades are feathered so that the wind passes easily without turning the rotor, and the brake is applied.

In calm conditions, the generator acts as a motor to turn the turbine slowly. This process is called **barring** and prevents the low speed shaft being distorted by the weight of the rotor. It explains the seemingly absurd sight of a wind turbine turning on a calm day.

15.127 Power from a Turbine

Consider a turbine in a wind of wind speed v m s^{-1}. The density of the air is ρ kg m^{-3}. The rotor sweeps an area of A m^2(*Figure 138*)

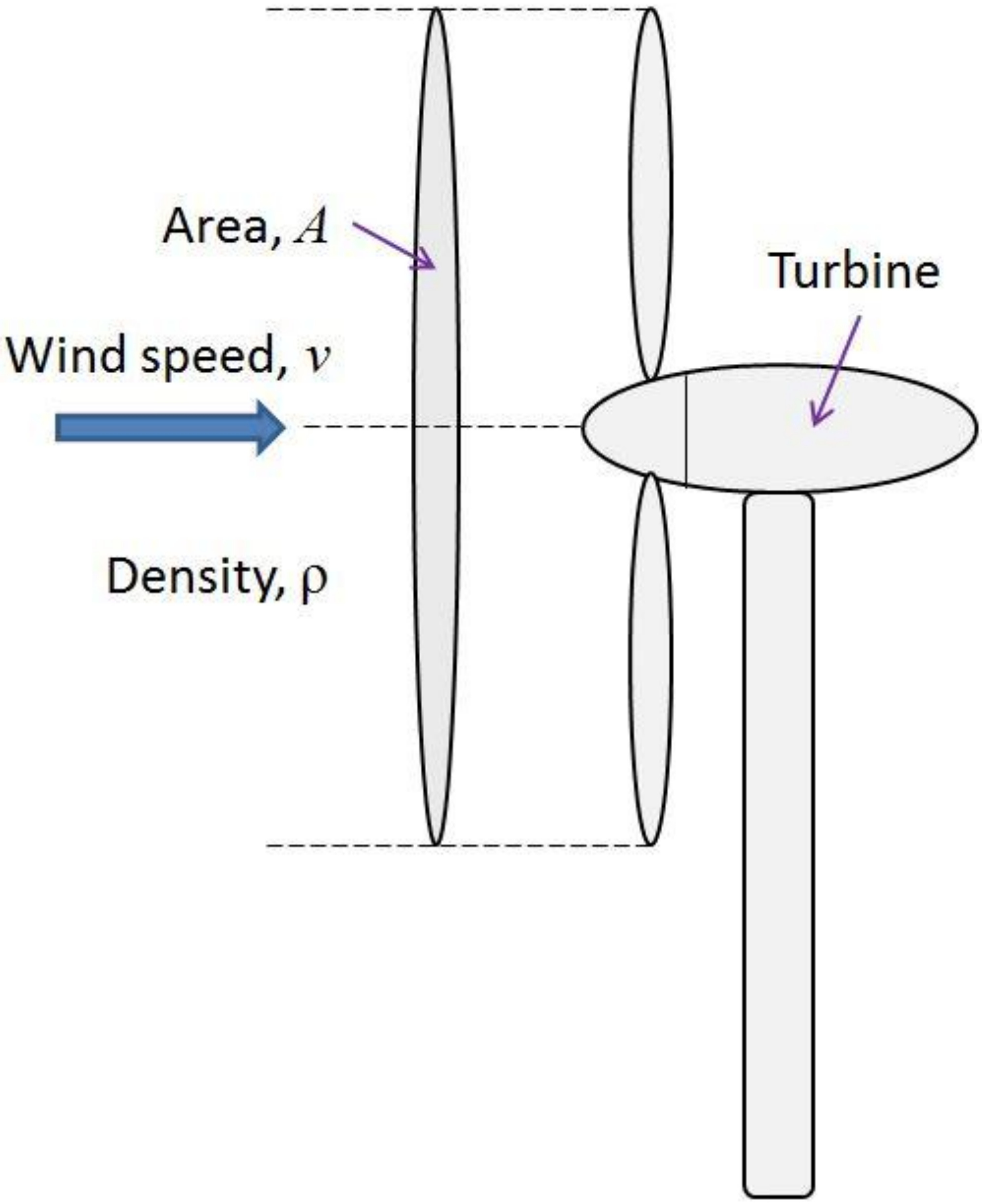


Figure 138 Working out the power of a wind turbine

Every second a cylinder of volume V m^3 of air passes the turbine. This cylinder has a length of v m and area A m^2.

$$V = Av \qquad \text{............. Equation 200}$$

The air has density ρ kg m^{-3}. We know that mass = density × volume. Therefore:

$$m = \rho A V$$ .... *Equation 201*

So, every second, $\rho A V$ kg of air passes. We know that the kinetic energy of any mass is:

$$E_k = \frac{1}{2}mv^2$$

.... *Equation 202*

The kinetic energy **every second** of this mass of air is therefore:

$$E_k = \frac{1}{2}(\rho A v)v^2$$

.... *Equation 203*

And this tidies up to:

$$E_k = \frac{1}{2}\rho A v^3$$

.... *Equation 204*

Remember that this is kinetic energy **every second**, which is **power**. So, our final equation is:

$$P = \frac{1}{2}\rho A v^3$$

.... *Equation 205*

There is a **maximum power** that can be absorbed by the machine. If the wind speed doubled from 10 m s^{-1} to 20 m s^{-1}, the machine would give out 8 times the power (in this case 6 MW), which is quite sufficient to overheat the generator and set it on fire. Wind speeds of 20 m s^{-1} (gale) are common on windy days.

To prevent damage on windy days, the turbine is shut down, and the blades are **feathered** so that the wind has no turning effect. The idea is shown here (*Figure 139*):

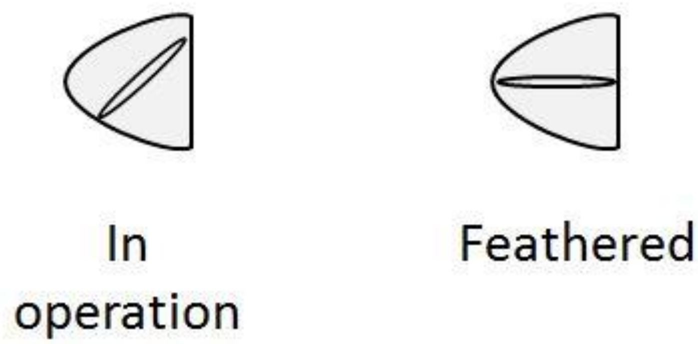


Figure 139 Feathering

Pilots of multi-engine aeroplanes do the same if one of the engines fails. If the propeller is not feathered, it will "windmill", trying to drive the failed engine. This will increase the drag and make the aeroplane much harder to control. If it's a single engine aeroplane, the pilot will still feather the propeller. This will increase the range as the pilot glides downwards to make the emergency landing. If the propeller is fixed pitch (the blades cannot be moved), feathering cannot be done.

Wind turbines do NOT extract all the energy of the wind. Theory suggests that the maximum power achieved is no more than 16/27 (59.3 %) of the wind power. Therefore, *Equation 205* can be modified to:

$$P = \frac{8}{27}\rho A v^3$$

.... *Equation 206*

(8/27 = 16/27 × 1/2)

Since wind is free, engineers don't worry too much about this. You will not be asked about this equation in the exam.

15.128 Hydroelectric Power

When water falls from a height, **potential** energy is turned to **kinetic** energy. Some of this kinetic energy can be extracted to generate electrical energy. Water power has been used since ancient times, so connecting a waterwheel to a generator was an obvious thing to do when generators were invented in the second half of the nineteenth century. The earliest hydroelectric power station was installed in 1878 by the British engineer and businessman, William George Armstrong (1810 - 1900) at Cragside, his house in the Northumberland town of Rothbury.

Hydro-electric power can be used to describe tiny installations like the one below (*Figure 140*), to enormous ones with a capacity of 22500 MW in China.

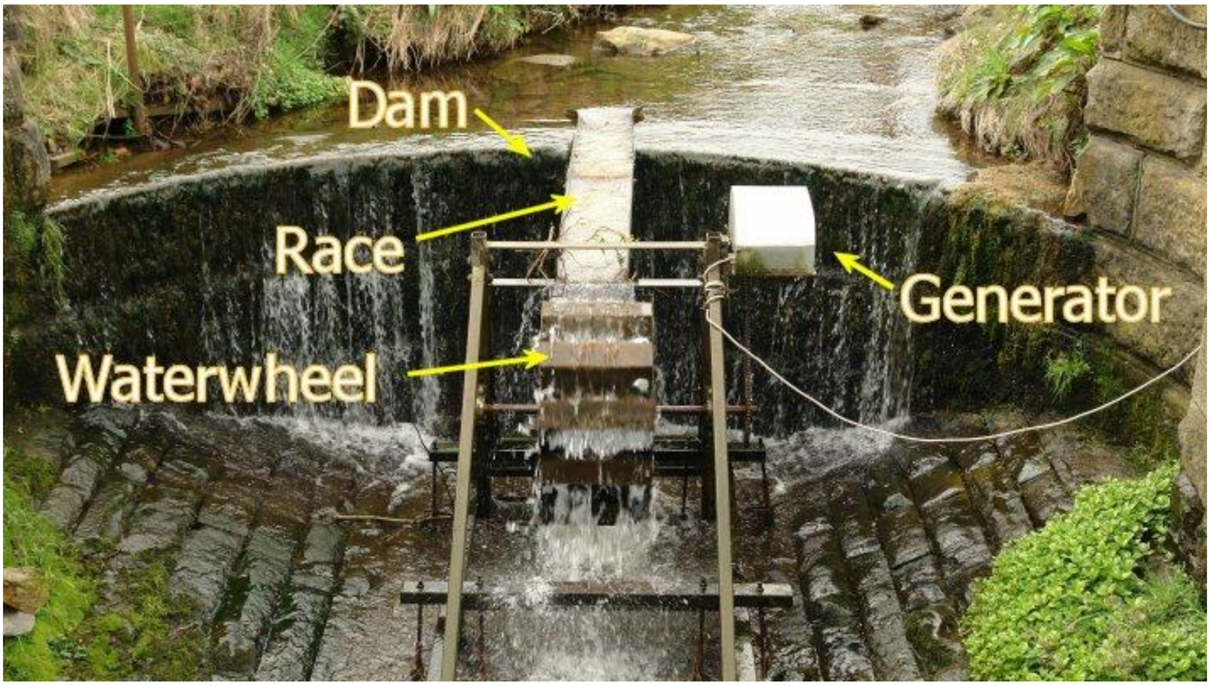


Figure 140 A tiny hydroelectric generator

The principle is the same. A **dam** stores water on a **reservoir**. The water has **gravitational potential energy**. Water is guided along a trough (or **race**) to a water turbine. In a larger installation, the water is taken to the turbine through large pipes called **penstocks**. The **kinetic** energy of the fast-flowing water is converted to **electrical** energy. Once the water has passed through the turbine, it flows through a **tailrace** to the river downstream of the dam. The picture (*Figure 141*) shows the idea:

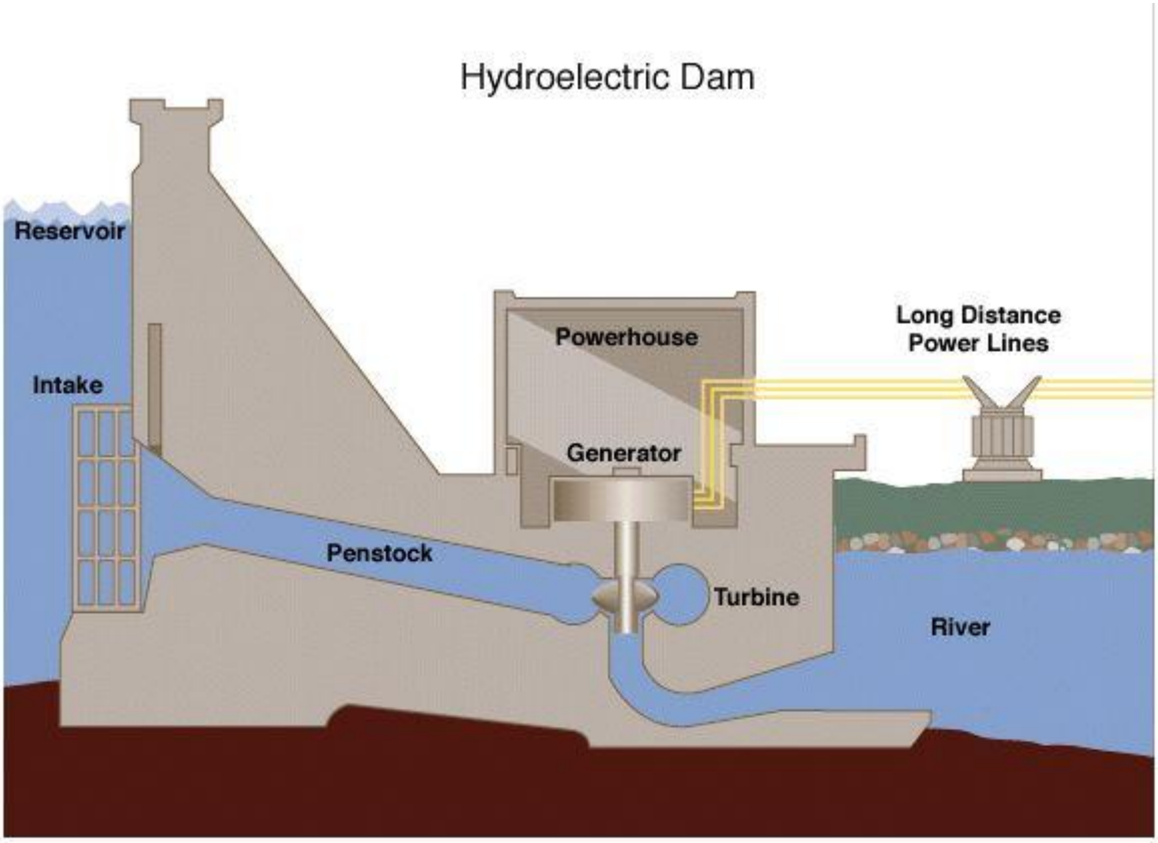


Figure 141 A hydroelectric power station (Image from Tennessee Valley Authority, Wikimedia Commons)

We will look at the physics of the hydroelectric power station using the diagram below (*Figure 142*):

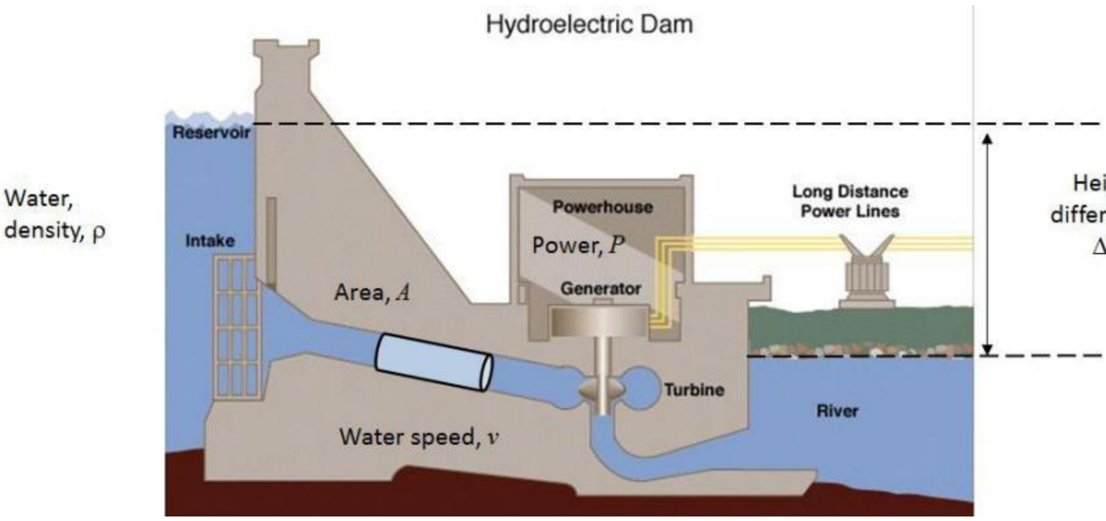


Figure 142 Power of a hydroelectric generating station

The height difference between the surface of the reservoir and the level of the river is h m. Water is flowing through the penstock of area A m^2 at a speed of v m s^{-1}. The flow rate is the volume, V m^3, of water flowing per second can be written as $\Delta v/\Delta t$, or r.

$$\frac{\Delta V}{\Delta t} = Av$$ ……….. *Equation 207*

So, we can write:

$$r = Av$$ ……….. *Equation 208*

The gravitational potential energy is:

$$E_p = mg\Delta h$$ ……….. *Equation 209*

This gets converted to kinetic energy:

$$E_k = \frac{1}{2}mv^2$$ ……….. *Equation 210*

By the Law of Conservation of Energy, we can write:

$$mg\Delta h = \frac{1}{2}mv^2$$ ………. *Equation 211*

The m terms obligingly cancel out:

$$g\Delta h = \frac{1}{2}v^2$$ ………… *Equation 212*

And rearranging:

$$v^2 = 2g\Delta h$$ ………… *Equation 213*

Now we know from the wind turbine that:

$$P = \frac{1}{2}\rho A v^3$$

.... *Equation 214*

And we can write this in an untidy form:

$$P = \frac{1}{2}(\rho A v)v^2$$

.... *Equation 215*

Now we can substitute for v^2:

$$P = \frac{1}{2}(\rho A v)2g\Delta h$$

.... *Equation 216*

And this tidies up to:

$$P = \rho A v g \Delta h$$ .... *Equation 217*

The term Av is the volume every second, so we can write:

$$P = \rho \frac{\Delta V}{\Delta t} g \Delta h$$

.... *Equation 218*

The term $\Delta v/\Delta t$ is the **flow rate**, r m^3 s^{-1}. So, we can now write an expression:

$$P = \rho r g \Delta h$$ .... *Equation 219*

This assumes that the power station is 100 % efficient, which it isn't. It may be 80 % (0.80) efficient. We can modify the relationship to take this into account:

$$P = \rho rg\Delta hk$$ .... *Equation 220*

The term k is a constant with a value between 0 and 1.

There are three main types of hydroelectric power station:

- The conventional hydroelectric power station (which we have discussed above);
- The tidal power station which uses the rise and fall of the tides;
- Pumped storage (in which water is pumped up from a low reservoir to a higher reservoir).

15.129 Tidal Power Stations

Britain's coasts have some of the highest **tidal ranges** in the world. It makes sense to try to extract energy from the huge volumes of water that daily surge up and down estuaries like that of the Severn. A causeway can be built to make a large lagoon as a reservoir. At a certain point in the causeway, a dam can be built with a turbine hall built into it, as shown in the diagram (*Figure 143*):

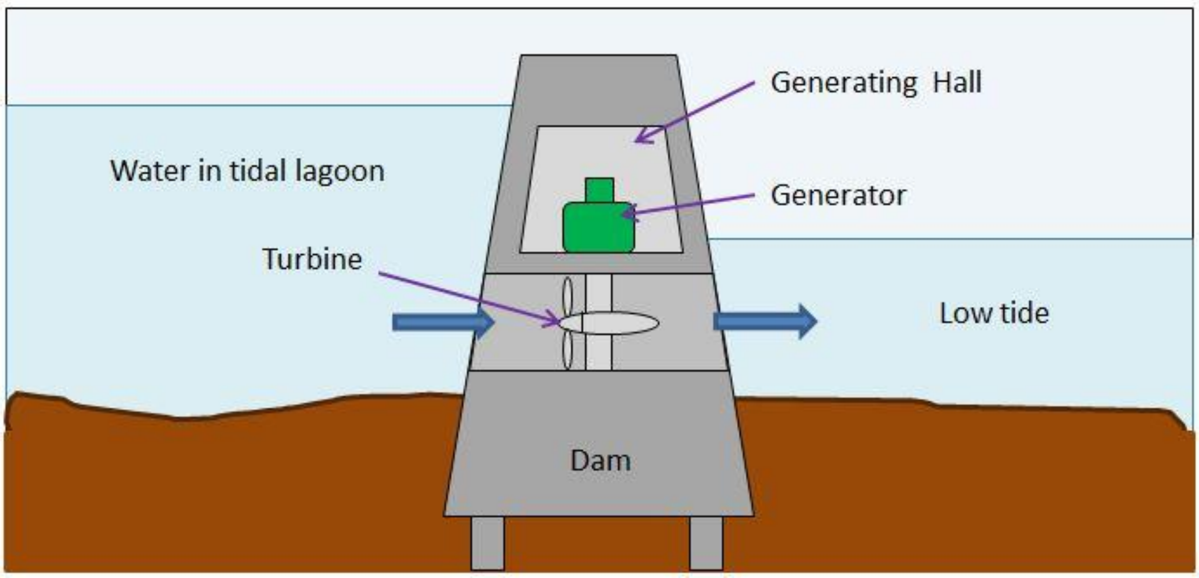


Figure 143 Tidal power station

Water flows into the **tidal lagoon** (or **storage pool**) on the incoming tide. As it does so, it turns a generator to generate electricity. The water is stored in the tidal lagoon, so that as the tide turns and the level of the sea drops, the turbines will turn the other way to produce electrical energy.

There is nothing new in this. **Tide mills** have been used in Europe for many centuries. The first tidal power station was built on the mouth of the Rance in France in 1966. The main problem is that such installations are very challenging and expensive to build.

In some areas where there are narrow channels and strong currents, a free-standing water turbine can be set up on a suitable foundation. In effect it's a water version of a wind-turbine. Such a device is shown, called a **sea-generato**r in the picture below (*Figure 144*):

Figure 144 Tidal generator (Image by Fundy - Wikimedia Commons)

These kinds of devices are clearly much less expensive to install.

15.1210 Pumped Storage Power Stations

In Wales there are two **pumped storage** power stations:

- Tan-y-grisiau (near Blaenau Ffestiniog), commissioned in 1963;
- Dinorwig (near Llanberis), commissioned in 1984.

Another pumped storage power station was considered for Exmoor in Devon, but the Central Electricity Generating Board did not proceed with the plans.

The power station installation is similar to that of a conventional hydroelectric scheme, except that the water flowing from the tailraces does not flow away down a river. It is stored in a lower reservoir. The idea is shown in this diagram (*Figure 145*):

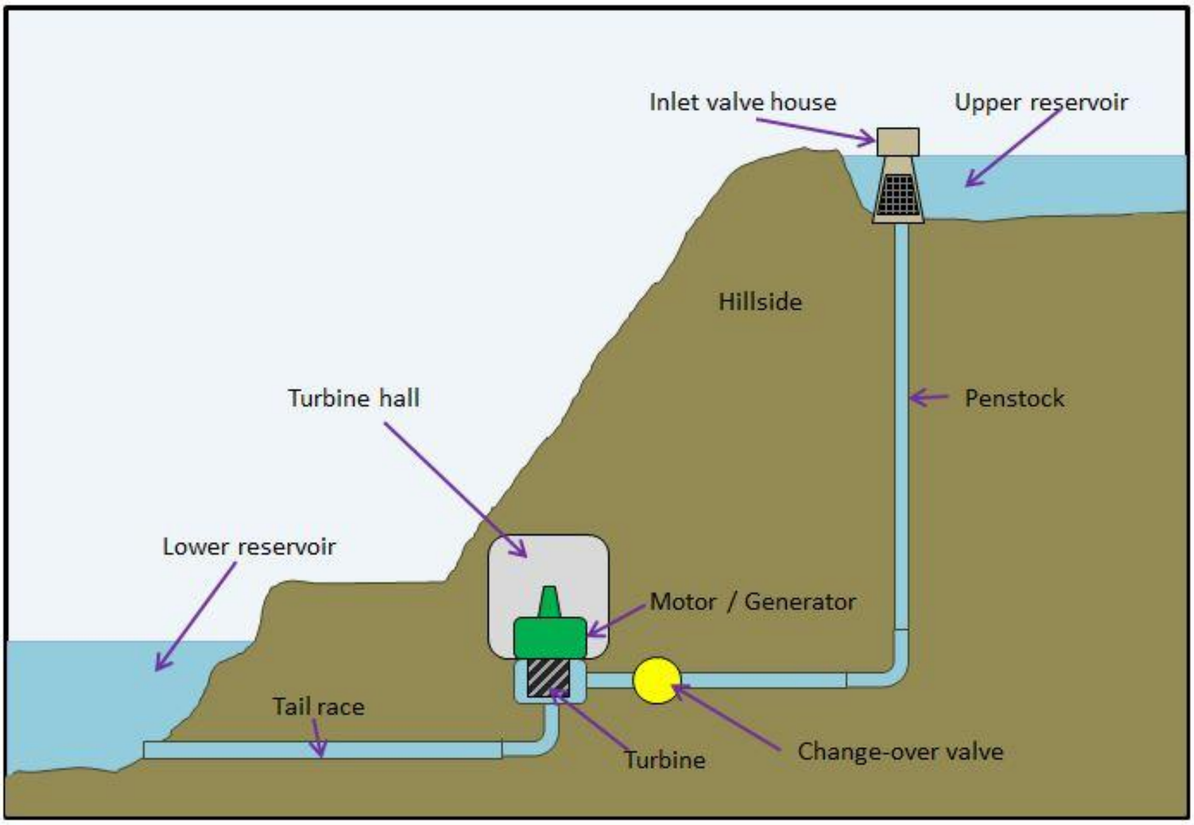


Figure 145 A pumped storage power station

The whole idea of this scheme is to soak up excess energy from the National Grid when the load is light (during the night when most people are in bed). Power stations give out all or nothing; it is difficult to get them to reduce their output in periods of reduced demand. Therefore, pumped storage power stations are a good way of maintaining the load.

During quiet periods, the generators become motors, and the turbine is a pump. So, water is pumped up from the lower reservoir to the upper reservoir. When the reservoir is full, the power station is on standby for a peak in demand. Within seconds, valves open allowing the water to fall from the upper reservoir. The water passes through turbine (which was the pump) and the motors become generators.

When demand has reduced, the generators are switched over to become motors again.

The efficiency of such a power station is about 70 %. Therefore, for each 100 MW h (megawatt-hours) used in pumping the water, 70 MW h are generated by the falling water.

15.1211 Nuclear Enrichment and Breeding

If you need to review the process of fission, have a look at Topic 12 Tutorial 7. In this section, we will look at the processing of uranium to make it useful in nuclear power station.

Uranium comes in a number of isotopes, the most common of which is:

$$^{238}_{92}U$$

The fissile isotope is:

$$^{235}_{92}U$$

This isotope represents 0.7 % of the total uranium.

Uranium is a heavy metal and decays by alpha decay. The half-life of uranium-238 is 4500 million years, while the half-life of uranium-235 is 704 million years. The fissile nature of uranium-235 is completely separate from its radioactive decay. All isotopes of uranium share the same chemical properties in that the metal reacts readily with other elements. Pure uranium metal rapidly gains a coat of uranium oxide. It is a dense and hard metal which is a poor conductor of electricity.

Uranium-235 is **fissile** when it captures a **thermal** neutron. Thermal neutrons have a **kinetic energy** equivalent to an infra-red photon, about 1 eV (1.6×10^{-19} J). The neutron is travelling at about 14 000 m s^{-1} - fast enough for us, but as far as particle movement is concerned, a slug. The uranium nuclei are not smashed by this interaction. They form a wobbly drop that splits into two or three fission products. When the uranium splits, a large amount of energy is released in a **chain reaction**. On average three neutrons are released, to be captured by other uranium-235 nuclei. In this case, the energy is released very quickly and in an uncontrolled way. The result is a powerful and destructive explosion.

If the chain reaction is controlled, so that one thermal neutron is captured by one nucleus, the heat can be used to heat up a coolant, which in turn boils water, to turn a steam turbine. This turns a generator to produce electricity in the conventional way. Please see Topic 12 Tutorial 8.

Uranium is mined as an ore. This has a low concentration of uranium (<1.0 %) and is quite useless for energy production. The ore is processed into more concentrated (60 %) uranium oxide called yellowcake. In yellowcake, the concentration of uranium-235 is low. Therefore, the probability of a neutron capture event is too low for there to be energy from fission. Therefore, further processing needs to be done to convert the uranium oxide into uranium hexafluoride for **enrichment**. Uranium hexafluoride is a crystalline solid at room temperature, to it needs to be heated to turn it into a gas. For enrichment the gas is passed into a **gas centrifuge** (*Figure 146*).

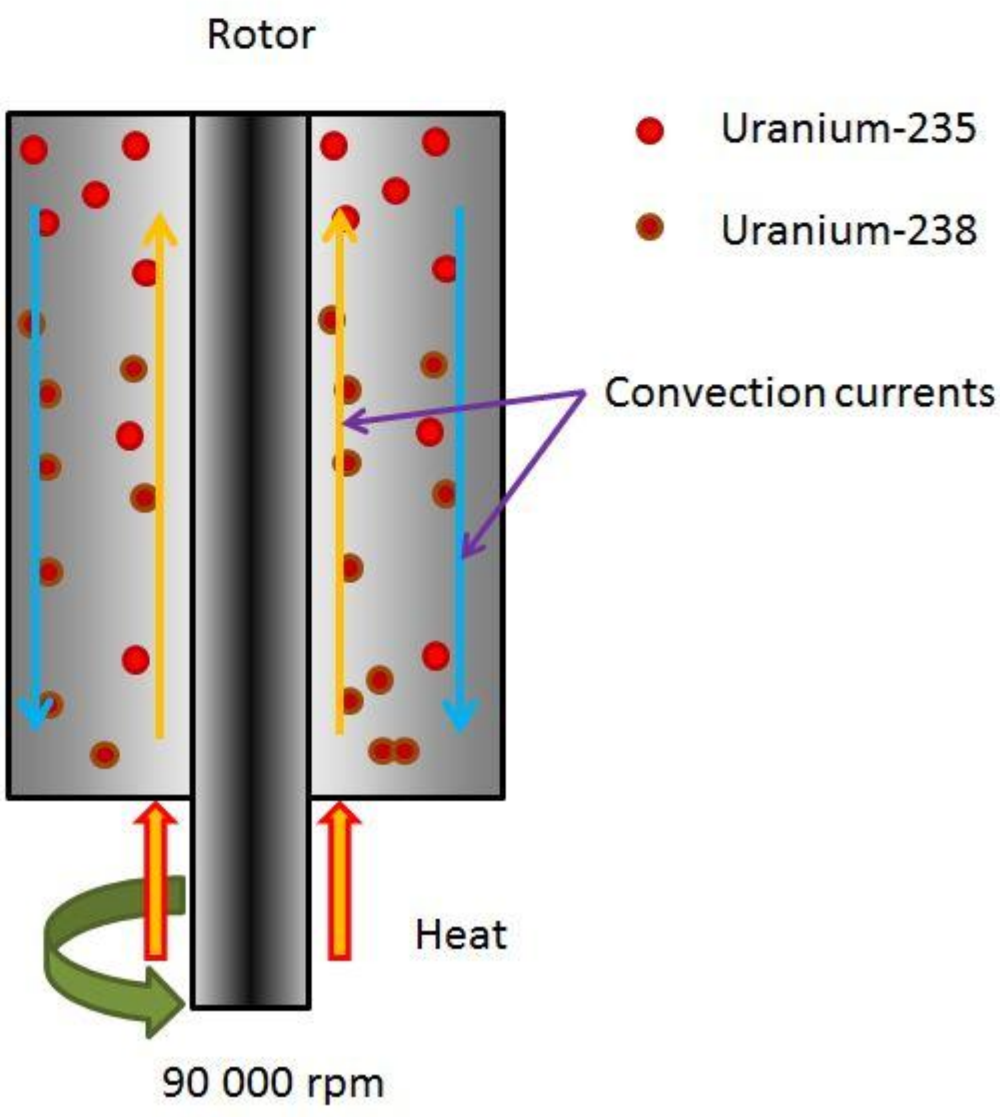


Figure 146 Gas centrifuge

In this machine, the uranium hexafluoride made with the heavier uranium-238 has a greater centripetal force, so the molecules pass towards the outside of the drum. The lighter uranium-235 hexafluoride molecules remain closer to the centre of the machine and can be extracted. Additionally, the heat applied causes convection currents in which the lighter uranium-235 hexafluoride molecules are carried to the top by the convection currents.

The rate of rotation 90 000 rpm (1500 s^{-1}) has to be very high to separate the two different hexafluorides, because the masses are very close. This compares with 1500 rpm for a washing machine, or 30 000 rpm for a gas turbine.

The enriched uranium needs to be 20 % uranium-235 for **reactor grade uranium,** while for highly enriched uranium (**weapons grade**), the concentration needs to be up to 80 %. The highly enriched uranium is also used in research reactors to produce isotopes for medical use.

The uranium is converted to an **oxide** before insertion into fuel rods. This is because uranium metal is very reactive, and melts at 1400 K, a temperature that can be reached in a reactor core. The oxide is compressed to form **pellets** which are, in turn, loaded into **fuel rods**.

The uranium-238 left over is called **depleted uranium**. It is stored as uranium hexafluoride (UF_6). The storage has to be done with a great deal of care as UF_6 is highly toxic and potentially damaging to the environment. It reacts readily with water vapour in the air to produce uranyl fluoride (UO_2F_2) and hydrogen fluoride (HF). Hydrogen fluoride reacts readily with water to form hydrofluoric acid which is highly corrosive. In the UK, it is estimated that there are 30 000 tonnes of stored uranium hexafluoride.

Metallic uranium is a dense metal, density 19.1×10^3 kg m^{-3}, compared with 11.34×10^3 kg m^{-3} for lead. It is reactive with oxygen, so the metal needs protection to separate it from the oxygen in the air. Uses have included or include:

- Shielding in industrial radiation cameras;
- Trim weights in aircraft. The early Boeing 747 jumbo jets suffered from **flutter** (a potentially destructive resonance in the wings) which was cured by application of uranium weights.
- Keel in a yacht. The heavy keel was thinner than a normal lead keel. However, the uranium keel was later replaced by a conventional lead one;
- Colouring of glass and enamel;
- Munitions.

The use of depleted uranium in munitions is controversial. While uranium is only weakly radioactive, it has high toxicity. Fragments can pollute an environment and can be ingested.

15.1212 Breeder Reactors

Plutonium is another radionuclide with fissile isotopes. It is a dull grey metal that is highly reactive and rapidly oxidises. It has a density of 16.63×10^3 kg m^{-3}. Plutonium has a number of isotopes including the following that decay with the following half-lives:

- Pu-238 - 88 years by alpha decay.
- Pu-239 - 24 100 years by alpha decay (fissile).
- Pu-240 - 6560 years by alpha decay or spontaneous fission.
- Pu-241 - 14 years by beta minus decay (fissile).
- Pu-244 - 82 million years by alpha decay or by spontaneous fission.

Therefore, plutonium isotopes are found only in trace amounts in nature.

The plutonium isotope used in reactors is **plutonium-239**. This isotope is produced in a reactor. The steps in production is shown in the following steps:

1. The uranium nucleus **captures** a neutron from a fission event:

$$^{238}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{239}_{92}\mathrm{U}$$

2. The uranium-239 decays with beta decay to **neptunium**:

$$^{239}_{92}\mathrm{U} \rightarrow ^{239}_{93}\mathrm{Np} + ^{\;\,0}_{-1}\mathrm{e}^{-} + \bar{\nu}_e$$

The half-life for this decay is 1410 s (23.5 min).

3. The neptunium decays by beta minus decay to **plutonium**.

The half-life of the decay to plutonium 239 is 2.35 days (2.03×10^5 s).

In a fast breeder reactor, there is a certain amount of uranium-235 the fission of which provides neutrons to be captured (as well as contributing to the energy of the reactor). While we learn that each fission event gives out 3 neutrons, 2 of which are absorbed, this figure is an average. So, some neutrons are captured by plutonium nuclei:

$$^{1}_{0}\mathrm{n} + ^{239}_{94}\mathrm{Pu} \rightarrow [^{240}_{94}\mathrm{Pu}] \rightarrow \text{fission products}$$

As with uranium-235, the fission products are random pairs, which release on average 3 neutrons. The fission products are the result of 73 % of the neutron capture events. The other 27 % result in a plutonium-240 nucleus in an **excited state**. It loses the excess energy by emission of a **gamma photon**.

$$^{1}_{0}\mathrm{n} + ^{239}_{94}\mathrm{Pu} \rightarrow ^{240}_{94}\mathrm{Pu} + ^{0}_{0}\gamma$$

The plutonium-240 decays by alpha decay by alpha decay.

The plutonium fuel builds up in the reactor and **doubles** every 10 years. So, after 10 years, the idea is that the plutonium fuel can be removed and placed in a second reactor. Up to 75 % of the uranium-238 is used in conversion to plutonium

Fast breeder transfer their energy by liquid sodium. The sodium melts at a temperature of 98 °C. Therefore, it is liquid at reactor temperatures. It does not slow down (**moderate**) the neutrons as water does. Sodium has a high **specific heat capacity**, so can transfer a lot of energy to a **heat exchanger** where water is boiled to turn a turbine and generate electricity. It is essential that the liquid sodium does not come in contact with the water, otherwise a violent reaction leading to an explosion will result.

15.1213 Fusion Power

The most powerful fusion bombs used reactions that are thought to go on in the Sun. If it could be controlled in the same way as a fission reaction can be controlled, it would be possible to get huge amounts of energy from very small amounts of hydrogen fuel. The only waste product would be helium which is an inert gas.

A possible fusion reaction is:

$$^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + \text{energy}$$

It is not simply a case of sticking some deuterium and tritium together and shaking it up. Each nucleus has to have sufficient energy to:

- Overcome electrostatic repulsion from the protons;
- Overcome the repulsive strong force which is found outside the region of the strong force.

This means that the gases have to be heated to a very high temperature, 100 million Kelvin. As all matter at this temperature exists as an ionised gas (**plasma**), it has to be **confined** in a very small space by powerful magnetic fields. This is called **magnetic confinement**.

A considerable amount of effort has been made to make fusion work to generate electricity. A fusion reactor would be made to boil water to turn a turbine. Fusion has occurred, but the energy put in to cook the gases enough to make them fuse is far greater than the energy got out by a fusion reaction. The prize of vast amounts of energy is still being sought. A possible plan for a fusion reactor is shown below. It is based on a **torus**, a ring shaped like a doughnut (*Figure 147*).

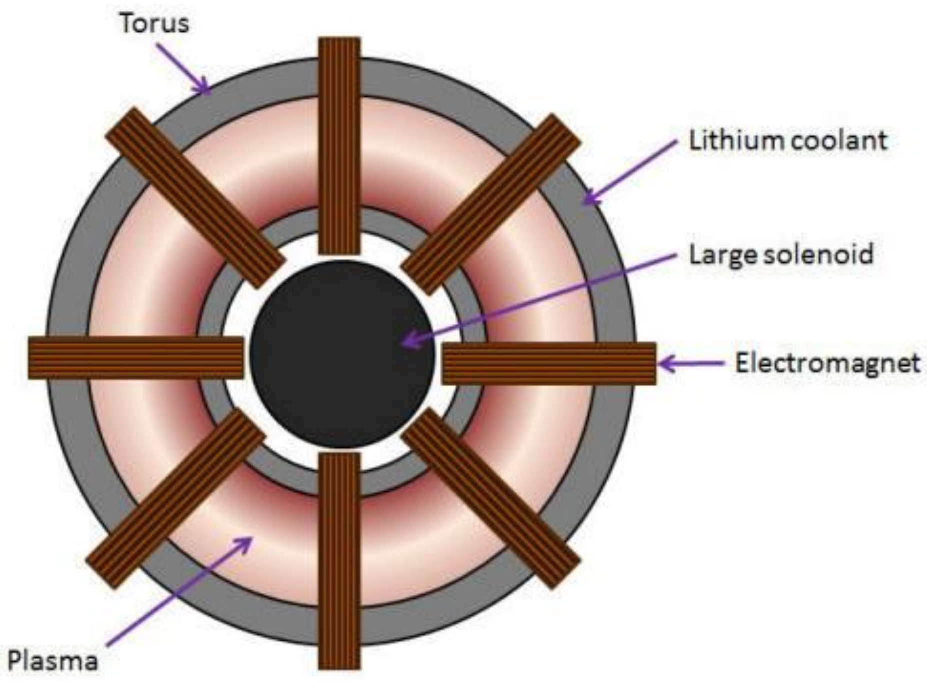


Figure 147 A torus

From the side, the torus looks like this (*Figure 148*):

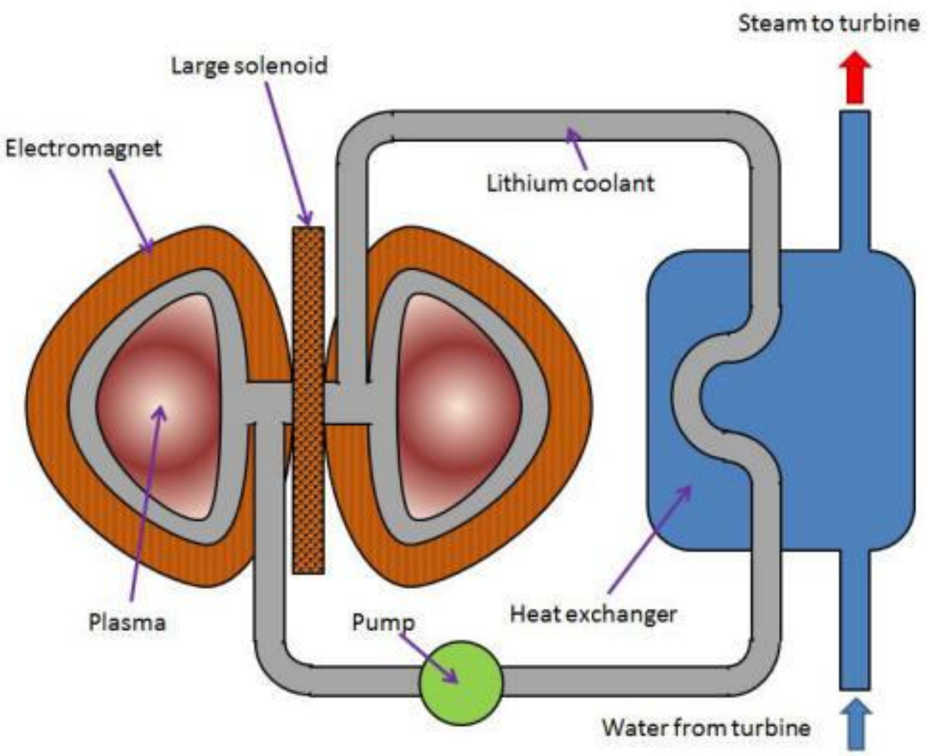


Figure 148 Side view of a torus

The **plasma** consists of nuclei that have been stripped of all orbiting electrons; they are **bare nuclei**. In the middle of the torus is a large **solenoid** (coil of wire) that acts as the **primary** of a **transformer** when an alternating current flows through it. The plasma acts as a single-turn **secondary**, so vast currents are possible. It is heated up and confined using strong magnetic fields, generated by the electromagnets. The temperature gets to about 100 million to 200 million Kelvin. More deuterium and tritium nuclei are injected at very high speed, and this causes the fusion. The plasma needs to be replenished continuously for it to be sustained.

Heat is removed from the torus using a **coolant** like liquid lithium, or water. This passes to a **heat exchanger** which boils water to steam to turn the turbines in the turbine hall to generate electricity.

That's the idea. However, there are problems that include:

- **Heating** the plasma to the temperature required takes a lot of energy.
- Keeping plasma **confined** with the magnetic field takes a lot of energy. Therefore, more energy is put in than go out.
- The plasma tends to **wriggle** and squirm in the magnetic field. If it touches the sides of the torus, it goes out immediately.
- **Neutrons** can be captured by the material of the torus. This will make the nuclei unstable, therefore radioactive. Radioactive decay will change the proton numbers, hence the elements.
- Neutrons with high kinetic energy which are not captured pass through the vessel, so containment buildings with thick walls are needed.

15.1214 Inertial Plasma Confinement

To get fusion, nuclei have to slammed together and held there by very high temperatures and pressures. In the earliest days of fusion, such conditions could only be achieved by the detonation of a fission device. The atoms were ionised into a plasma by the intense heat of a fission explosion and confined by the intense pressure of the explosion. The fusion reaction would happen in an uncontrolled manner, releasing huge amounts of energy. The amount of hydrogen nuclei would fill a small party balloon but release enough energy to cause immense damage to a wide area.

Figure 149 A fusion bomb (deactivated)

The third bomb from the left is one such device. It is a genuine fusion bomb, but it's now in a museum and has been deactivated. It would have been dropped from a high-flying bomber.

This would be useless for controlled fusion. We have seen how plasma can be confined using magnetic fields and it is not easy to do. An alternative method to holding the plasma with strong magnetic fields is to slam the hydrogen nuclei together with **laser beams**. This is called **inertial confinement**.

A small capsule containing deuterium and tritium is exposed to very intense laser light. The light has energy of 10 000 J in 1×10^{-9} s (*Figure 150*).

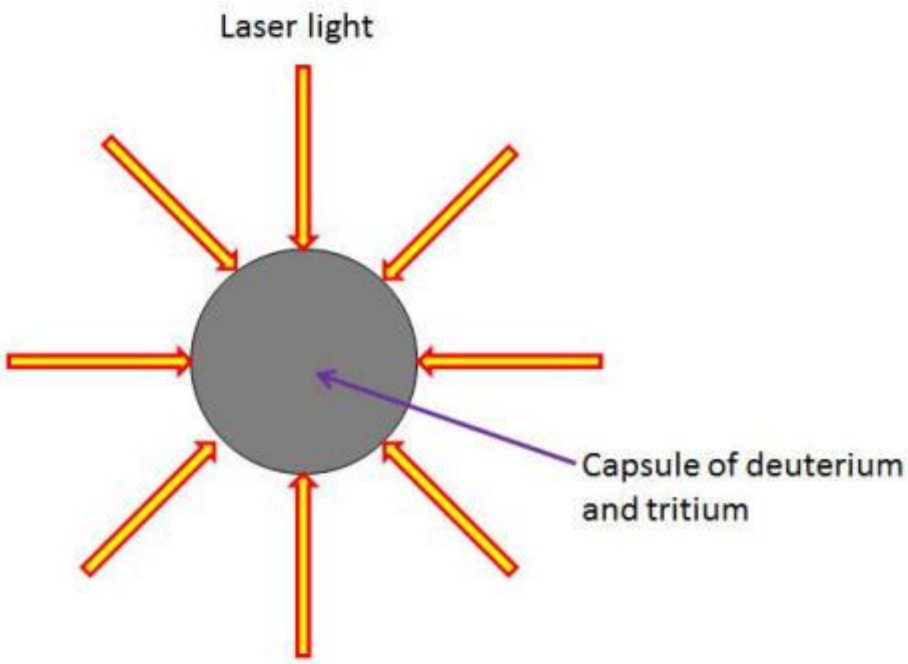


Figure 150 Inertial confinement

The intense heating effect from the laser driver means that the temperature rises very rapidly to 1 × 10^8 K. The result of this is that surface explodes like this, compressing the capsule with intense pressure (*Figure 151*):

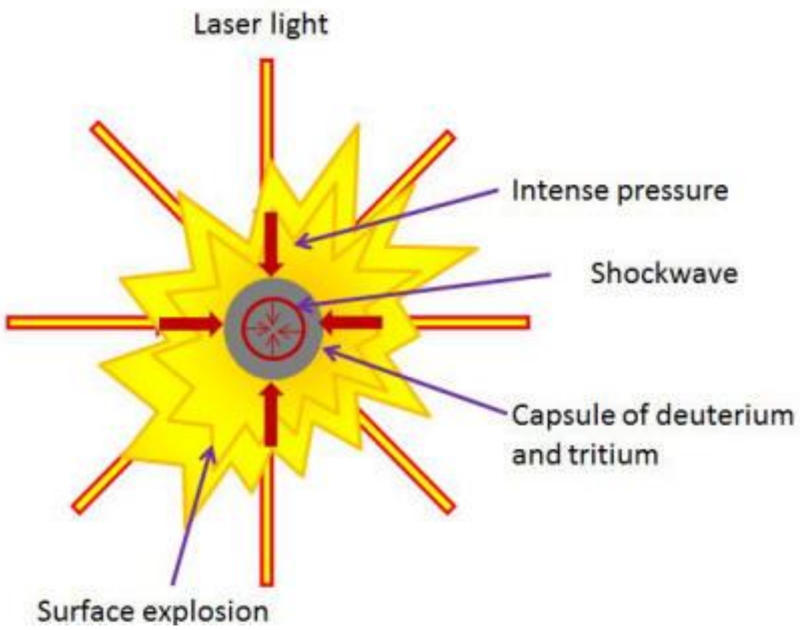


Figure 151 Action of inertial confinement

The compression arises due to Newton III (for every action, there is an equal and opposite reaction). Since there is an outward pressure, there must be an inwards pressure. The intense pressure forms a **shockwave** that is sufficient to push the nuclei together, overcoming the **strong force** that would normally push them apart. This enables the fusion to occur, releasing a large amount of energy (*Figure 152*).

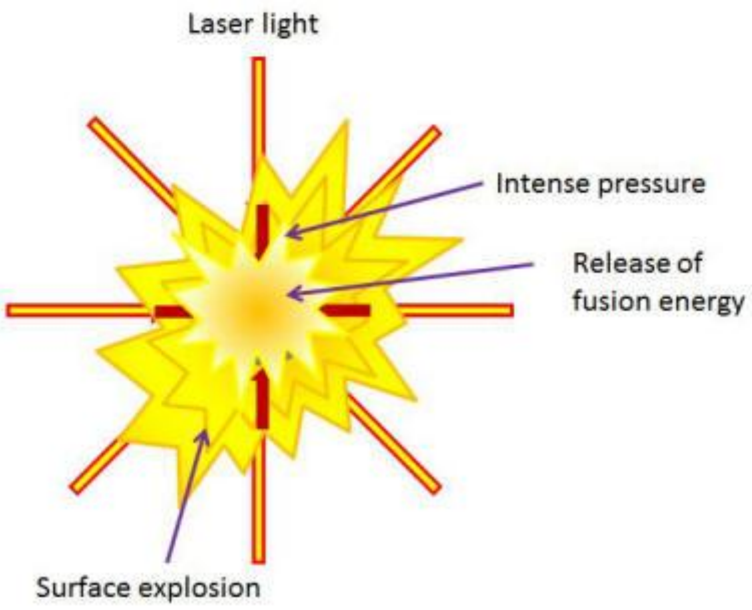


Figure 152 Fusion reaction

The shockwave compresses the material right in the centre and the fusion reaction propagates through the body of the capsule more rapidly than the capsule can expand. This explains why the confinement is **inertial**.

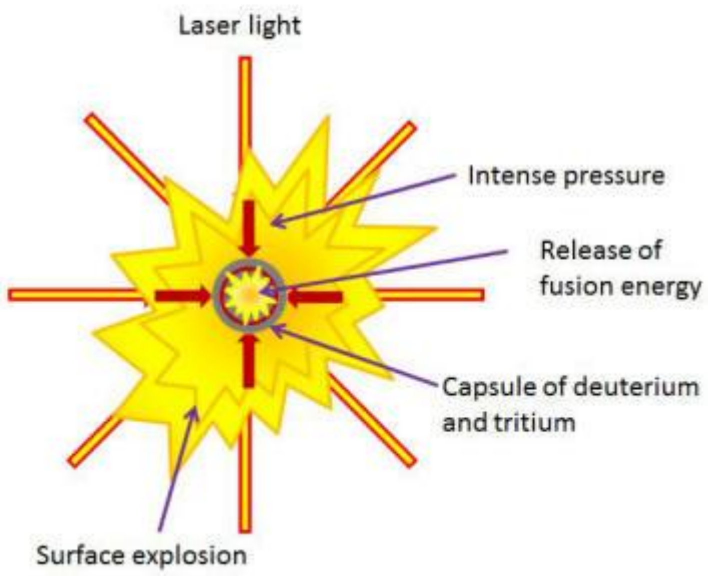


Figure 153 Release of fusion energy

In this case, the lasers act as **direct** drivers.

There is another way of doing achieving the compression. Lasers are directed onto the inner surface of a gold plated hollow capsule which contains the fuel pellet of deuterium and tritium. The gold-plated capsule is called a **hohlraum** (German - *hohl* - hollow; *raum* - "space"). It bathes the fuel pellet with **soft** (low energy) **X-rays**, which causes the surface explosion. The lasers act as **indirect** drivers.

15.1215 Gravitational Confinement

Plasma can be confined due to the force of gravity. However, this can only occur in very large bodies of plasma, such as those found in stars. There is a **minimum** size for a body of gas to compress the plasma sufficiently to allow fusion reactions to occur. The star Proxima Centauri is a **red dwarf**. It has a radius of 2.0×10^5 km, about 14 % of that of the Sun. Its mass is 12 % of the mass of the Sun. It is little bigger than the gas giant Jupiter.

In theory, for fusion to start in a star, the mass needs to be about 7 % of the mass of the Sun. Some astronomers think the Jupiter is a star that failed to ignite.

Questions

Tutorial 15.12

15.12.1

How does fusion differ from fission?

15.12.2

$$^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n} + \text{energy}$$

Data to use:

Mass of deuterium nucleus = 3.3425×10^{-27} kg

Mass of tritium nucleus = 6.6425×10^{-27} kg

Mass of helium nucleus = 6.6465×10^{-27} kg

Mass of a neutron = 1.675×10^{-27} kg

What is the energy in J and eV released in this reaction above?

15.12.3

The Earth is 150 million km from the Sun. The value of the intensity of light at the equator is 1400 W m^{-2}.

The radius of the Sun is 6.96×10^{8} m

Modelling the Sun as a point source, work out the intensity of the light at the surface of the Sun.

15.12.4

How many photovoltaic cells in *Figure 132* would be needed to give out a voltage of 240 V?

15.12.5

What is the maximum current this array in *Figure 133* could give out?

15.12.6

A typical photovoltaic array gives out a power of 4.8 kW at a voltage of 240 V. Each cell gives 2.5 A at 0.5 V.

Show that about 3500 cells are required for this array.

How should they be arranged?

15.12.7

The average intensity of sunshine in the UK is about 500 W m^{-2} (Intensity (W m^{-2}) = Power (W) ÷ area (m^2)

The area of a solar cell is 25 cm^2. The cell is arranged so that it can give out a voltage of 1.2 V.

(a) What is the absolute maximum power that the cell could provide if it were 100 % efficient?

(b) In reality the cell is 10 % efficient. What is the maximum power obtained.

(c) What is the charging current if the voltage is 1.2 V?

(d) How long does it take to charge up a 600 mAh Ni-MH battery?

15.12.8

A wind turbine has blades of length 20.0 m. A wind is blowing with a steady speed of 10.0 m s^{-1}.

(a) Calculate the maximum power available from the machine. Give your answer to an appropriate number of significant figures.

(b) What assumption have you made?

Density of air is 1.2 kg m^{-3}.

15.12.9

The wind turbine in Question 15.12.8 can deliver a maximum 1.0 MW in continuous operation.

What is the wind speed that will deliver this?

15.12.10

A water turbine discharges into a river, the surface of which is 60 m below the surface of the reservoir that feeds it. It is fed by a penstock (water pipe) of diameter 1.0 m and the water is regulated by a valve to flow at 10 m s^{-1}.

(a) Calculate the water flow. Give the correct unit.

(b) Calculate the power from the generator if the machine is 60 % efficient.

15.12.11

Give two advantages and two disadvantages of a tidal power station with a storage pool.

15.12.12

When a pumped storage is in pumping mode, each motor pumps 21 $m^3 s^{-1}$ against a change in level of 300 m.

(a) If water has a density of 1000 kg m^{-3}, calculate the power of each motor. Assume that the value of the constant k = 1.

(b) In fact, the motors each have a power of 75 MW. What is the efficiency of the motor-pump unit?

15.12.13

The formula for uranium hexafluoride is UF_6. The relative atomic mass of fluorine is 19.

Calculate the masses of the hexafluoride compounds of two isotopes of uranium. Express it as a percentage.

(The answer is NOT 1.26 %)

15.12.14

The neptunium decays by beta minus decay to **plutonium**.

Write down the equation that describes this decay.

15.12.15

Waste materials consist of helium and neutrons. These are referred to as "ash". What happens to the ash?

Tutorial 15.13 Fuel Cells

Welsh Board and Eduqas Syllabus

Contents

15.131 Fuels for Vehicles

Internal combustion engines can be run on fuels other than petrol or diesel. Will this reduce emissions like carbon dioxide and nitrogen oxides?

Some vehicles have been converted to run on **liquefied petroleum gas** (LPG). In static installations, **methane** is produced by biogas digesters and fed to a stationary engine and burned there to produce energy. The same is done with the methane produced in land-fill sites. In the Second World War, buses and lorries were converted to run on carbon monoxide from the town gas supply. These carried distinctive large bags of gas on their roofs.

The **calorific value** (energy per unit mass) of methane is slightly greater than that of petrol or diesel:

Fuel	Calorific Value / MJ kg^{-1}	Density / kg m^{-3}
Hydrogen	130	0.083
Methane	50	0.80
LPG	49	1.7
Petrol	45	750
Diesel	45	830

The calorific value is sometimes called the **specific energy**.

The **energy density** of a fuel is the **energy per unit volume** (J m^{-3}). We can convert the calorific value to the energy density.

$$\text{Energy density (J m}^{-3}) = \text{Calorific value (J kg}^{-1}) \times \text{Density (kg m}^{-3})$$

Worked Example What is the energy density of diesel fuel?
Answer Energy density = 45×10^6 J kg^{-1} × 830 kg m^{-3} = **3.7×10^{10} J m^{-3}** = 37 GJ m^{-3}.

Answer Question 15.13.1 and consider this. Perhaps fuel should be sold by the kilogram, and fuel consumption be measured in kilograms per 100 km.

Apart from hydrogen, the fuels above are **carbon based**, which means that carbon dioxide is emitted from the vehicles. Also, **nitrogen oxides** (NO_x) are produced in the conditions of high temperature and pressure in the cylinders.

15.132 Hydrogen as a Fuel

Data from the table above suggest that **hydrogen** would make an excellent fuel for an internal combustion engine. It has nearly three times the calorific value of diesel. So why isn't it used as a car fuel? It also emits water as an exhaust.

- It is expensive to make, as hydrogen needs to be produced by electrolysis;
- Although the energy per kilogram is impressive, 1 kg of hydrogen takes up a lot of space. To confine it to a small enough space to fit into a car would require very high pressure.

Your answer to Question 15 13.3 should tell you that very high pressures would be needed in the fuel tank of a hydrogen powered vehicle to give it a reasonable range. A 50-litre tank that will hold hydrogen at that intense pressure would be very heavy indeed with thick stainless steel walls. It would have a mass that was a significant fraction of the car's mass.

Hydrogen powered vehicles with an internal combustion engine are limited. Hydrogen filling stations are very few. Additionally, the heat of combustion and pressure in the cylinders will also give rise to nitrogen oxides. Therefore, our dream of a zero-emissions vehicle is as far away as before. Or is it?

15.133 Fuel Cells

One problem with internal combustion engines is that their **efficiency** is quite low, even if it has improved over recent years. An **electric motor** is more efficient. How do we convert hydrogen to electricity. We produce hydrogen by passing electricity through an **electrolyte**. Can we reverse the process to get electricity back?

The answer is a **fuel cell**. It takes in hydrogen as a fuel and its waste is water. The diagram below shows the idea (*Figure 154*):

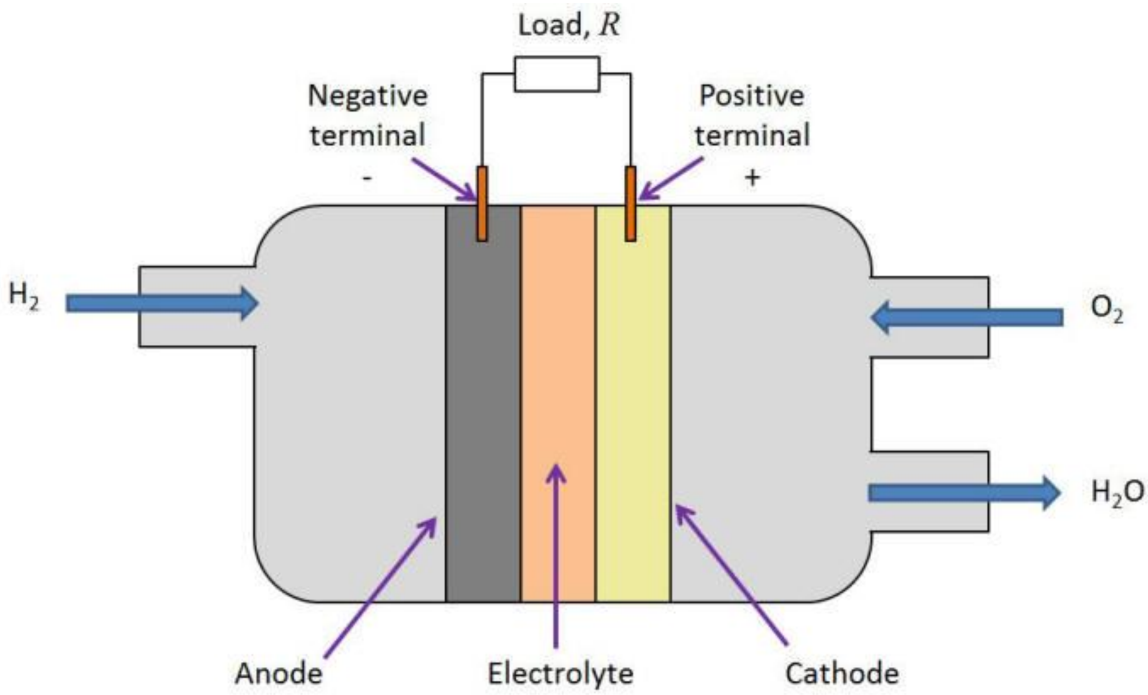


Figure 154 A Fuel cell

Hydrogen gas (H_2) passes into the cell on the left hand side of the fuel cell. The hydrogen molecules hit the **anode**, which is made from **platinum**, a catalyst. The hydrogen molecules split into **hydrogen ions** and **electrons**:

$$H_{2(g)} \rightarrow 2H^{+}{}_{(aq)} + 2e^{-}$$

The **electrolyte** allows **protons** to cross it but is contained by a **polymer membrane** that is **impermeable** to electrons. The electrons are left on the anode. The **excess** of electrons makes the anode **negative**.

On the right-hand side **oxygen** comes in. The oxygen atoms hit the **cathode** which is made from **nickel** (also a catalyst), which facilitates the combination of the oxygen atoms with the hydrogen ions:

$$4H^{+}{}_{(aq)} + O_{2(g)} \rightarrow 2H_2O_{(g)}$$

The electrons have come through the **external circuit**, which passes through the **load**. In a car, this would be a **motor**. The water is given off as steam, as the reaction occurs at a temperature of about 130 °C.

The voltage of such a cell is about 0.7 V. A car traction motor operates at a typical voltage of 350 V. So, the hydrogen cells have to be placed in series to make a high enough voltage.

A typical current from a fuel cell is about 0.8 A (800 mA). To provide the currents needed in a typical electric car, the cells would need to be set up in parallel arrays.

15.134 Fuel Cells in Cars

To make a viable unit to power a car, a very large number of individual fuel cells are needed. In these examples a DC motor was used as the model. In modern electric cars, the motors are **3-phase AC motors** which have to have **inverters** to control them

There are many advantages to using fuel cells including:

- They use hydrogen which, unlike carbon-based fuels, is not toxic to the environment;
- The waste product is water.
- The device has no moving parts;
- The efficiency is high;
- Electricity for the electrolysis of hydrogen can be generated using renewable sources;
- Generating the electricity and making the hydrogen can be done anywhere;
- Nitrogen takes no part in the reactions, so there are no nitrogen oxides produced;
- There is no memory effect, unlike batteries;
- Refuelling is much quicker than charging a battery on an electric car.

There are disadvantages:

- Hydrogen is explosive;
- Hydrogen has to be stored in high-pressure cylinders;
- The fuel cells are expensive;
- There are few hydrogen filling stations (the nearest filling station to where I live is in Doncaster, 120 km away);
- Hydrogen powered vehicles are very expensive (a fuel-cell version of a petrol car costs about £60000 (€66000) compared with £25000 (€27000) for the petrol equivalent).

Questions

Tutorial 15.13

15.13.1

If the calorific values of both petrol and diesel are both the same, why does a petrol car burn more fuel (for example a petrol car might burn 6 litres per 100 km, while the equivalent diesel car burns 5 litres per 100 km)?

15.13.2

Why are petrol and diesel the most common fuels used in vehicles, rather than the gases?

15.13.3

A car petrol tank has a capacity of 50 litres. Use data from the table above to answer the questions.

(a) Show that the mass of petrol is about 38 kg.

(b) 1 mole (6.0×10^{23} particles) has a mass of 2.0×10^{-3} kg. Calculate how many moles there are in 1 kg of hydrogen.

(c) Calculate the number of kilograms of hydrogen that would give the same range as the 50 litre tank of petrol.

(d) How many moles is this?

(e) 1 mol of any gas occupies 0.024 m^3 at room temperature and pressure. What is the volume taken by the number of moles you worked out in part (d).

(f) Show that the pressure in the fuel tank is about 3000 atmospheres (atm).

15.13.4

How many fuel cells are needed in series to make a voltage of 350 V?

15.13.5

A particular car has a traction motor rated at a maximum power of 75 kW.

(a) Calculate the current needed.

(b) How many parallel arrays would be needed to provide this current?

(c) Use your answer to question 4 to work out the total number of fuel cells needed.

15.13.6

(*Challenge*)

An electric car is travelling along a straight and level road at a constant speed of 100 km h^{-1}. It draws a current of 200 A from a hydrogen fuel cell. The current is produced by electrons from the anode.

(a) Write down an equation that shows how electrons are produced from hydrogen gas.

(b) Work out how many electrons are taken every second.

(Electronic charge = 1.60×10^{-19} C)

(c) How long does it take for the car to use 2.0 g of hydrogen gas (H_2)?

(d) How far will the car travel at this speed?

(e) State the assumption made in this calculation and discuss whether the car would travel this far on 2.0 g of H_2. Give an estimate of the true distance the car would travel.

Tutorial 15.14 Thermal Conduction

Welsh Board and Eduqas Syllabus

Contents

This tutorial is for students of the Welsh Board and Eduqas

15.141 Conduction Basics

You will remember that metals are good thermal **conductors**, while non-metals, liquids, and gases tend to be rather poor conductors (or good **insulators**). If you have forgotten it (or were off task in that lesson), you can revise it in your GCSE notes.

Conduction in any material can be explained using a **model** of molecules (represented by the balls in the picture below) joined with other molecules that are connected with bonds that act like springs (*Figure 155*).

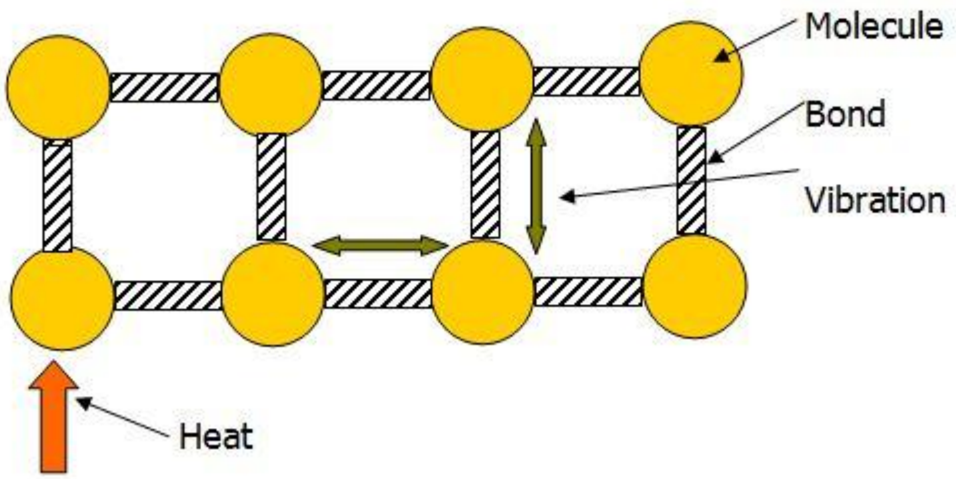


Figure 155 Simple model of conduction

We used a similar model to explain how wires obeyed Hooke's Law for tensile forces.

If we apply heat, the molecules vibrate with **bigger vibrations** and set their neighbours vibrating with bigger vibrations. These pass on the vibrations to their neighbours in turn. The bigger the vibrations, the hotter the material. If the vibrations are passed on easily, the material is a good conductor.

However, this model does not explain why metals have a much better **conductivity** than non-metals. The answer lies in the fact that metals can be modelled as being a **lattice** of ions in a **sea of free electrons** that can move easily between the ions (*Figure 156*).

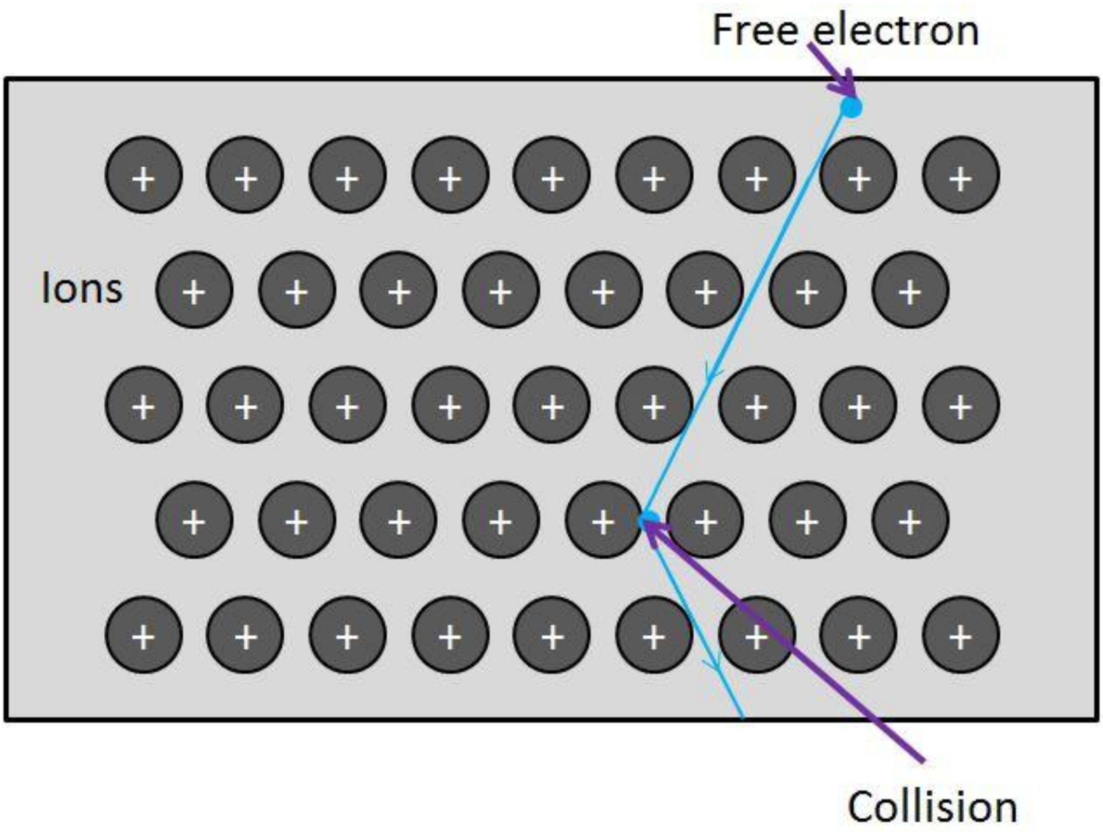


Figure 156 Free electron moving about a lattice of ions

As the metal **ions** are heated, the vibrations gain a **bigger amplitude**. Therefore, there is a **greater chance** that the free-moving electrons will **collide** with the vibrating ions. The electrons then move off to other parts of the lattice and collide with other ions. The electrons transfer energy to the ions, and these will vibrate with a larger amplitude. Thus, the heat energy flows more rapidly from the hot part of the metal to the cool part of the metal.

15.142 Measuring Conduction

We will look at how we can identify the factors that are involved in thermal conduction, and how they relate to each other. Consider a block of material that is mounted all the way around its edges with a perfect insulator (shown in the side view in *Figure 157*).

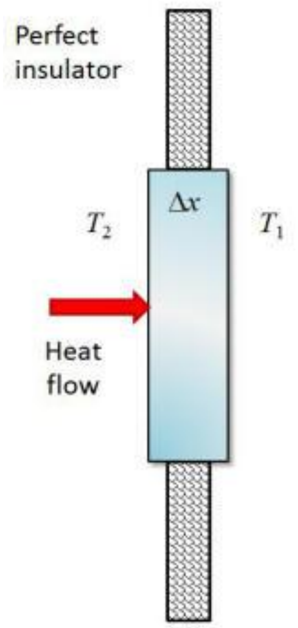


Figure 157 A block of conductive material mounted in a perfect insulator

The only way that heat can pass is through the largest face that has an area A. The block has a thickness of Δx. The heat flow is **constant**, and the temperatures are at constant values (called **steady state**). The heat flow is from **hot to cold**, since heat never flows from cold to hot. The block looks like this (*Figure 158*) from the front (not showing the perfect insulator).

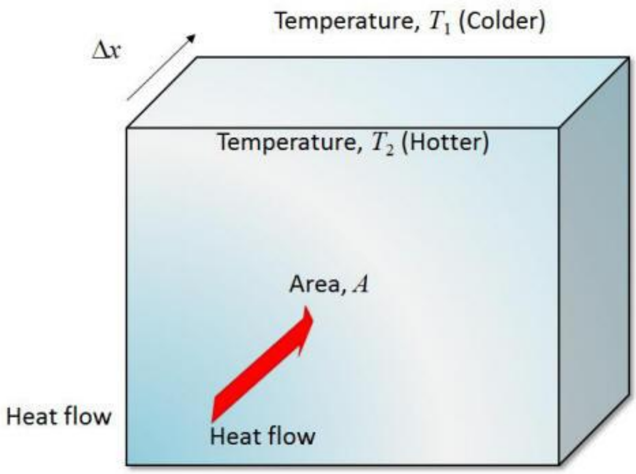


Figure 158 Block of conductive material seen face on

Heat flow is defined as the **amount of energy that passes in unit time**. It is given the physics code:

$$\frac{\Delta Q}{\Delta t}$$

The units are Joules per second (J s^{-1}) or Watt (W).

Since the hot area is a distance x from the cold area, there is a **temperature gradient**:

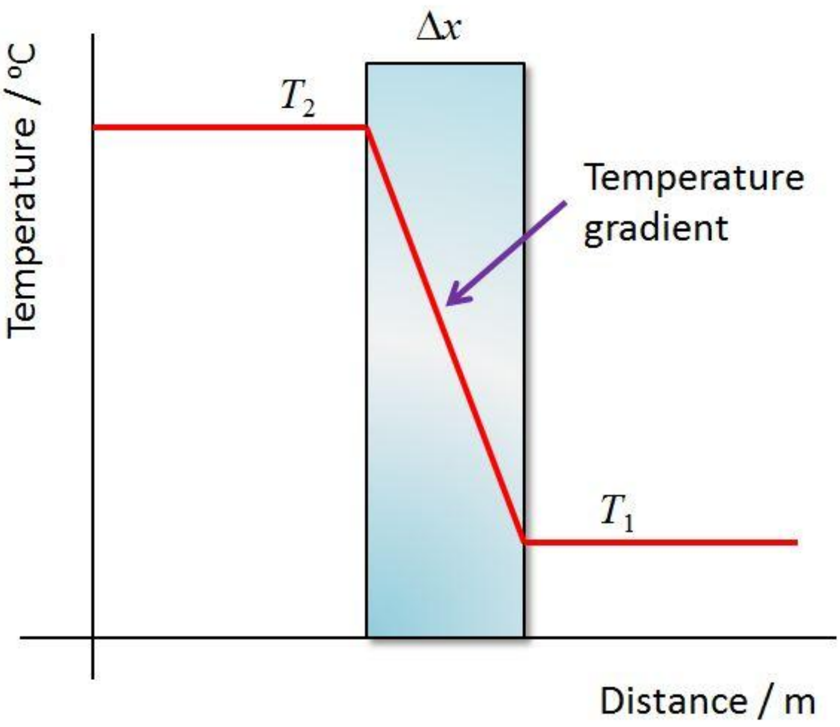


Figure 159 Temperature gradient

The temperature gradient is given the physics code:

$$\frac{\Delta T}{\Delta x}$$

And the units are Kelvin per metre (K m^{-1}). Temperature gradient is related to the temperatures by:

$$\frac{\Delta T}{\Delta x} = \frac{T_1 - T_2}{\Delta x}$$

.... *Equation 221*

We can say that the heat flow depends on two factors:

- Area of the block;
- Temperature gradient.

And we can write this as:

$$\frac{\Delta Q}{\Delta t} \propto A \frac{\Delta T}{\Delta x}$$

.... *Equation 222*

We then add in a **constant of proportionality**, λ ("lambda", a Greek lower case letter 'l'):

$$\frac{\Delta Q}{\Delta t} = -\lambda A \frac{\Delta T}{\Delta x}$$

.... *Equation 223*

The term λ is the physics code for **thermal conductivity**, which is a property of the material. Thermal conductivity is defined as the **heat flow per unit length per unit temperature**. The units for thermal conductivity are W m^{-1} K^{-1}.

The thermal conductivity has a **negative** sign because the thermal gradient is negative. The heat flow is positive, so the extra negative sign cancels out the negative thermal gradient.

In the syllabus this equation is written slightly differently:

$$\frac{\Delta Q}{\Delta t} = AK\frac{\Delta \theta}{\Delta x}$$ .... *Equation 224*

Note that the minus sign is ignored, as the temperature gradient is expressed as simply the temperature difference per unit length, without reference to the flow of heat from hot to cold. Because it's a temperature difference, you can use degrees Celsius, rather than Kelvin. Also, the thermal conductivity is given the code *K*, rather than l.

Here are some typical thermal conductivities for different materials.

Material	***Thermal conductivity / W m^{-1} K^{-1}***
Copper	385
Aluminium	238
Steel	60
Concrete	1.5
Brick	1.0
Glass	0.80
Polythene	0.50
Wood	0.30
Air	0.026
Insulating Foam	0.023

The answer to Question 15.14.2 gives a result that is actually rather higher than what would be measured. This is because it assumes that the surface of the glass is actually at the same temperature as the room and the outside. This is not the case, as there is a thin layer of air on either side of the glass. Air is a rather poor conductor of heat, so the temperature gradient is rather less (*Figure 160*).

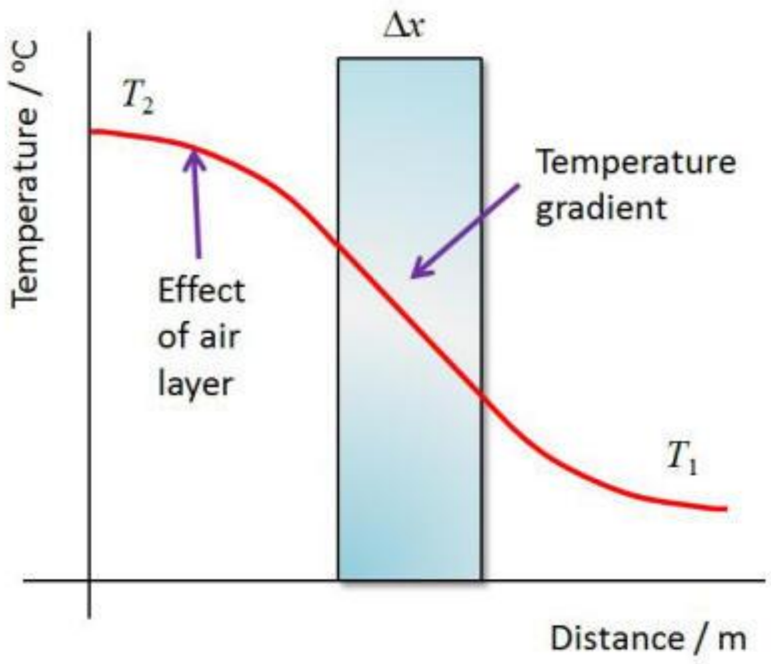


Figure 160 True temperature gradient is slightly curved

15.143 U-values

Architects and heat engineers do not use the thermal conductivity as shown above. Instead, they use the **U-value** which is based on the measured heat flow out of a window or wall. The U-value is defined as:

the thermal conductivity per unit length.

We will explore the relationship between the thermal conductivity and the U-value.

The U-value is measured as:

the rate of heat flow per unit area per unit temperature change.

In words, the relationship is:

$$\text{U} - \text{value} = \frac{\text{rate of energy flow}}{\text{area} \times \text{temperature difference}}$$

In physics code:

$$U = \frac{\Delta Q}{A \Delta t \Delta \theta}$$

.... Equation 225

This can be rearranged to give:

$$\frac{\Delta Q}{\Delta t} = U A \Delta \theta$$

.... Equation 226

The units for the U-value are watts per square metre per Kelvin (W m^{-2} K^{-1})

Here are some U-values:

Building Elements	***U-Value / W m^{-2} K^{-1}***
Solid Brick Wall	2.0
Cavity Wall (no insulation)	1.5
Insulated Wall	0.18
Single glazing	4.8 to 5.8
Double glazing	1.2 to 3.7
Triple Glazing	1.0
Solid timber door	3.0
UPVC	2.2

In some building regulation documents, the **R-value** is used. The R-value is simply the **reciprocal** of the U-value, and the units are K m^2 W^{-1}.

The assumption made in the question above is that the window is mounted in a wall that is a perfect insulator, i.e. has a U-value of 0.

15.144 Electrical Analogy for Thermal Conductivity

We treat a **building element** like a window or a wall like a thermal conductor. The higher the thermal conductivity, the more heat flows through the element. It's like electricity. The higher the conductivity (or, strictly speaking, the **conductance**) of an electrical component, the greater the current for a given voltage. We can model thermal conductance like electricity flowing through resistors.

The table below shows the comparisons:

Thermal Conduction	***Electrical Conduction***
Heat flow ($\Delta Q/\Delta t$)	Current (I)
Temperature Difference ($\Delta\theta$)	Voltage (V)
Thermal conductivity (k)	Electrical conductivity (σ)
Distance of flow (Δx)	Distance of flow (l)
Relationship: $\frac{\Delta Q}{\Delta t} = AK\frac{\Delta\theta}{\Delta x}$	Relationship $I = \frac{\Delta Q}{\Delta t} = A\sigma\frac{V}{l}$

The electrical quantities we are interested in are voltage and current. However, we don't use resistance. Instead, we use **conductance**. Conductance is the reciprocal of resistance.

$$G = R^{-1}$$ .... *Equation 227*

It has the physics code G and the units are Siemens (S). In some textbooks you might see Ω^{-1} or even "mho" ("ohm" written backwards). From this we can write familiar equations in terms of the conductance:

$$I = VG \text{ .... } \textit{Equation 228}$$

$$G = I/V \text{ .... } \textit{Equation 229}$$

$$V = I/G \text{ .... } \textit{Equation 230}$$

The physical property **conductivity** is likewise the reciprocal to **resistivity**:

$$\sigma = \rho^{-1} \text{ .... } \textit{Equation 231}$$

The units for conductivity are Siemens per metre (S m^{-1}).

When we combine series resistors, we know that:

$$R_{tot} = R_1 + R_2 + R_3 \ldots + R_n \text{ .... } \textit{Equation 232}$$

For parallel resistors:

$$R_{tot}^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1} \ldots + R_n^{-1} \text{ .... } \textit{Equation 233}$$

Now that we know that conductance is the reciprocal of resistance, we can write equations for series and parallel conductors.

For series conductors:

$$G_{tot}^{-1} = G_1^{-1} + G_2^{-1} + G_3^{-1} \ldots + G_n^{-1} \text{ .... } \textit{Equation 234}$$

For parallel conductors:

$$G_{tot} = G_1 + G_2 + G_3 \ldots + G_n \text{ .... } \textit{Equation 235}$$

The same can be done for thermal conductors.

15.145 Combining U-Values

If we have two building elements, for example a window set in a wall, heat can go through both the wall and the window. There are two paths. They are like the two parallel branches of an electrical circuit. Since we are using the analogy of **conductance**, we can say that the U-values add up (*Figure 161*):

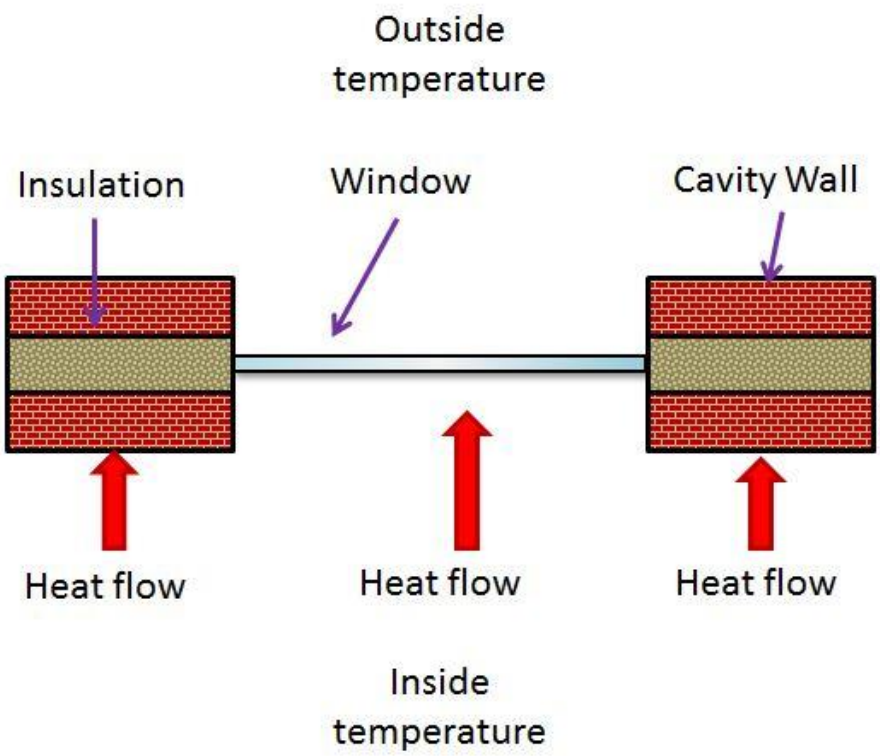


Figure 161 Heat flow through a window and two walls.

Suppose for the sake of simplicity, that the areas of the window and the walls were all the same. We use the analogy of parallel conductors (*Figure 162*):

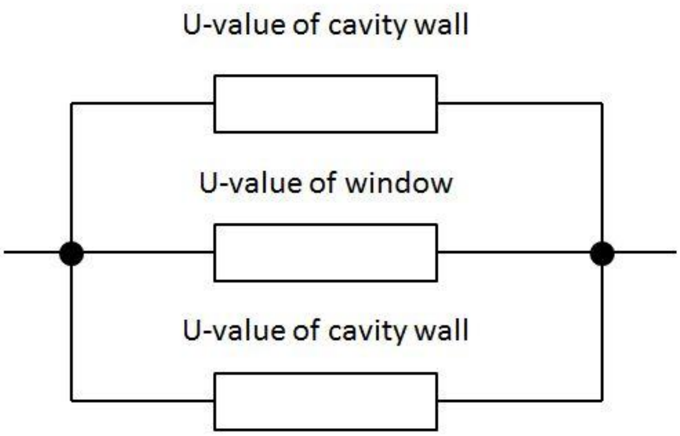


Figure 162 Conductance of parallel resistors as an analogy to heat conduction

Rarely do we have a window and a wall panel that are exactly the same area. So, we need to take into account the **area** that each element has. The product of the U-value and the area gives us the **heat flow per unit temperature**:

$$\frac{\Delta Q}{\Delta t \Delta \theta} = UA$$

.... *Equation 236*

The heat flow per unit temperature has units of W K^{-1}.

We can work out the U-value of a building element like a double glazed door by working out the area of the UPVC frame and the glass panels. Consider a door like this (*Figure 163*):

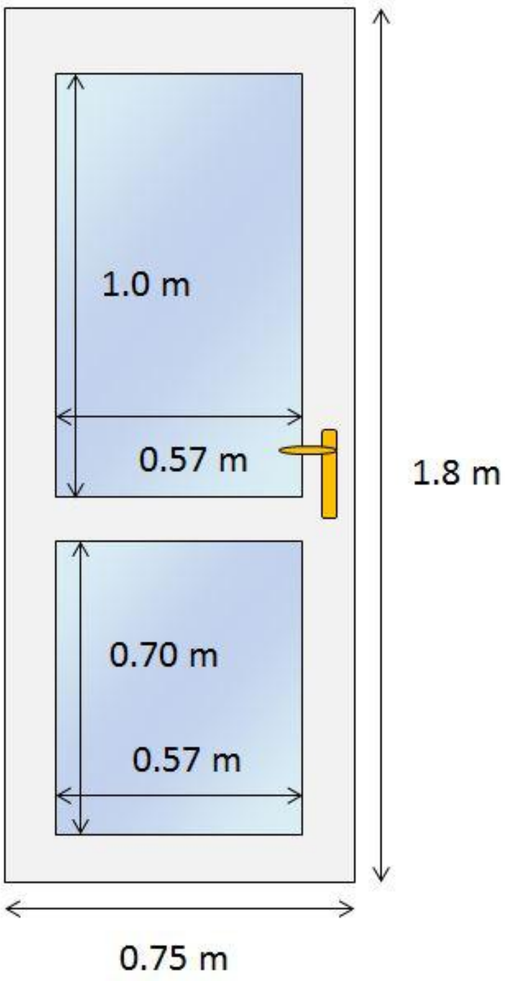


Figure 163 Glass panelled door

Worked Example

The door in the diagram above (*Figure 163*) consists of two double glazed panels of U-value 1.9 W m^{-2} K^{-1} of the dimensions shown. It is surrounded by UPVC of U-value 2.2 W m^{-2} K^{-1}.
What is the total U-value of the door?

Answer

Work out the total area of the door:

Area of the door = 1.8 m × 0.75 m = 1.35 m^2

Work out the areas of each glass pane:

Area of top pane = 1.0 m × 0.57 m = 0.57 m^2
Area of bottom pane = 0.70 m × 0.57 m = 0.40 m^2
Total area of glass = 0.97 m^2

Now work out the area of the frame:

Area of frame = 1.35 m^2 - 0.97 m^2 = 0.38 m^2

Heat flow per unit temperature for the glass = 1.9 W m^{-2} K^{-1} × 0.97 m^2 = 1.843 W K^{-1}.
Heat flow per unit temperature for the frame = 2.2 W m^{-2} K^{-1} × 0.38 m^2 = 0.836 W K^{-1}.

The heat flows add up, therefore:

Total heat flow per unit temperature for the door =
1.843 W K^{-1} + 0.836 W K^{-1} = 2.679 W K^{-1}

U-value of door = heat flow per unit temperature ÷ area = 2.679 W K^{-1} ÷ 1.35 m^2 =
1.98 W m^{-2} K^{-1} = **2.0 W m^2 K**$^{-1}$ (2 s.f. are appropriate here)

15.146 U-Value and Thermal Conductivity

Earlier on in the tutorial, we defined the U-value as:

the thermal conductivity per unit thickness of the material.

We can also measure it as:

the rate of heat flow per unit area per unit temperature change.

We know that the heat flow is related to thermal conductivity with this relationship:

$$\frac{\Delta Q}{\Delta t} = AK\frac{\Delta \theta}{\Delta x}$$

.... *Equation 237*

and heat flow is related to U-value by:

$$\frac{\Delta Q}{\Delta t} = UA\Delta \theta$$

.... *Equation 238*

Therefore:

$$U = \frac{K}{\Delta x}$$

.... *Equation 239*

You can see that your answer to Question 15.14.7 is much higher than the U-value of a single glazed window, which was 5.1 W m^{-2} K^{-1}. This is because there is a layer of air either side of the window. Air is a poor conductor of heat. We will ignore convection. Consider our 6 mm pane of glass (*Figure 164*):

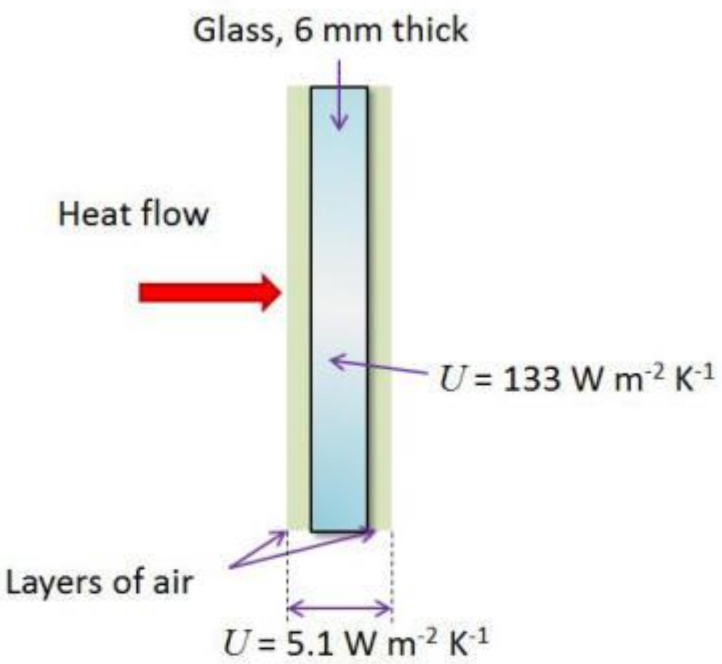


Figure 164 Heat flow through a pane of glass

We have two layers of air, both of which have unknown thickness. However, we can work out the U-value of the two layers of air. We will assume that the layers of air are the same thickness. We are also ignoring the effects of convection. We model the thermal conduction as **current** passing through **series resistors**. The idea is shown below (*Figure 165*):

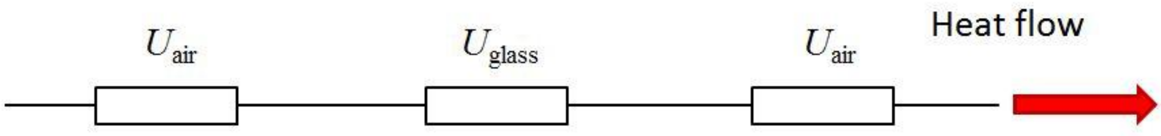


Figure 165 Modelling heat flow through layers of air as well as glass

For series resistors, the **reciprocals** of the conductance add up:

$$G_{tot}^{-1} = G_1^{-1} + G_2^{-1} + G_3^{-1} ... + G_n^{-1}$$ .... *Equation 240*

Therefore:

$$U_{tot}^{-1} = U_{air}^{-1} + U_{glass}^{-1} + U_{air}^{-1}$$ .... *Equation 241*

So, using the values from *Figure 164*, we can substitute:

$$5.1^{-1} = 133^{-1} + 2(U_{air}^{-1})$$

and rearrange:

$$2(U_{air}^{-1}) = 5.1^{-1} - 133^{-1} = 0.196 - 0.00752 = 0.189$$

Turn it all upside down to get the final answer:

$$1/2U_{air} = 0.189^{-1} = 5.30 \text{ W m}^{-2} \text{ K}^{-1}$$

Therefore, each layer of air has a U-value of **10.6 W m^{-2} K^{-1}**.

It is entirely possible for house to be cooler inside than outside. I wrote these notes during the very hot July of 2018, and this was often the case that my house was several degrees cooler inside than it was outside.

U-values are an essential part of **modern building regulations** and there are several on-line tools for working out overall U-values for a whole house, taking into account not only walls, doors, and windows, but also the floors, ceilings and the roof. We want to keep the house warm at minimum cost because:

- we want to have money to spend on other things (as the rent/mortgage allows);
- heating houses leads to emission of carbon dioxide (our carbon footprint);
- a warm house is welcoming;
- a warm house requires less maintenance.

A cold house is not just unpleasant to be in. A cold house leads to condensation which leads to deterioration in the decoration of rooms. It encourages mould that not only damages the furniture and fitting, it also can cause health problems. When heating a house, we are paying to keep ourselves warm, not to provide a heated platform for jackdaws and feral pigeons to warm their bottoms on the roof.

Questions

Tutorial 15.14

15.14.1

Show that the units for thermal conductivity are W m^{-1} K^{-1}.

15.14.2

A single-glazed window is made of glass 6.0 mm thick and is 1.3 m high and 2.0 m wide. The temperature in the room is 20 °C while the temperature outside is -5.0 °C.

(a) Show that the temperature gradient is about 4200 K m^{-1}

(b) Looking up a suitable value on the table on Page 252, work out the heat flow through the window.

15.14.3

A single-glazed window is made of glass 6.0 mm thick and is 1.3 m high and 2.0 m wide. The temperature in the room is 20 °C while the temperature outside is -5.0 °C.

(a) The room at is at a steady temperature. The temperature outside is -5.0 °C. The heat flow through the window is measured at 300 W. If the U-value for the glass is 5.1 W m^{-2} K^{-1}, calculate the temperature in the room.

(b) Which data item is not relevant?

15.14.4

We know that the relationship for electrical resistivity is:

$$\rho = \frac{AR}{l}$$

(a) What is the equivalent relationship for electrical conductivity?

(b) Work out the equivalent relationship for thermal conductivity, K.

(c) Draw a diagram to indicate the analogous terms between the two.

15.14.5

The U-value of the insulated cavity wall is 0.18 W m^{-2} K^{-1}. The U-value of the single glazed window is 5.2 W m^{-2} K^{-1}.

What is the total U-value?

15.14.6

The temperature outside is 10 °C while the temperature inside is 21 °C.

What is the heat loss through the door in the example above on Page 259?

15.14.7

In Question 15.14. 2, you looked at a single-glazed window that was 6.0 mm thick. The thermal conductivity was 0.80 W m^{-1} K^{-1}.

What is the U-value?

15.14.8

What is the thickness of the layer of air either side of the glass window described on Page 261?

The thermal conductivity of air is 0.026 W m^{-1} K^{-1}

6. More Quantum Concepts

Tutorial 15.15 Spin

Pre-U and IB Syllabuses only

Contents

This tutorial attempts to explain the concept of particle spin, which is not easy to understand. Work through the tutorial carefully.

Many of the sources contradict each other and are hard to follow, which made the preparation of this tutorial rather difficult.

15.151 What is Particle Spin? (*Extension for Pre-U students only*)

Elementary particles such as electrons and quarks possess a property called **spin**, as do composite particles such as hadrons and atomic nuclei. Spin is an intrinsic property, just like mass and charge. Spin is a quantum property. It has a value of half-integer or integer. That means values of ½, or 0, or 1. Other half integer or integer values are theoretically possible.

$$s = \frac{n}{2}$$

.... *Equation 242*

The spin also has a sign, indicating that is a vector quantity. Spin can be +½ or -½, or +1 or -1. The sign could be considered as **up** (positive) or **down** (negative), or clockwise (positive) or anticlockwise (negative), depending on how you want to think about it.

The concept of spin is not easy, but I hope these notes will give you an idea of what it's about.

15.152 Stern Gerlach Experiment

Spin of electrons was first mooted as a result of the **Stern-Gerlach** Experiment. You are NOT expected to know any details of the experiment at this level. The experiment was set up like this (*Figure 166*):

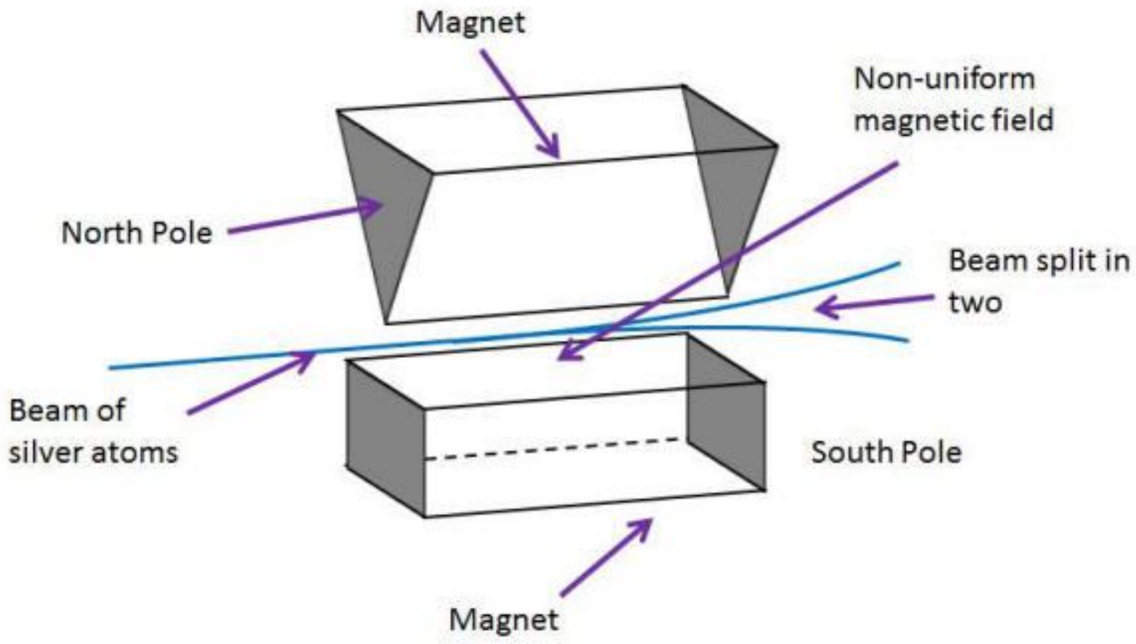


Figure 166 Stern-Gerlach Experiment

Silver atoms from a furnace were injected into the apparatus. The result was surprising. The atoms split into two beams. One half went upwards, while the other half went downwards. No atoms went to the left or right. If the apparatus was turned on its side, half of the atoms went to the right, and the other half went to the left. None went up or down.

The key points are that:

- If there is a deflection due to the magnetic field, the atoms must be acting as little magnets.
- The silver atom is neutral, so that there is no way that the magnetic effect is due to a charge.
- Silver is not a magnetic material.

Somehow the silver atoms were behaving like little magnets.

We can stop the downward beam by putting a screen in the way like this (*Figure 167*):

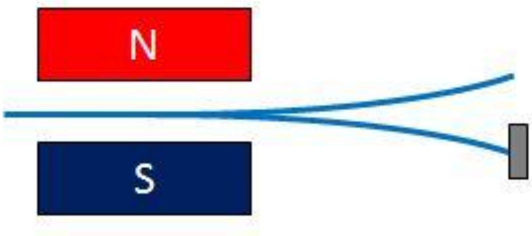


Figure 167 Blocking the downward beam

The we pass the upward beam into another pair of magnets that produce a non-uniform field. We would expect the upward beam to be deflected upwards only. Instead, we get the beam split into two like this (*Figure 168*):

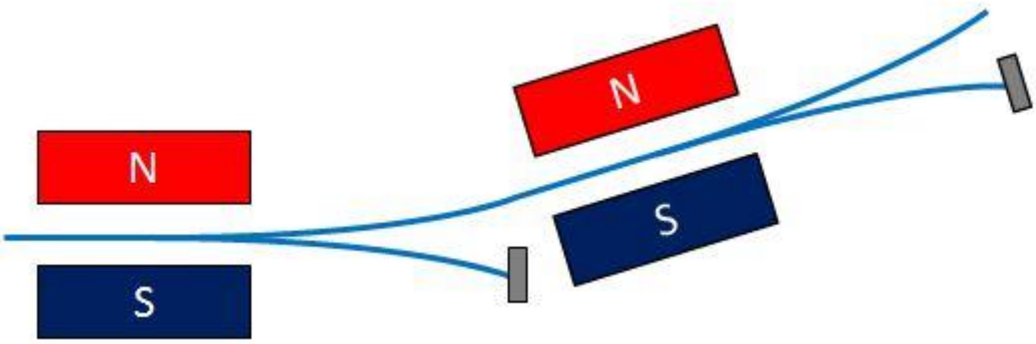


Figure 168 ...and again

If we do this again and again, we see the same effect.

The neutral silver atom has a single outer shell electron. It must be this electron that is responsible for the effect. This was proved later by doing the same experiment with **hydrogen atoms**.

15.153 How do we explain this?

We know that charged particles are deflected when they **move** through a magnetic field. A **stationary** charged particle will not interact with a magnetic field at all. We also know that moving charges generate a magnetic field. The deflection is due to the interactions between the magnetic fields.

We can conclude that the silver atoms (and hydrogen atoms) in the experiment must have some kind of magnetic property. In fact, the atoms form a magnetic **dipole**, which has a north pole and a south pole. The magnetic field is just like a bar magnet. However, the atoms are neutral. Therefore, a charge has got to be moving in a rotary motion to make the magnetic field. (if it wasn't moving in a circle, the electron would separate from the atom.) If something is moving in rotary motion it has to have **angular momentum**, just like an object moving in a straight line has linear momentum.

From rotational mechanics, we know the formula for angular momentum:

Angular momentum = moment of inertia × angular velocity

In code:

$$L = I\omega$$ .... *Equation 243*

[L - angular momentum (kg m^2 s^{-1}); I - moment of inertia (kg m^2); ω - angular velocity (rad s^{-1})]

The units for L are kg m^2 s^{-1}. Since the radian is a dimensionless unit, it gets left out. This is consistent with the accepted quantum number of spin, which is derived from the **Shortened Plank's Constant**, $\hbar$. The term $\hbar$ (with the slash through it ("h-bar")) is called the **shortened Planck Constant**, and this is related to h, the **Planck's Constant** (6.63 × 10^{-34} J s) by:

$$\hbar = \frac{h}{2\pi}$$ .... *Equation 244*

We can, of course, give a value for $\hbar$:

$\hbar$ = 1.06 × 10^{-34} J s

If we go back to base units:

1 J = 1 kg m^2 s^{-2}

Therefore, if we multiply the energy by time, we get:

$$1 \text{ J s} = 1 \text{ kg m}^2 \text{ s}^{-2} \times 1 \text{ s} = \text{kg m}^2 \text{ s}^{-1}$$

So, we can say that the shortened Planck Constant is an angular momentum.

15.154 Two Possible Classical Models

Consider the silver atom (*Figure 169*):

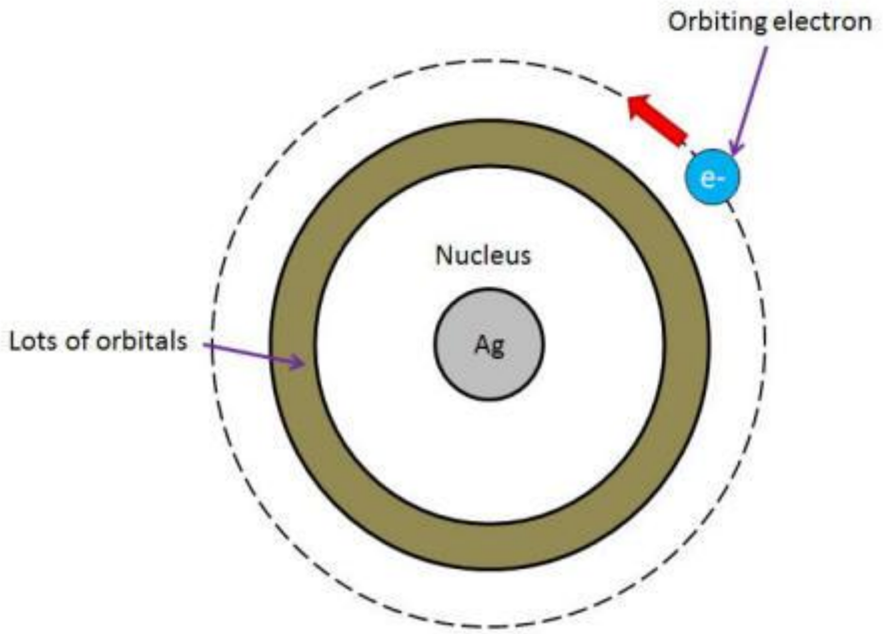


Figure 169 A silver atom

Could the magnetic field be caused by the orbiting electron in the outer shell? The first thing to say is that the orbit is not like a planet going around a star. The orbit is all over the place and it's best modelled as a **hollow sphere**. The moment of inertia is given by:

$$I = \frac{2}{3}mr^2$$

.... *Equation 245*

Silver is a heavy transition metal (with a relative atomic mass of 107). Its diameter is 172 pm (1.72×10^{-10} m). Therefore, the radius is 8.60×10^{-11} m. The mass of a single electron is 9.11×10^{-31} kg.

Electron spin is quantised. It can only have two values:

$$+\hbar/2 \text{ or } -\hbar/2$$

The value of the spin for an electron is:

$$\mathbf{5.28 \times 10^{-35}\ J\ s}$$

Let's work out the moment of inertia of the electron in its orbit:

$$I = 2/3 \times 9.11 \times 10^{-31}\ \text{kg} \times (8.60 \times 10^{-11}\ \text{m})^2 = \underline{\mathbf{4.49 \times 10^{-51}\ kg\ m^2}}$$

If the angular momentum of the electron is 5.28×10^{-35} J s, we can work out the angular velocity:

$$\omega = L/I = 5.28 \times 10^{-35}\ \text{J s} \div 4.49 \times 10^{-51}\ \text{kg m}^2 = \underline{\mathbf{1.18 \times 10^{16}\ rad\ s^{-1}}}$$

We can work out the linear speed of the electron using:

$$v = \omega r \quad \text{............ } \textit{Equation 246}$$

$$v = 1.18 \times 10^{16}\ \text{rad s}^{-1} \times 8.60 \times 10^{-11}\ \text{m} = \underline{\mathbf{1.02 \times 10^{6}\ m\ s^{-1}}}$$

However, this does not explain how the electron paths in the magnetic field are only upwards and downwards. The random movement of the electron as it goes around the orbit would mean that there could be any number of paths the silver atoms could take. There is every possibility that the resulting little magnetic dipoles are perfectly horizontal, meaning that they would not be deflected at all. This is not consistent with the real observation.

We can model the spin in an electron in a very simplistic way as the **particle spinning on its axis** (*Figure 170*).

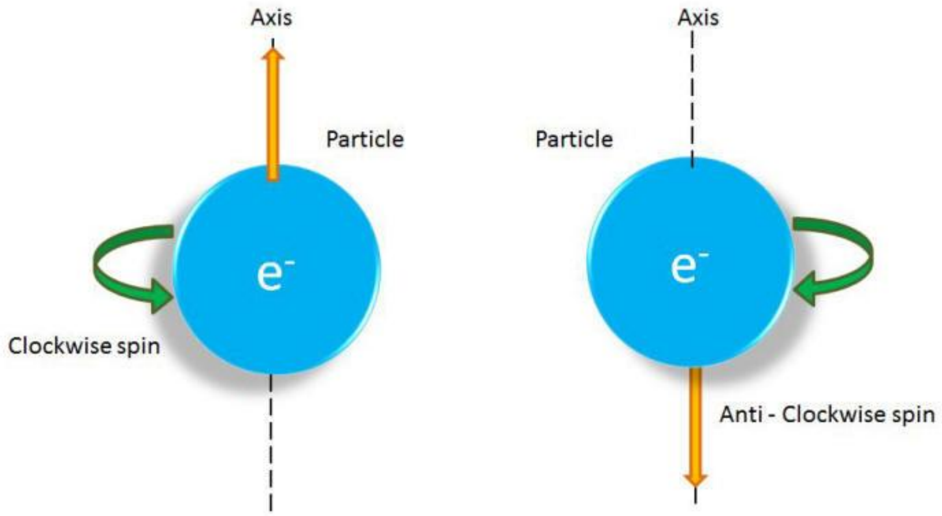


Figure 170 Spin

In this case we can model the electrons as solid spheres. Let's assume that the clockwise electron moves **up**, and the anticlockwise goes **down**. We would be right in thinking that not all the magnets are pointing perfectly upwards or downwards. The orientation is random. It is tempting to think that the little magnets formed as a result of the magnets flipping upwards, or downwards, depending on their orientation. However, the electrons are spinning and act like **gyroscopes**. Gyroscopes resist change in orientation due their angular momentum. Let's look at this further (*Figure 171*):

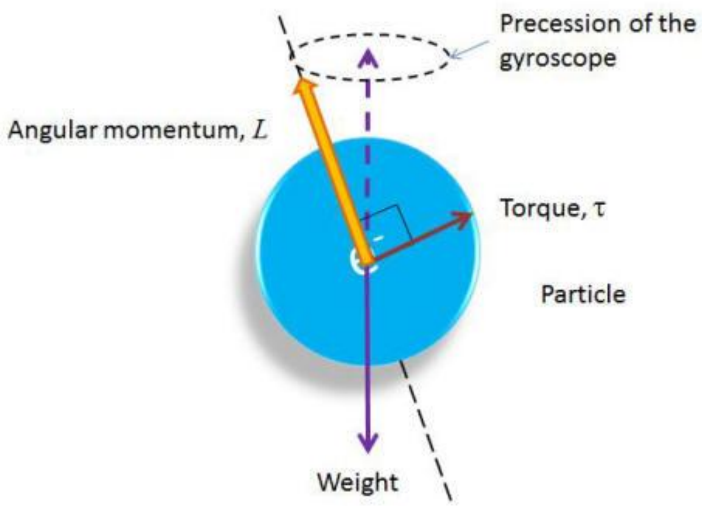


Figure 171 Gyroscopic effect of electron spin

The vector of the angular momentum is always perpendicular to the torque acting on the spinning ball (which we are using to model the spinning electron). The value of the torque remains constant, but the direction is constantly changing. The gyroscope **precesses** in a circle that is on a horizontal axis. The vertical component of the angular momentum vector points up (as in this case) or down. The precession of the angular momentum vector produces the upwards component that results in an upwards magnetic field. Or the downwards component makes a downwards magnetic field.

The moment of inertia is given by:

$$I = \frac{2}{5}mr^2$$

.... *Equation 247*

We can do similar calculations as before. Electron spin is quantised. It can only have two values:

$$+\hbar/2 \text{ or } -\hbar/2$$

The value of the spin for an electron is:

5.28×10^{-35} J s

The classical electron radius is given as 2.82×10^{-15} m

Let's work out the moment of inertia of the electron in its orbit:

$$I = 2/5 \times 9.11 \times 10^{-31} \text{ kg} \times (2.82 \times 10^{-15} \text{ m})^2 = 2.90 \times 10^{-60} \text{ kg m}^2$$

If the angular momentum of the electron is 5.28×10^{-35} J s, we can work out the angular velocity:

$$\omega = L/I = 5.28 \times 10^{-35} \text{ J s} \div 2.90 \times 10^{-60} \text{ kg m}^2 = 1.82 \times 10^{25} \text{ rad s}^{-1}$$

We can work out the linear speed of the electron using:

$$v = \omega r$$ .... *Equation 248*

v = 1.82× 10^{25} rad s^{-1} × 2.82 × 10^{-15} m = 5.13 × 10^{10} m s^{-1}

This means that the outer rim of the electron is travelling at a linear speed of 170 times the speed of light. This is not possible. Therefore, this model has a fatal flaw.

We can't say that one electron is going clockwise or anticlockwise as such. We cannot view the particle like this, let alone decide what is the top or the bottom. Electrons are not neat little blue marbles marked with e^-; they exist in a cloud of probability. Also, the model as shown can give other false results, for example, the size of an electron is bigger than an atom - clearly not true.

15.155 Quantum Model of Spin

The term **spin** has been carried over into the quantum nature of particles. As said above, electrons are not neat little blue marbles. They exist as little lumps of energy in a cloud of probability. The closer we get to an electron, the lower the probability of catching it, so we don't know the precise nature of the little brute. The precise nature of what gives an electron (or any other particle) the property of spin is not well understood. However, there are things we can say about it. Spin has the following properties:

- Spin can have values that are integers (whole numbers) and half integers (an odd whole number divided by 2);
- The direction can change, but the particle cannot be made to spin faster or slower;
- Spin is described by a quantum number, s, which is defined by:

$$s = \frac{n}{2}$$ .... *Equation 249*

where n is any whole number.

Particles with spin have a value and a direction. The SI units for spin are kg m^2 s^{-1}, but in particle physics the spin is given a dimensionless quantum number, with a positive or negative sign depending on the direction they are rotating in. The way this is done is by dividing the angular momentum by the **Shortened Plank's Constant**, $\hbar$. As we saw above, it is related to h, the **Planck's Constant** (6.63×10^{-34} J s) by:

$$\hbar = \frac{h}{2\pi}$$

.... *Equation 250*

We can, of course, give a value for $\hbar$:

$\hbar = 1.06 \times 10^{-34}$ J s

Electron spin is quantised. It can only have two values:

$+\hbar/2$ or $-\hbar/2$

We can divide these by $\hbar$ to write:

$+1/2$ or $-1/2$

The value of the spin for an electron is:

5.28×10^{-35} J s

Rather than use the value of the spin, we use integer and half integer values. We saw above that we can have spin values of 0, ½, 1, 1½, 2, etc. There are relationships that show this, but you are not expected to know them at this level. You will study them at university.

At this point, it is sufficient to say that electrons have two states of spin 1/2. They can be up (+) or down (-) as shown in *Figure 172*.

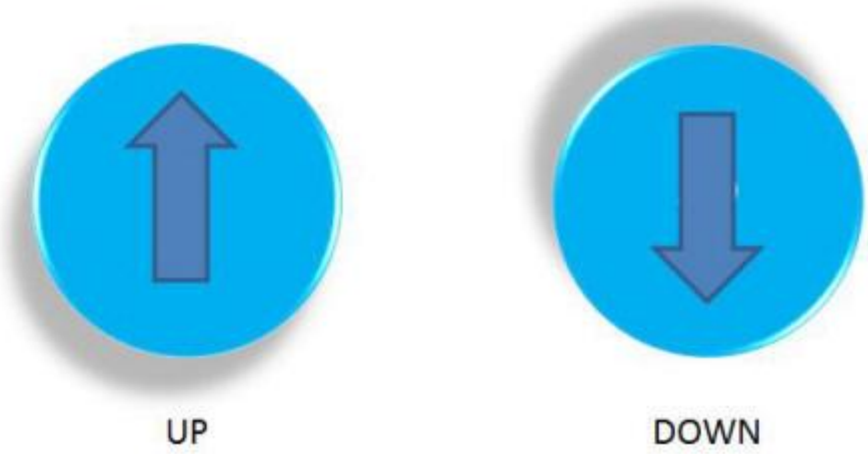


Figure 172 Up and down spin

Electrons are **elementary** particles called **fermions** with a spin of ½. Elementary particles with spins of 3/2, or 5/2 are not known to exist.

Two of the same fermions with a half-integer spin cannot occupy the same quantum state. This is called the **Pauli Exclusion Principle**. This means that if two electrons are in the same shell, their quantum numbers have to be different. While charge and mass have to be the same, the spin numbers have to be different.

Spin is **conserved** in the same way as angular momentum is conserved. This means that if the angular momentum is changed in a particle interaction, there has to be a corresponding angular momentum change elsewhere. But the quantum number of spin appears not to have to be conserved in the same way as lepton number, charge, or baryon number.

Note: the syllabus says that spin in antiparticles is opposite to the spin in particles. There seems to be quite a lot of confusion among users or different forums about this. Some users say that this is the case. However other sources I have looked at say that the spin in particles and antiparticles is the same.

15.156 Spin and Particles

Fermions such as quarks and leptons have a spin of ½. Elementary particles with spins of 3/2, or 5/2 are not known to exist.

Bosons have a spin of 1. It is thought that the graviton has a spin of 2, while the Higgs Boson has a spin of 0. Otherwise, bosons have a spin of 1. Helium-4 can show similar properties to bosons.

Photons have spin of 0.

Quarks all have a spin of 1/2. They are fermions.

Mesons are **composite** particles made of one quark and one antiquark. Rather confusingly, they can have spins of -1, 0, and +1. If the spin vectors are aligned upwards, the spin is +1. If the spin vectors are aligned downwards, the spin is -1. Of one spin vector is aligned upwards and the other is aligned downwards, the spin is 0.

Baryons are composite particles of three quarks (or three antiquarks if they are anti-baryons). Each one of the quarks can have a spin of +1/2 or -1/2. Therefore, the four following spins are possible:

Quark 1	*Quark 2*	*Quark 3*	*Spin*
-½	-½	-½	-3/2
+½	-½	-½	-1/2
+½	+½	-½	+1/2
+½	+½	+½	+3/2

Although **neutrons** are by their nature uncharged, spin from the quarks allow neutrons to interact with magnetic fields.

Antiparticles have the same spin as particles. Therefore, an antibaryon will still have any one of these spin states.

Spin is an important part of **quantum mechanics**, a branch of physics that carries this description, "If you think you understand quantum mechanics, you don't." You will have the chance to study it at university.

Questions

Tutorial 15.15

There are no questions for this tutorial.

Tutorial 15.16 The Quantum Atom

Pre-U and IB Syllabuses only

Contents

In Topic 3 Tutorial 4, we saw how electrons in excited atoms go up to certain energy levels. About a microsecond later, they fall back to lower energy levels releasing photons of very specific wavelengths. We also saw how a photon that excited an electron had to have exactly the right energy. A fraction less, the photon is not absorbed; a fraction more, it is not absorbed either. This explains how we see specific wavelengths in an emission spectrum.

We developed the theory further in Topic 15 Tutorial 2 to look at the Bohr Model of the hydrogen atom.

You may wish to review these tutorials before going on.

15.161 The Hydrogen Line Spectrum

The visible line spectrum of hydrogen consists of just four lines of different colours, like this (*Figure 173*):

Figure 173 Visible spectrum of hydrogen

However, there are many more spectral lines that are caused by transitions between different energy levels (*Figure 174*):

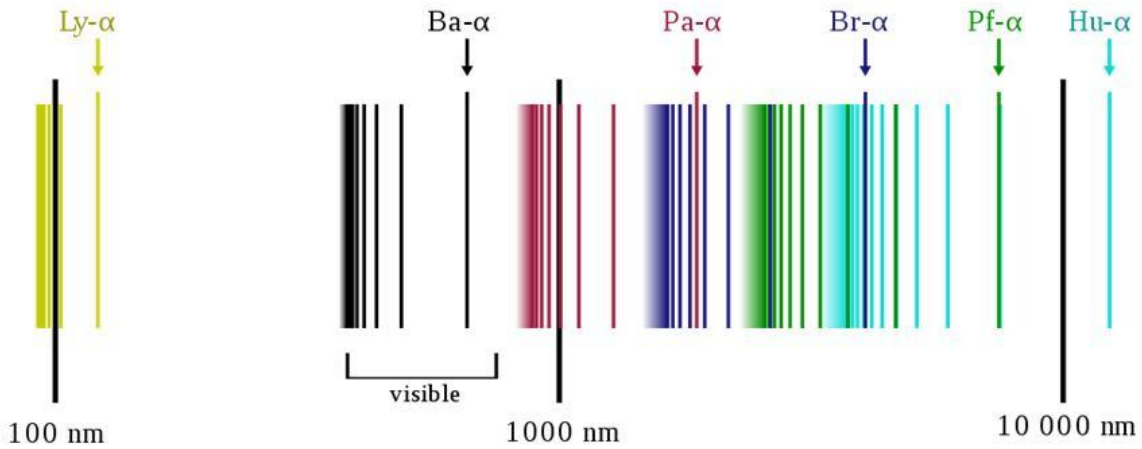


Figure 174 Spectral line in hydrogen (Image by OrangeDog, Wikimedia Commons)

The transitions give photons of the shortest wavelength in the ultra-violet region (100 nm). These are the most energetic. Wavelengths of 1000 nm are in the infra-red region, while wavelengths of 10 000 nm (1 × 10^{-5} m or 10 mm) are in the far infra-red region, getting on towards the microwave region. These transitions result in photons of the least energy.

When electrons are excited, they will go to the level of the exciting radiation, provided that the exciting energy is of exactly the right energy. Within 1 μs, they will fall back to lower energy levels, emitting photons.

If the exciting radiation is 13.6 eV or above, the electrons will be removed, and the hydrogen atom will be ionised. However, electrons will be attracted back to the ion. These electrons can go from the 0 eV level (the ionisation level, where the electron can escape) all the way down to the ground state ($n = 1$, -13.6 eV) in one big leap, or in many different steps. We also know that the rungs of the energy ladder are not equally spaced. Therefore, we can see that many photons of different wavelengths can be observed. An electron dropping down can pause on any rung of the energy ladder on their way to the ground state. The lower the rung, the higher the energy of the photons emitted. The photons of the highest energy result from transitions from higher energy levels all the way down to the ground state. Therefore, a **series** of wavelengths in the UV region is observed.

When observing ionised hydrogen, we observe all the emitted lines at the same time. This is because there are many thousands of millions of atoms being ionised. Also, the way electrons fall down the electron levels can be in any combination.

If the electrons pause at the level above the ground state ($n = 2$), photons in the visible region are observed. These form another series of wavelengths. The idea is shown in this diagram (*Figure 175*):

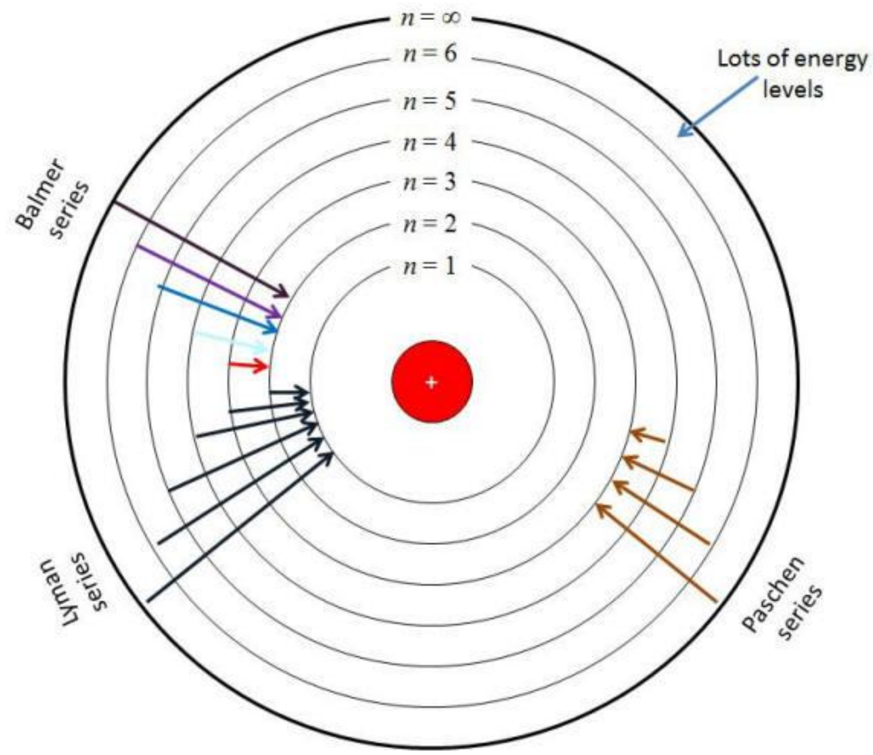


Figure 175 Transition series in hydrogen

Note that the levels in the diagram are regularly spaced for clarity. In reality the levels are not evenly spaced.

The transitions ending at $n = 1$ are called the **Lyman series**, after the American physicist, Theodore Lyman (1874 - 1954), who discovered them. Transitions that end at $n = 2$ are the **Balmer series**, named after Johann Balmer (1825 - 1898), a Swiss physicist. Transitions that end at $n = 3$ are called the **Paschen series**, after the German physicist, Friedrich Paschen (1865 - 1948).

Note that when the electron reaches its end level, it will still fall to the **ground state**, contributing to the **Lyman series**.

15.162 Lyman Series

All the photons emitted in the Lyman series are in the **UV region**. This is because the transition from $n = 2$ to $n = 1$ is from -3.41 to - 13.6 eV, a leap of 10.19 eV. A transition from $n = 3$ to $n = 1$ would give a photon of more energy, hence shorter wavelength. The main Lyman lines are shown below (*Figure 176*):

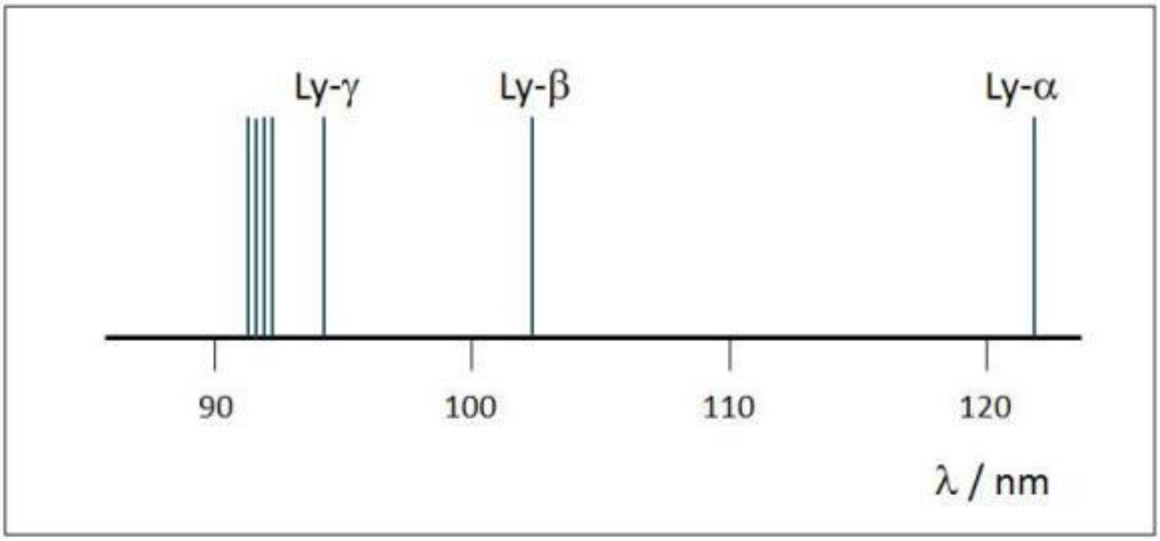


Figure 176 Lyman lines

The lines are marked **Ly** (for Lyman - really?), with a Greek letter, for example Ly-α. In the energy ladder, the lines are the result of these transitions (*Figure 177*):

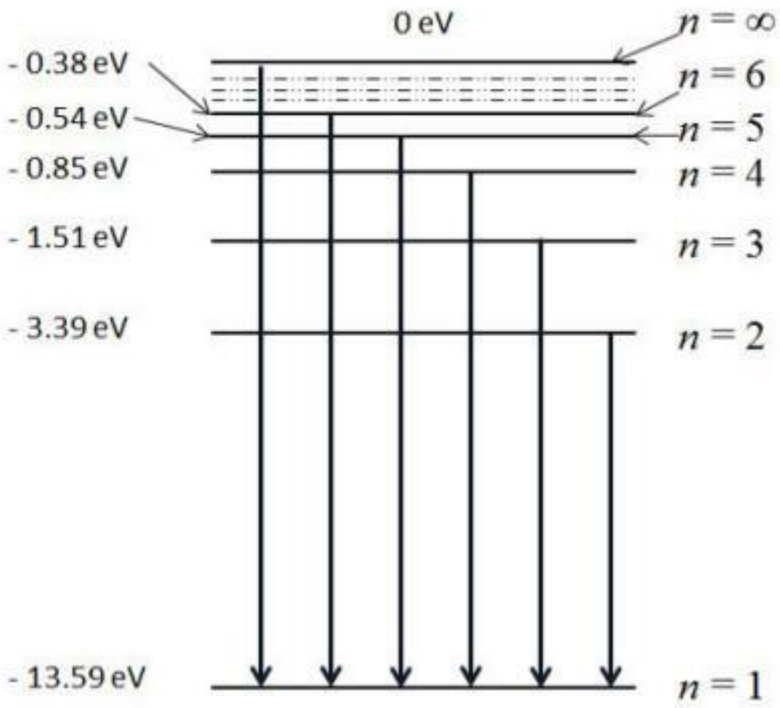


Figure 177 Lyman transitions

<u>Worked example</u>

Calculate the wavelength of a photon emitted when an electron falls from the levels $n = 6$ to $n = 1$.

<u>Answer</u>

We need to work out the value of the energy change as the electron falls:

$$\Delta E = E_1 - E_2$$

$$\Delta E = -0.38 \text{ eV} - -13.59 \text{ eV} = 13.21 \text{ eV}$$

We need to convert this to joules:

$$\Delta E = 13.21 \times 1.60 \times 10^{-19} \text{ J eV}^{-1} = 2.11 \times 10^{-18} \text{ J}$$

We now need to use:

$$E = \frac{hc}{\lambda}$$

Therefore:

$\lambda = (6.63 \times 10^{-34} \text{ J s} \times 3.00 \times 10^{8} \text{ m s}^{-1}) \div 2.11 \times 10^{-18} \text{ J} =$ **<u>9.41×10^{-8} m</u>** $= 94.1$ nm

15.163 Balmer Series

The photons emitted in the **Balmer series** are in or around the visible light region. All the transitions end at $n = 2$. The lines are marked with **Ba-** with a Greek letter. Their place on the energy ladders are shown (*Figure 178*):

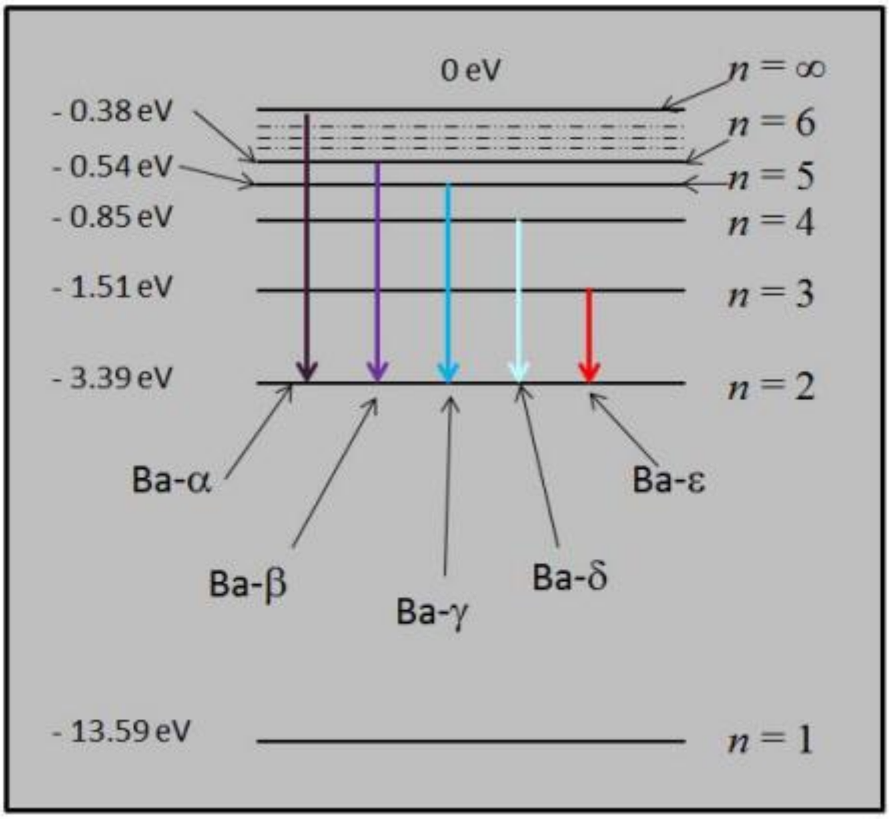


Figure 178 Balmer Transitions

The grey background is to make the blue-green line Ba-δ stand out a bit more.

The line Ba-α has a wavelength of 367 nm. Violet light has a minimum wavelength of 380 nm, so the shorter wavelength indicates that the Ba-α line is in the **UV region**. This is consistent with the observation of just four visible Balmer lines.

15.164 Paschen Series

As with the two series above, the lines are marked with the letters **Pa-**, with a Greek letter. The transitions end up at n = 3. They are in the infra-red region. The idea is shown below (*Figure 179*):

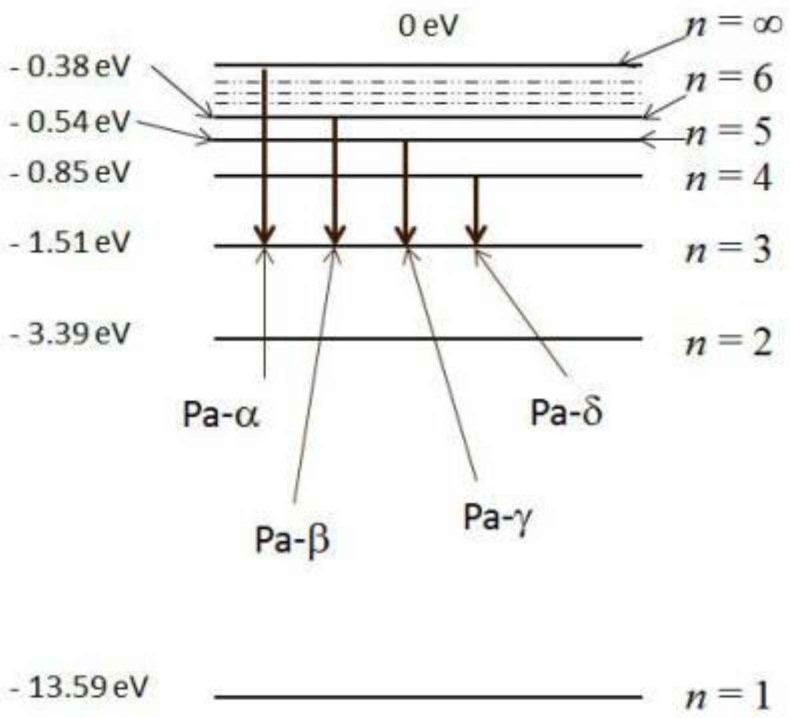


Figure 179 Paschen transitions

15.165 Other Series

There are series that end at n = 4 (**Brackett Series**), $n = 5$ (**Pfund Series**) and $n = 6$ (**Humphreys Series**). The lines get fainter, as these transition are increasingly rare events. Transitions at $n = 7$ have been found in the far infra-red region.

15.166 Equation for the Energy Levels

The electron energy levels in the excited hydrogen atom do not form a ladder with equally spaced rungs. They are very irregularly spaced. The **energy at a certain level** can be predicted using a simple equation:

$$E_n = \frac{E_0}{n^2}$$

.... Equation 251

- E_n is the energy at a certain energy level (eV).
- E_0 is the ionisation energy (eV).
- n is the energy level.

The spacing gets smaller and smaller, as would be expected with the energy being proportional to $1/n^2$.

In the hydrogen atom, E_0 is 13.6 eV. So, we can rewrite the equation:

$$E_n = \frac{13.6 \text{ eV}}{n^2}$$

.... Equation 252

We can model this on a spreadsheet:

Energy Level	Energy level / eV
1	-13.60
2	-3.40
3	-1.51
4	-0.85
5	-0.54
6	-0.38
7	-0.28
8	-0.21
9	-0.17
10	-0.14

We can plot these data a **bar-graph** (*Figure 180*). Note that a line graph would not be appropriate here due to the **quantum** nature of the energy levels. The energy levels are integers. You could, of course, plug numbers into the equation where $n = 1.25$, and get a number out of it (E_n = 8.704 eV), but that number would be meaningless.

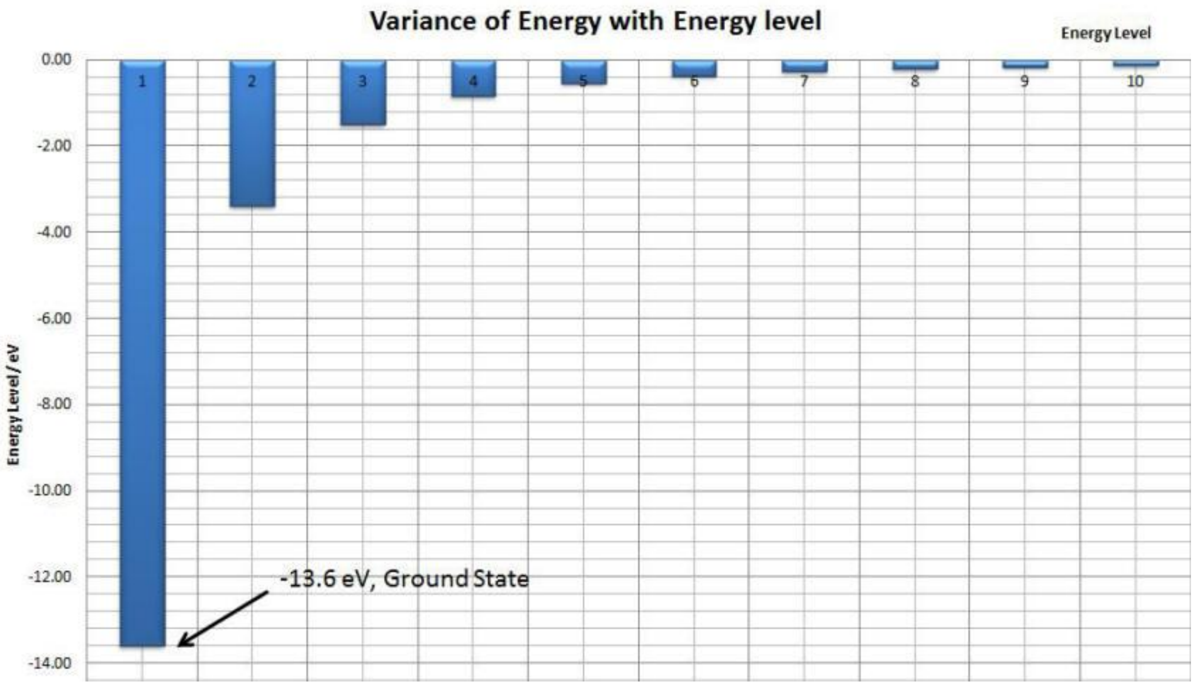


Figure 180 Data plotted as a bar graph

Notice that the signs are negative. This means that work is got out when the electron falls from the **excited state** to the **ground state**. The equation above is positive, showing the work required to raise an electron from an energy level to the ionised state.

15.167 Electron Standing Waves (*)

This will be examined only in the extension questions. It is difficult.

Electron behaviour is impossible to describe using classical physics. An orbiting electron can be in two places at once. It can appear anywhere within a shell without appearing at any point in between. There are certain points in which electrons have a high probability of appearing, and where the probability of them appearing is very low. We can model this as a standing wave (*Figure 181*).

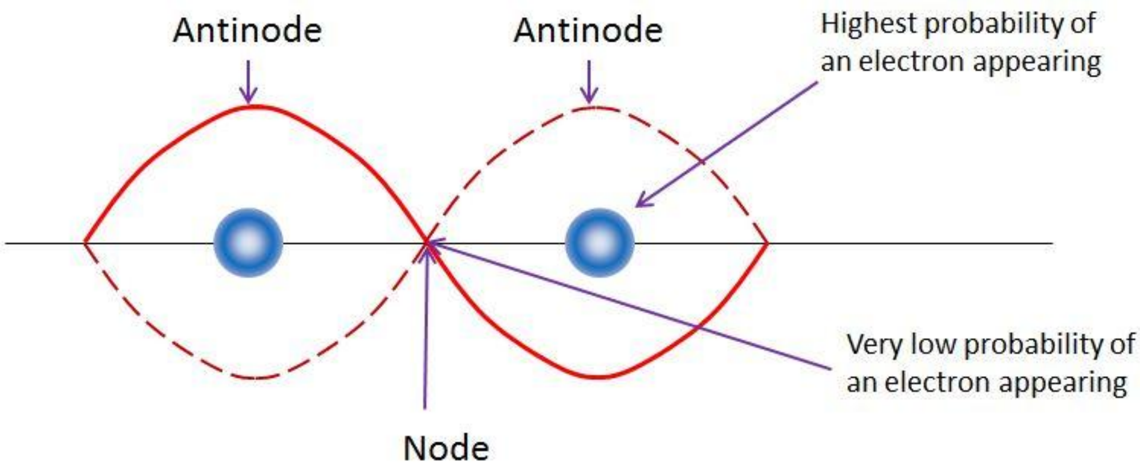


Figure 181 Electron standing waves

We can use much of what we know about standing waves to explain the standing wave behaviour of electrons. Standing wave behaviour is covered in Topic 7 Tutorial 4. You may wish to revise it before continuing.

In a resonating string, the line represents the position of the string in the standing wave. With electrons, the area enclosed by the line presents the probability of finding the electron. We will use the same ideas from standing waves in strings to help us to understand the way electron standing waves work. The classic way that we represent a standing wave in a string is a standing wave in **one dimension** (*Figure 182*).

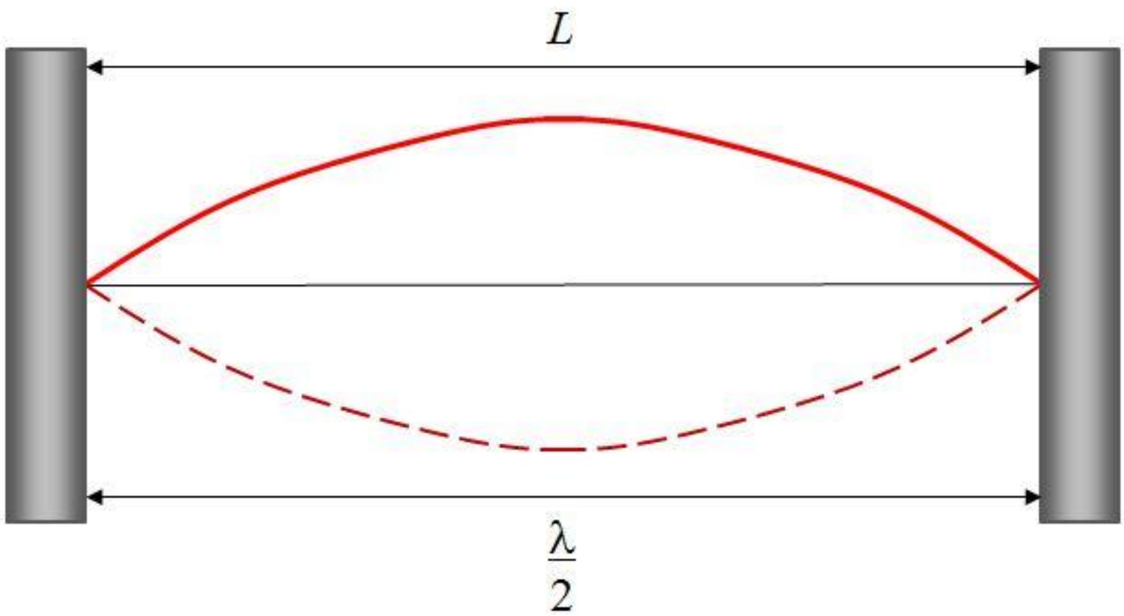


Figure 182 Standing wave in a string

We know that:

$$\lambda = 2L$$ .... *Equation 253*

We also know the **de Broglie relationship** that links de Broglie wavelength and **linear** momentum:

$$\lambda = \frac{h}{mv}$$ .... *Equation 254*

Rearranging:

$$mv = \frac{h}{\lambda}$$ .... *Equation 255*

So, we can write:

$$mv = \frac{h}{2L}$$ .... *Equation 256*

or

$$p = \frac{h}{2L}$$ .... *Equation 257*

We can also write a relationship between kinetic energy and momentum:

$$E_k = \frac{1}{2}mv^2$$ .... *Equation 258*

and momentum is given by:

$$p = mv$$ .... *Equation 259*

So, we can write:

$$E_k = \frac{p^2}{2m}$$ .... *Equation 260*

If we combine *Equations 257* and *260*, we get:

$$E_k = \frac{h^2}{8mL^2}$$ .... *Equation 261*

So what?

We know that all the energy in an electron is **kinetic**, so we can work out what the energy is for the fundamental standing wave.

Let us assume that the hydrogen atom has a diameter of 1.06×10^{-10} m. The electron standing wave looks like this (*Figure 183*):

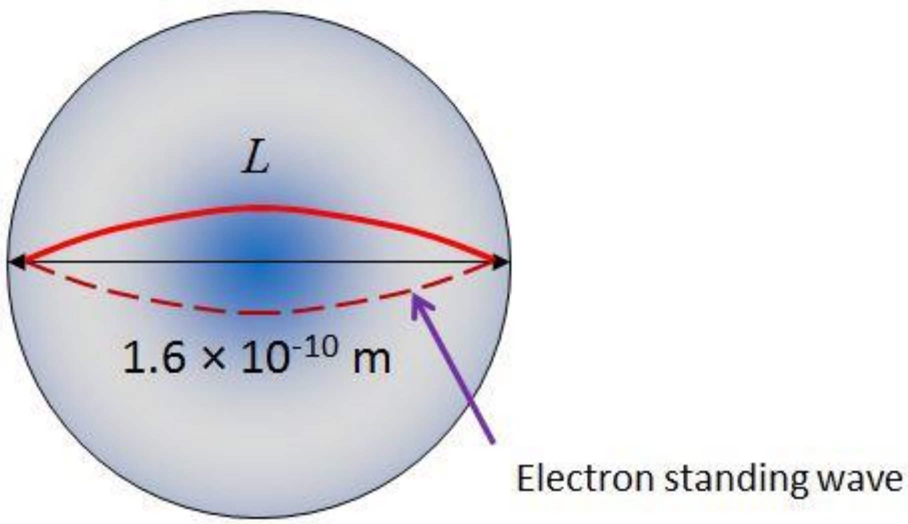


Figure 183 Electron standing wave

<u>Worked example</u> Calculate the energy of the electron standing wave in eV shown above. (Mass of an electron = 9.1×10^{-31} kg; Planck's constant = 6.6×10^{-34} J s)
<u>Answer</u> Use: $$E_k = \frac{h^2}{8mL^2}$$ $E_k = (6.6 \times 10^{-34}\text{ J s})^2 \div (8 \times 9.1 \times 10^{-31}\text{ kg} \times (1.6 \times 10^{-10}\text{ m})^2) = 2.34 \times 10^{-18}$ J $E_k = 2.34 \times 10^{-18}\text{ J} \div 1.6 \times 10^{-19}\text{ J eV}^{-1}$ = **<u>15 eV</u>**. This is very approximate to the real ionisation energy of the hydrogen. The correct answer is 13.6 eV

Let us suppose that we have two waves in the box (*Figure 184*):

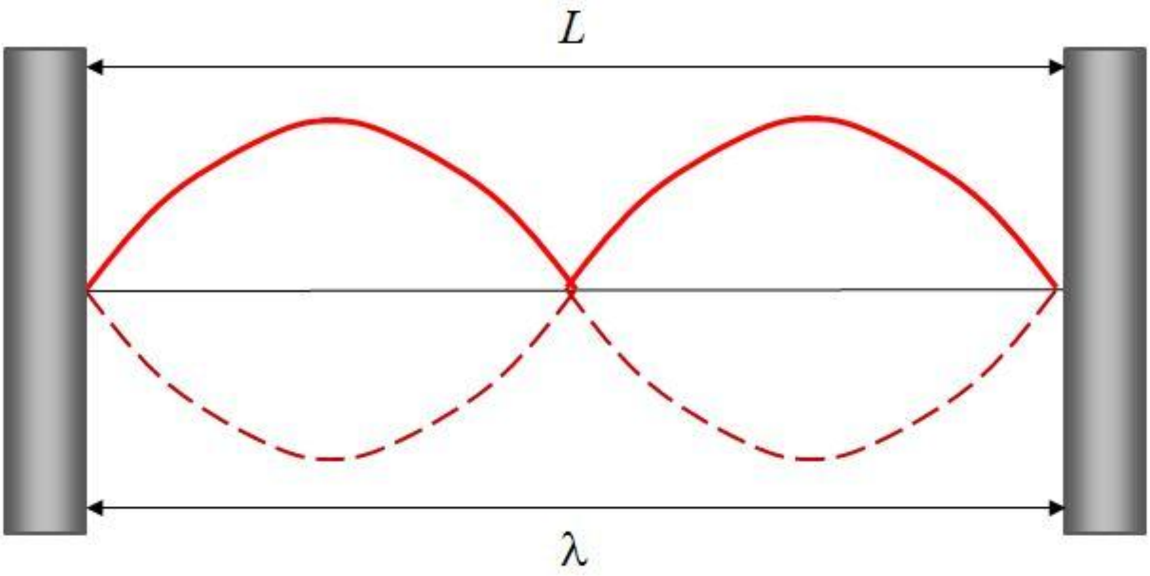


Figure 184 Two standing waves

By a similar argument to that above, we can write an equation for the energy of the two standing wave loops (one wavelength):

$$E_k = \frac{h^2}{2mL^2}$$

.... *Equation 262*

This is because we get one whole wavelength into the box instead of 1/2 wavelength. If the wavelength has a lower energy, it has a longer wavelength, so the length of the wave in the box increases.

<u>*Worked Example*</u> An excited electron in an atom falls from the ionised state to the *n* = 2 state (0 eV to -3.4 eV). (a) Calculate the energy in the photon emitted. (b) Use the equation above to calculate the size of the box needed to accommodate the electron in its excited state, assuming the box is one whole wavelength in size. (Mass of an electron = 9.1×10^{-31} kg; Planck's constant = 6.6×10^{-34} J s)
<u>*Answer*</u> (a) ΔE = 0 eV - -3.4 eV = 3.4 eV (work is got out) Convert the energy to joules: $E = 3.4 \text{ eV} \times 1.6 \times 10^{-19} \text{ J eV}^{-1} = 5.44 \times 10^{-19}$ J (b) Rearrange the equation above to: $$L^2 = \frac{h^2}{2mE_k}$$ $L^2 = (6.6 \times 10^{-34} \text{ J s})^2 \div (2 \times 9.1 \times 10^{-31} \text{ kg} \times 5.44 \times 10^{-18} \text{ J}) = 4.40 \times 10^{-20} \text{ m}^2$ $L = (4.40 \times 10^{-19} \text{ m}^2)^{0.5}$ = <u>**2.1 × 10⁻¹⁰ m**</u>

The box swells as the electron gets excited. This is because it accommodates two longer standing wave loops of lower energy.

The difficulty is that this model explains the idea of electron standing waves just like those on a string. However, the numerical analyses above do not give convincing consistencies with the wavelengths and energy. In reality the waves loops are formed on the **circumference** of the electron shell, not across its diameter. Another problem is that the **nucleus** is at the same point as where the probability of the electron being found is the highest.

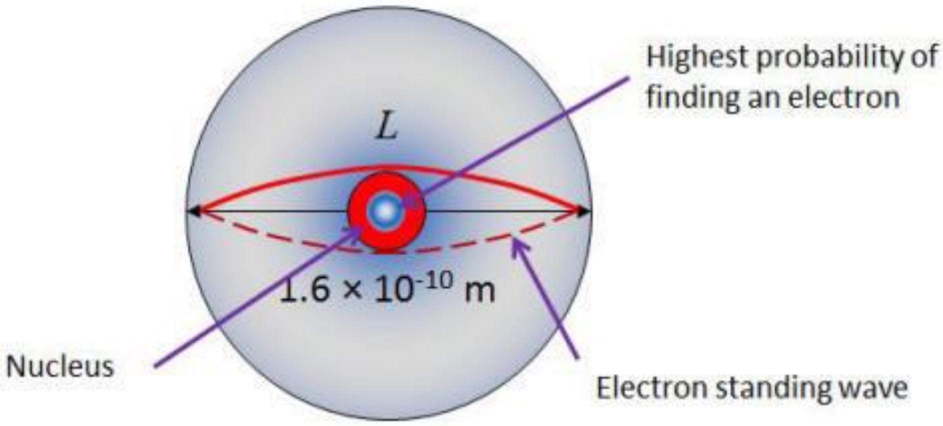


Figure 185 Probability wave

This could even result in an electron capture event.

15.168 The Bohr Model and Electron Standing Waves (*)

This helps us to understand the idea of electron standing waves, but it only shows what happens in one-dimension. A more sophisticated model is one that shows us what happens in when we see two dimensions. This uses the Bohr Model (*Figure 186*):

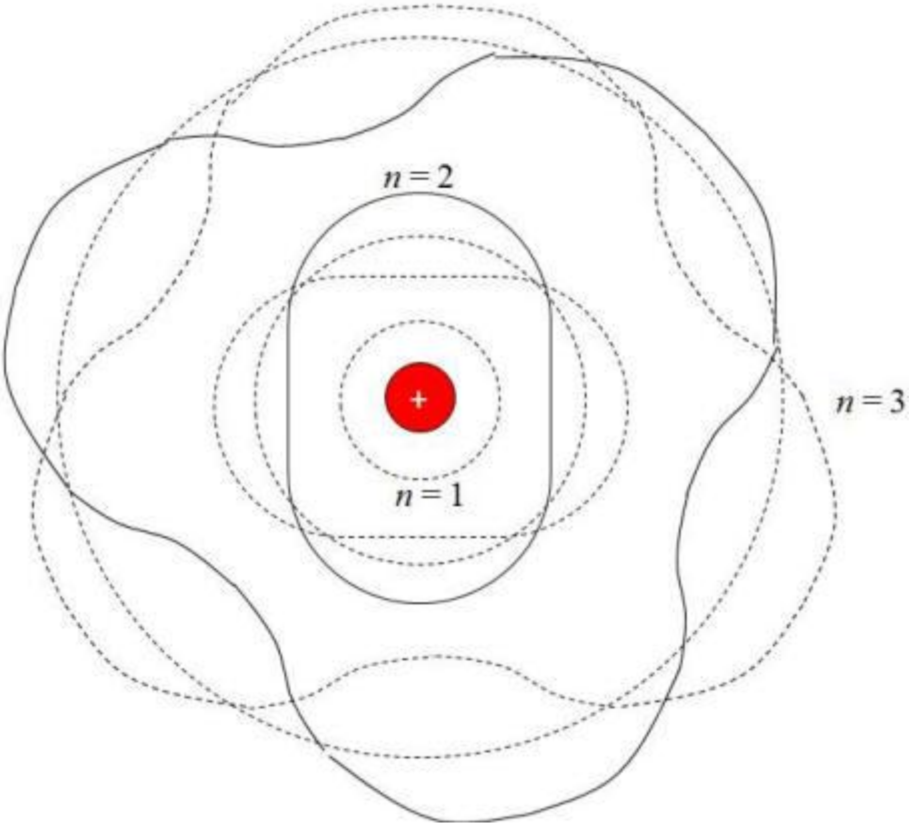


Figure 186 Standing waves and the Bohr model

The $n = 1$ resonance is circular and represents a single standing wave loop (i.e. half a wavelength). The $n = 2$ resonance represents two standing wave loops (i.e. one whole wavelength). The $n = 3$ resonance has three standing wave loops (not very well drawn, but it will have to do). It shows 3/2 wavelengths.

For the **Bohr model** electronic resonance patterns, the radius is the **Bohr Radius**, a_0, where:

$$a_0 = 0.0529 \text{ nm} = \mathbf{5.29 \times 10^{-11} \text{ m}}$$

The wavelength of the first resonance, $n = 1$, is given by:

$$\lambda_1 = 2\pi r_1 = 6.28 a_0$$ .... *Equation 263*

The wavelength of the second resonance, $n = 2$, is given by:

$$2\lambda_2 = 2\pi r_2$$ .... *Equation 264*

If n doubles, the radius according to the Bohr Model increases four times. We have before that the energy goes down four times as n goes from 1 to 2. Therefore, the wavelength increases by four times. So, we can say:

$$2\lambda_2 = 8\pi r_1$$ .... *Equation 265*

Therefore:

$$\lambda_2 = 4\pi r_1 = 12.57 a_0$$ .... *Equation 266*

The wavelength of the second resonance, $n = 3$, is given by:

$$3\lambda_3 = 2\pi r_3$$ .... *Equation 267*

Since the wavelength goes up as n^2, we can write:

$$3\lambda_3 = 18\pi r_1$$ .... *Equation 268*

Therefore:

$$\lambda_3 = 6\pi r_1 = 18.85 a_0$$ .... *Equation 269*

15.169 Angular Momentum in Electron Orbits (*)

In the Bohr model, standing waves occur around the circumference of the electron shell. For a standing wave to form, the circumference has to be the same distance as a **whole number** of **standing wave loops**. When we looked at how electrons occurred in 1-dimension, we used the **linear momentum** in the de Broglie wavelength equation, i.e.:

$$\lambda = \frac{h}{mv}$$ .... *Equation 270*

We know that in the quantum atom, the wavelengths are quantised, meaning that they have to have a very specific value. The same is true for the standing wave loops. Therefore, the circumferences of the electron shells have to be quantised as well. The **wavelength** of the **standing wave** at any energy level is given by:

$$2\pi r = n\lambda_n$$ .... *Equation 271*

To work with a de Broglie wavelength of an electron orbiting the nucleus of an atom, we need to use the **angular momentum** for the **circular motion**:

$$L = mvr$$ .... *Equation 272*

- L = Angular momentum (kg m^2 s^{-1}).
- v = velocity (m s^{-1}).
- r = radius (m).

Do not use:

$$L = I\omega$$

for angular momentum as this refers to rotational motion, not circular motion.

Note that L is used as the physics code for both angular momentum, and the length of a standing wave loop. Be careful what you are referring to.

So, we can combine the equation for angular momentum and the de Broglie equation:

$$L = mvr = \frac{rh}{\lambda}$$ .... *Equation 273*

From the equation:

$$2\pi r = n\lambda_n$$ .... *Equation 274*

we can write:

$$\lambda_n = \frac{2\pi r}{n}$$ .... *Equation 275*

We can then substitute *Equation 275* into the *Equation* 273 for the angular momentum:

$$mvr = \frac{rh}{\left[\frac{2\pi r}{n}\right]} = \frac{nrh}{2\pi r} = \frac{nh}{2\pi}$$ .... *Equation 276*

We will use this relationship in the Bohr Orbit.

15.1610 Classical Electron Orbit (*)

In Physics 4, we studied centripetal force in Topic 8 Tutorial 1. We also looked at the forces between charges in Topic 9 Tutorial 4. You may wish to review these.

Consider an electron of charge e orbiting a nucleus of proton number Z. As the electron is travelling in a circular path of radius r, it is subject to a centripetal force F. Since the charges are opposite, the force will be negative, i.e., attractive. However we would have negative signs on both sides, which cancel out.

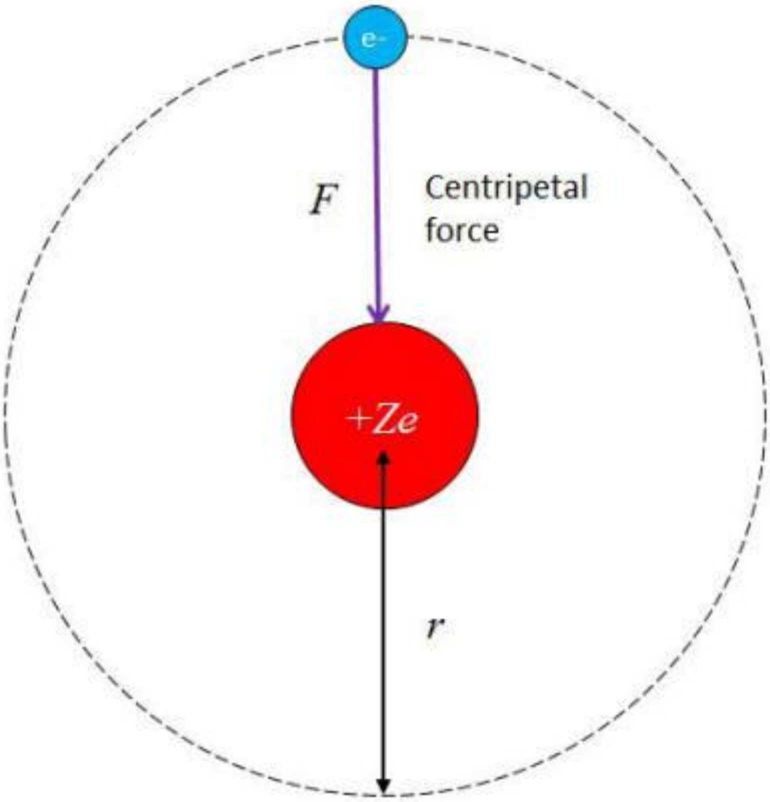


Figure 187 Centripetal force acting on an electron

From circular motion, we can write:

$$F = \frac{mv^2}{r}$$

.... *Equation 277*

From electric fields, we can write:

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

.... *Equation 278*

And we can combine *Equations 277* and *278* to write:

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

.... *Equation 279*

Cancelling:

$$mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r}$$

.... *Equation 280*

The electron in orbit has two kinds of energy, **kinetic** (because it's moving) and **potential energy** (the work that can be got out as a result of the centripetal force). We can work out the kinetic energy first:

$$E_\mathrm{k} = \frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

.... *Equation 281*

We can work out the potential energy from the attraction between the two charges:

$$E_\mathrm{p} = \frac{-Ze^2}{4\pi\varepsilon_0 r}$$

.... *Equation 282*

The minus sign is important to the argument, so don't miss it out.

The total energy of the orbiting electron around the hydrogen nucleus is:

$$E_\mathrm{tot} = E_\mathrm{k} + E_\mathrm{p} = \frac{Ze^2}{8\pi\varepsilon_0 r} + \frac{-Ze^2}{4\pi\varepsilon_0 r} = \frac{-Ze^2}{8\pi\varepsilon_0 r}$$

.... *Equation 283*

15.1611 The Bohr Orbit (*)

The Bohr orbit combines the energy from the classical argument above with the quantisation of the angular momentum, i.e.:

$$E_k = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

...........*Equation 284*

and

$$mvr = \frac{nh}{2\pi}$$

.... *Equation 285*

Kinetic energy is related to momentum by:

$$E_k = \frac{p^2}{2m}$$

.... *Equation 286*

By a similar argument we can related the kinetic energy to the angular momentum:

$$E_k = \frac{L^2}{2mr^2} = \frac{(mvr)^2}{2mr^2}$$

.... *Equation 287*

Now we can substitute for mvr:

$$E_k = \frac{(nh)^2}{(2\pi)^2 \times 2mr^2} = \frac{n^2h^2}{8\pi^2mr^2}$$

.... *Equation 288*

This gives us an expression for the quantisation for the angular momenta for the energy levels. We can now equate this last equation to:

$$E_k = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

.... *Equation 289*

and write:

$$\frac{Ze^2}{8\pi\varepsilon_0 r} = \frac{n^2h^2}{8\pi^2 mr^2}$$

.... *Equation 290*

Cancelling:

$$\frac{Ze^2}{8\pi\varepsilon_0 r} = \frac{n^2h^2}{8\pi^2 mr^2}$$

.... *Equation 291*

Now we make *r* the subject:

$$r = \frac{8\pi\varepsilon_0 n^2h^2}{8\pi^2 mZe^2} = \frac{\varepsilon_0 n^2h^2}{\pi mZe^2}$$

.... *Equation 292*

Now we substitute this for r in the equation:

$$E_k = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

.... *Equation 293*

which gives us:

$$E_k = \frac{Ze^2}{8\pi\varepsilon_0\left[\frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}\right]} = \frac{Ze^2 \times \pi m Z e^2}{8\pi\varepsilon_0 \times \varepsilon_0 n^2 h^2} = \frac{Z^2 m e^4}{8{\varepsilon_0}^2 n^2 h^2}$$

.... Equation 294

This looks quite a difficult result to take in until we realise that many of these terms are constants:

- e - electronic charge = 1.60 × 10^{-19} C.
- m - mass of an electron = 9.11 × 10^{-31} kg.
- e_0 - permittivity of free space = 8.85 × 10^{-12} F m^{-1}.
- h - Planck's constant = 6.63 × 10^{-34} J s.

Hydrogen has a proton number of 1. Let's consider the first level, $n = 1$. So, we can put these numbers in:

E_k = (1 × 9.11 × 10^{-31} kg × (1.60 × 10^{-19} C)^4) ÷ (8 × (8.85 × $10^{-12 F}$ m^{-1})^2 × 1^2 × (6.63 × 10^{-34} J s)^2)

= (9.11 × 10^{-31} kg × 6.55 × 10^{-76} C^4) ÷ (8 × 7.82 × 10^{-23} F^2 m^{-2} × 4.40 × 10^{-67} J^2 s^2)

= (59.67 × 10^{-107} kg C^{-4}) ÷ (275 × 10^{-90} F^2 J^2 s^2 m^{-2})

= 0.217 × 10^{-17} J = **21.7 × 10^{-19} J**

Now we can convert this to eV

E = 21.7 × 10^{-19} J ÷ 1.60 × 10^{-19} J eV^{-1} = **13.6 eV**

Notice that there was a power of ten that was greater than 100. This will not work in a calculator. This is where the brain is quite useful...

In this calculation we used $n = 1$. We can easily modify this for any value of n, to give this relationship:

$$E_n = \frac{13.6 \text{ eV}}{n^2}$$

.... Equation 295

We can rearrange the equation:

$$E = \frac{Z^2 m e^4}{8 {\varepsilon_0}^2 n^2 h^2}$$

.... Equation 296

to:

$$\frac{E n^2}{Z^2} = \frac{m e^4}{8 {\varepsilon_0}^2 h^2}$$

.... Equation 297

The right hand side of this equation consists of constants. The constants work out to the **Rydberg Constant**, which has a value of **21.7 × 10^{-19} J** or 13.6 eV. If $Z = 1$, we can rearrange to have E_n on the left hand side to give:

$$E_n = \frac{13.6 \text{ eV}}{n^2}$$

.... Equation 298

This relationship works well in hydrogen. The idea was that it should work for any proton number. But it doesn't.

Questions

Tutorial 15.16

There are no questions for this tutorial.

Tutorial 15.17 Interpreting Quantum Theory (*)	
Pre-U and IB Syllabuses only	
Contents	
15.171 The Double Slit Experiment	15.172 Wave Function
15.173 The Copenhagen Interpretation	15.174 Feynman's Sum-over-Histories
15.175 The Many Worlds Interpretation	15.176 Schrödinger's Cat
15.177 Uncertainty Principle	15.178 Determinism in Physics
15.179 Einstein and Quantum Theory	

The material in this tutorial will only be examined in Paper 3 Section 2.

15.171 The Double Slit Experiment

Interference of light passing through two close-set parallel slits was first demonstrated by Thomas Young (1773 - 1829) in 1801. From the section on Waves, we know that when a **laser** is passed through two slits, an interference pattern is observed (*Figure 188*):

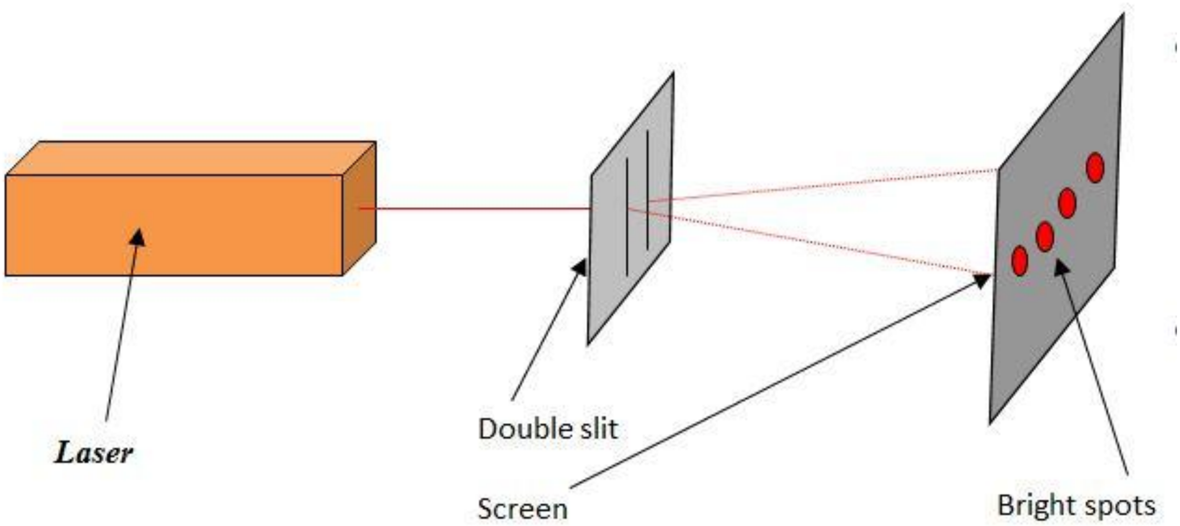


Figure 188 Young's Double slits

The pattern observed on the screen was like this (*Figure 189*):

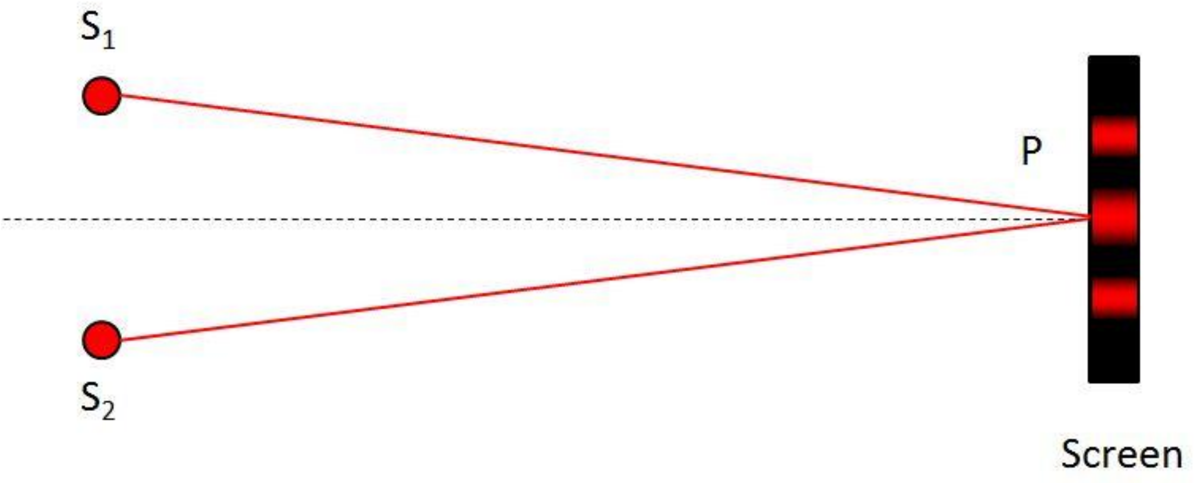


Figure 189 Pattern observed in Young's slits

You may wish to revise this in Topic 7 Tutorial 7, and a more detailed discussion is available in Topic 15 Tutorial 7.

Physicists thought that electrons were particles. If the electrons passed through double slits, the slits would act as a **stencil** to give a pattern like this (*Figure 190*):

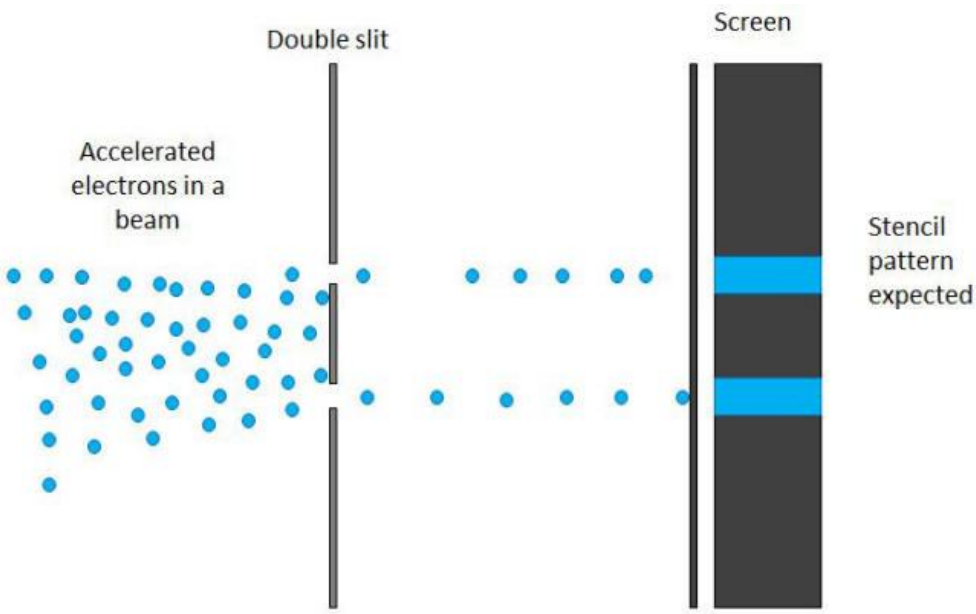


Figure 190 If electrons were particles

Instead, as we saw in Topic 3 Tutorial 6, accelerated electrons could be made to form diffraction patterns (*Figure 191*):

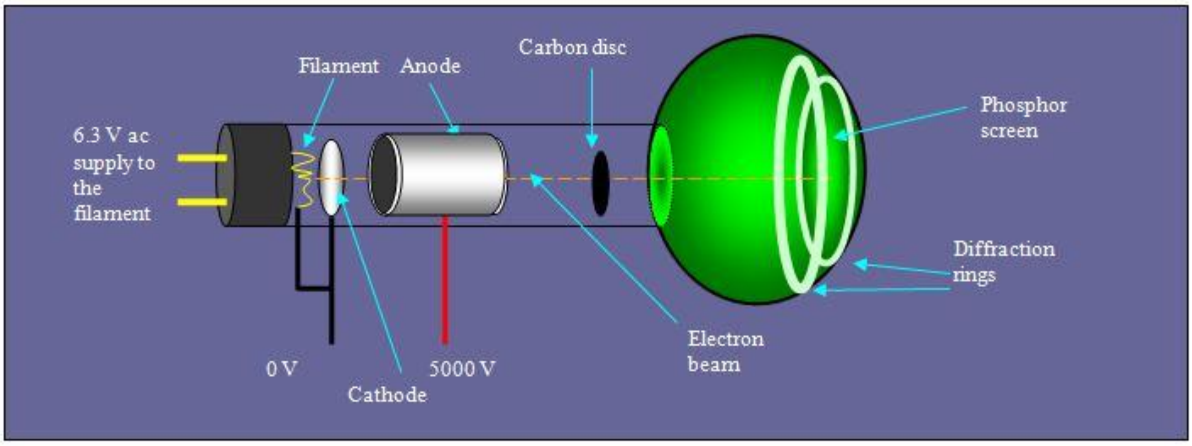


Figure 191 Diffraction patterns formed be electrons

We saw that if waves had particle properties (photons), it was very reasonable to assume that particles has wave properties. This led to the concept of **wave-particle duality.** It would also be reasonable to suppose that if electrons could diffract, they could **interfere**. And if we set up a similar apparatus to the above to show two slit interference, we see an **interference pattern**. The idea is shown below (*Figure 192*):

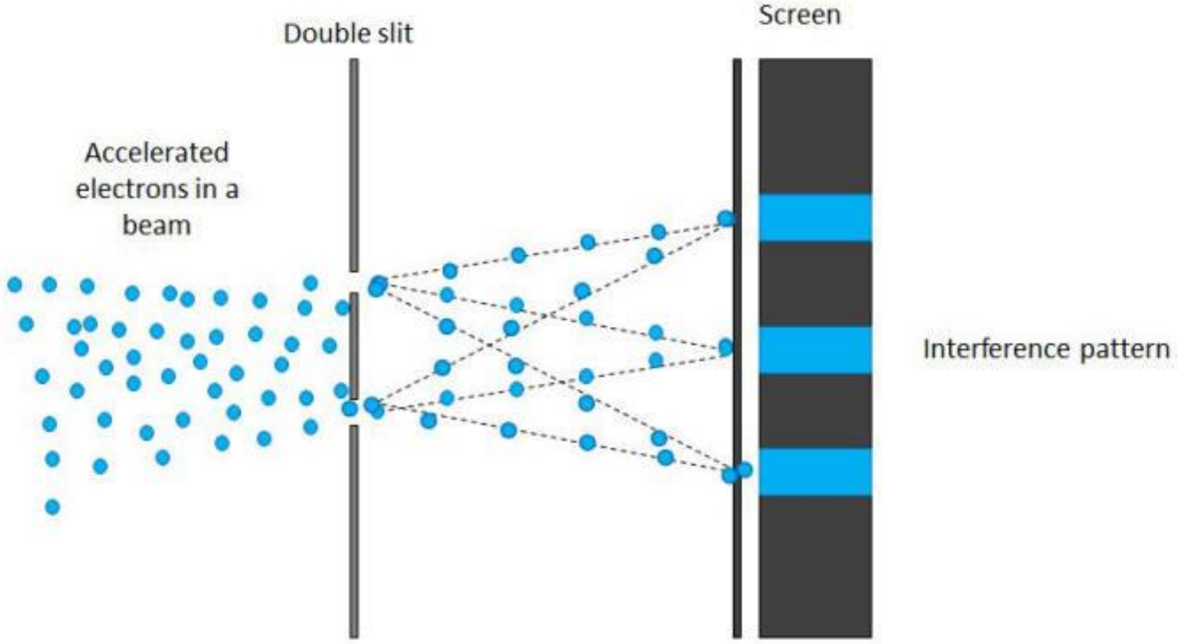


Figure 192 Interference pattern made by electrons passing through double slits

The same effect is observed with **neutrons**. (Neutrons, being neutral cannot be accelerated by electric fields and do not interact with magnetic fields either. Instead, they are be emitted from **fission reactions** and formed into beams. An alternative way to produce a more intense beam is to strip them from their partnering protons by a **deuteron-stripping reaction**. We won't go into this here.)

So far, so good, but know it's going to get weird...

It could be argued that many thousands of electrons jostling against each other could somehow interfere. Would the pattern be different if one single electron at a time was fired at the screen? This is what they saw (*Figure 193*):

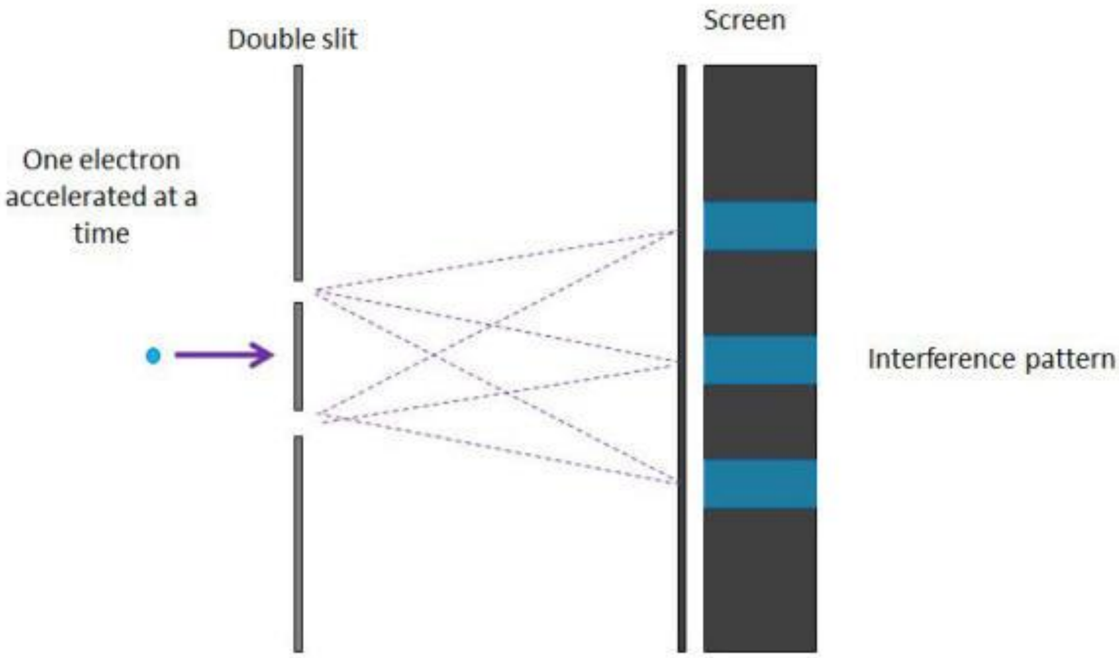


Figure 193 Interference pattern made by a single electron passing double slits

If the electron was a particle, it would hit the double slit and be absorbed. There would be no pattern at all. Even if the electron passed through one or other of the slits, the pattern would be that of a stencil. Instead, there was an **interference** pattern, suggesting that the electron was in two places at once. We could say that the electron was a wave, which would be consistent with wave-particle duality. However, if an electron was tracked, the pattern observed was that of the **stencil**, i.e. (*Figure 194*):

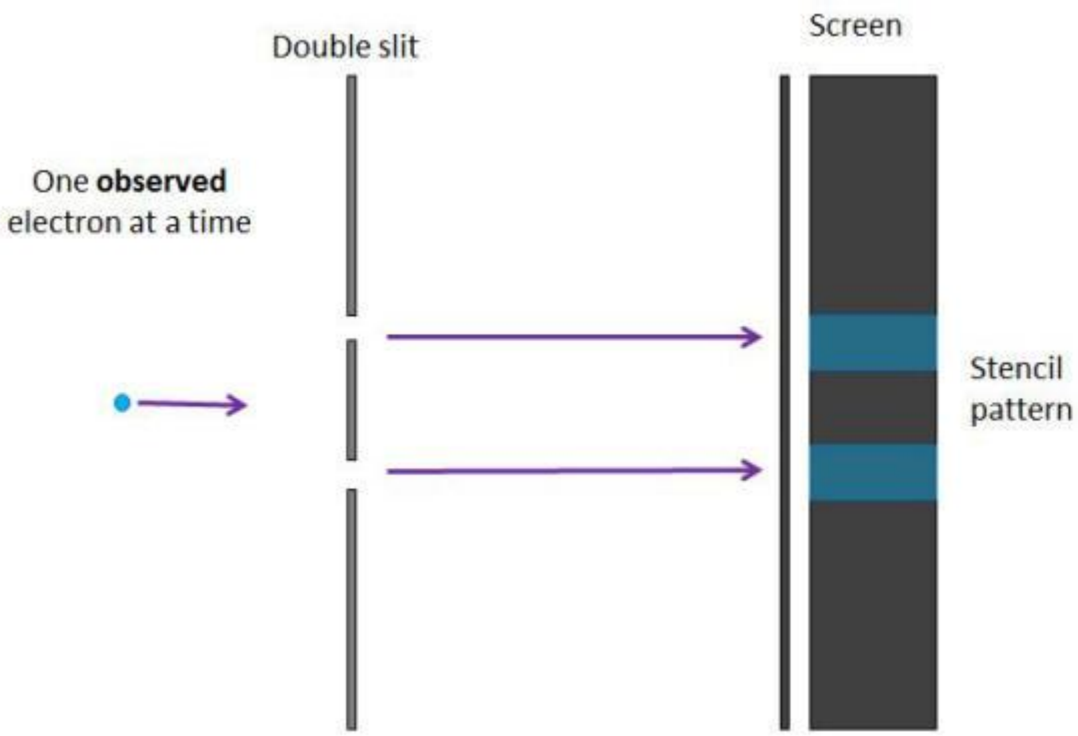


Figure 194 Tracking a singles electron gives a stencil pattern.

It was hard to explain such a contradiction, but in 1920, an attempt was made by Niels Bohr and Werner Heisenberg.

15.172 The Wave Function

The state of quantum systems is represented by a **wave function**, given the physics code Ψ, ("Psi", a Greek capital letter 'Ps', as in *psycho*). The wave is not mechanical, electromagnetic, or electrical. Wave function unites the wave and particle properties of quantum particles like electrons using a complex mathematical equation which is beyond what you need to know. The amplitude represents a **probability** of finding a particle in a particular place.

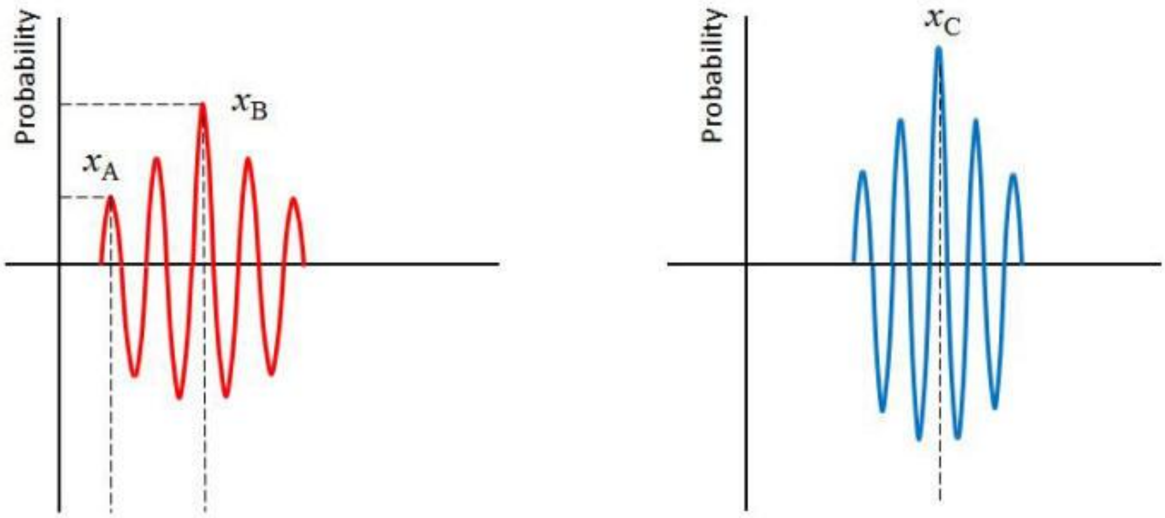


Figure 195 Probability waves

Therefore, the probability of finding a particle at x_C is greater than the probability of finding the particle at x_B, which is greater than the probability of finding the particle at x_A. The "wavelength" or "period" has no significance.

The **wave function** describes the probability of a particle being at a point. It is given the Physics code Ψ. It represents the **amplitude** of a probability wave, or a de Broglie matter wave. The units for Ψ are $m^{-1.5}$. The amplitude of such a wave does not have any specific physics significance. However, the term Ψ^2 (Psi-squared) is the **probability per unit volume**. The units for Ψ^2 are m^{-3}.

Consider an electron orbiting a hydrogen nucleus (a proton) at the Bohr radius (5.29×10^{-11} m). The idea is shown below:

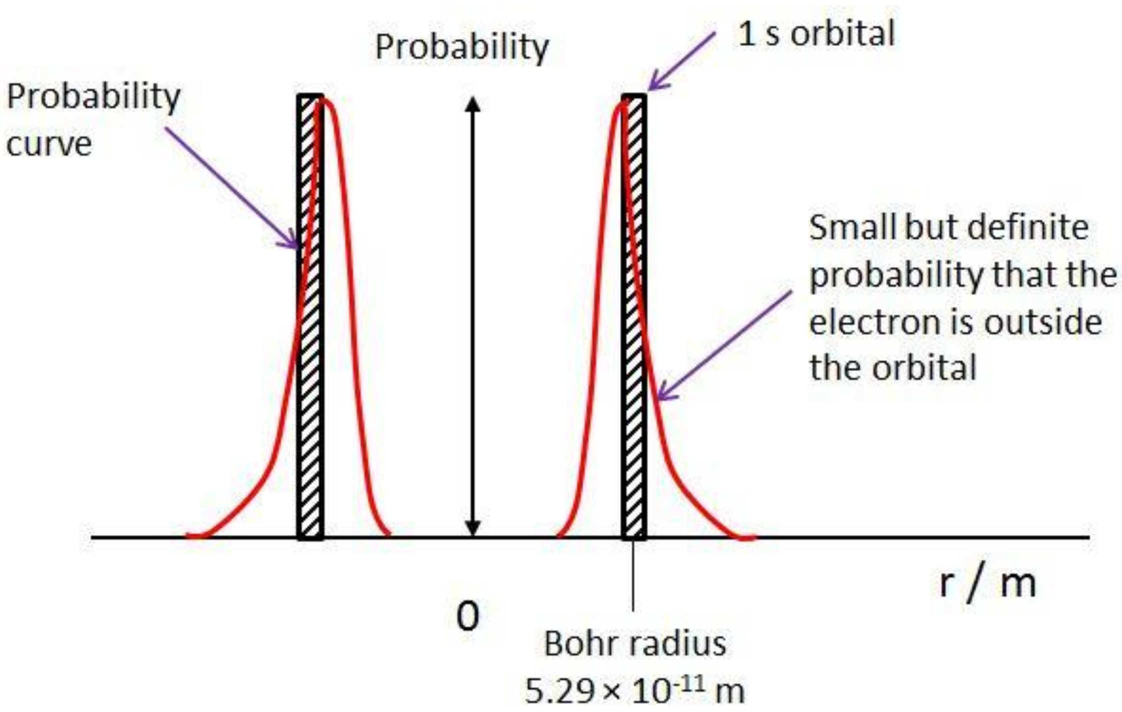


Figure 196 The Bohr radius

At the Bohr radius, the probability of finding the electron is highest. However, there is a small probability that the electron can be inside the Bohr radius, or outside the Bohr radius. Therefore, the cloud is not smooth billiard ball (nor is it light blue). It is fuzzy. The probability can be worked out using the equation:

$$P(r) = |\Psi^2|\Delta V$$

.... *Equation 299*

The terms are:

- $P(r)$ - the probability of finding the electron at a radius, r (no units).
- Ψ^2 - the square of the wave function, which is the probability per unit volume (m^{-3}).
- ΔV - the difference in the volume between where the electron is and the volume at the Bohr radius (m^{-3}).

To get a value for Ψ, we need to use the Schrödinger equation, which is challenging to say the least. It is a differential equation that has different solutions in different circumstances. It also has complex numbers ($i^2 = -1$). You will meet it at university, but its discussion is beyond the scope of these notes. For the electron in the ground state the solution is this equation:

$$\Psi(r) = \frac{1}{\pi^{0.5}a^{1.5}}e^{-r/a}$$

.... *Equation 300*

The key terms are:

- a - the Bohr radius (5.29×10^{-11} m).
- r - the radius of where the electron is (m).

So, let's put some numbers in:

<u>*Worked example*</u> An electron is orbiting a hydrogen nucleus at the Bohr radius. What is the value of the wave function at the Bohr radius? Give the unit. Hence work out the probability density.
<u>*Answer*</u> We will break up the equation into its bits and work out the value of each bit and then put it all together. $(p^{0.5} \times (5.29 \times 10^{-11})^{1.5}) = 6.82 \times 10^{-16}\ m^{-3/2}$ $e^{-5.29 \times 10^{-11} \div 5.29 \times 10^{-11}} = e^{-1} = 0.367$ $\Psi = (6.82 \times 10^{-16}\ m^{-3/2})^{-1} \times 0.367 = 5.38 \times 10^{14}\ m^{-3/2}$. Probability density = Ψ^2 = <u>**2.90×10^{29} m^{-3}**</u>.

This answer seems to be very high, but remember that it's a probability **density**, probability per unit volume.

Now let's consider the probability of finding an electron that has moved from a radius of 5.29×10^{-11} m to a radius of 5.31×10^{-11} m.

Worked example
An electron is orbiting a hydrogen nucleus at a radius of 5.31×10^{-11} m. What is the value of the wave function at this radius? Hence work out the probability density.
Answer We will break up the equation into its bits and work out the value of each bit and then put it all together. $(p^{0.5} \times (5.29 \times 10^{-11})^{1.5}) = 6.82 \times 10^{-16}\ m^{-3/2}$ $e^{-5.29 \times 10\text{^}-11 \div 5.31 \times 10\text{^}-11} = e^{-0.996} = 0.369$ $\Psi = (6.82 \times 10^{-16}\ m^{-3/2})^{-1} \times 0.369 = 5.41 \times 10^{14}\ m^{-3/2}$. Probability density = $\Psi^2 = 2.93 \times 10^{29}\ m^{-3}$.

Again, this number seems very large, but let's see what happens when put it into the equation (*Equation 299*):

$$P(r) = |\Psi^2|\Delta V$$

Worked Example
What is the probability of finding at electron outside the 1s orbital (the Bohr radius) at a radius of 5.31×10^{-11}.
Answer We need to work out the change in volume: $V = \frac{4}{3}\pi r^3$ At the Bohr radius: $V_B = 4/3 \times \pi \times (5.29 \times 10^{-11}\ m)^3 = 6.20 \times 10^{-31}\ m^3$ At the new radius: $V_r = 4/3 \times \pi \times (5.31 \times 10^{-11}\ m)^3 = 6.27 \times 10^{-31}\ m^3$

ΔV = 6.272 × 10^{-31} m^3 - 6.200 × 10^{-31} m^3 = 0.072 × 10^{-31} m^3

Now substitute:

$P(r)$ = 2.93 × 10^{29} m^{-3} × 0.072 × 10^{-31} m^3 = **0.0021**

This gives us a one in five hundred chance of finding the electron outside the 1s orbital.

(Note: Finding numerical values of Ψ has been incredibly difficult. The sources that I have searched have given long and learned expositions in mathematics that have not been that helpful. I have not yet seen a value for Ψ anywhere, so my calculation may well be completely wrong. If you can do better, please tell me (without complicated mathematics) and I will give you credit for it. Your explanation should be understandable for a pre-university student.)

15.173 Copenhagen Interpretation

This was devised by Niels Bohr and Werner Heisenberg and was given the name as Copenhagen is where they worked. It is the most widely accepted interpretation of the observations above.

The **Copenhagen Interpretation** went along these lines. If a quantum system, for example an electron, is left to its own devices, **superposition** could occur between the wave functions of the electron as it occupied its quantum space. This interference patterns could be formed (*Figure197*).

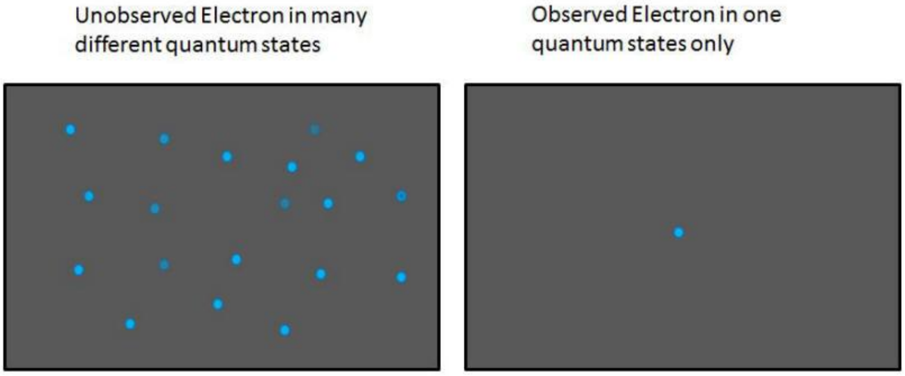


Figure 197 Possible interference patterns

If, however, we try to track the electron and measure its behaviour, it immediately reverts to being a particle. The wave-function **collapses** and the electron becomes a particle which follows the rules of classical physics.

Needless to say, not all physicists accepted the Copenhagen Interpretation. Nowadays it would be called a "fudge".

15.174 Feynman's Sum-over-Histories

This explanation was proposed by Richard Feynman (whom we know from his diagrams of particle interactions). Consider a particle going from A to B starting at time t_{A} and getting there at time t_{B}. In classical terms, the particle passes from A to B by the shortest route (*Figure 198*):

Figure 198 Particle passing from A to B

In the Feynman explanation, the particle can go any way it likes:

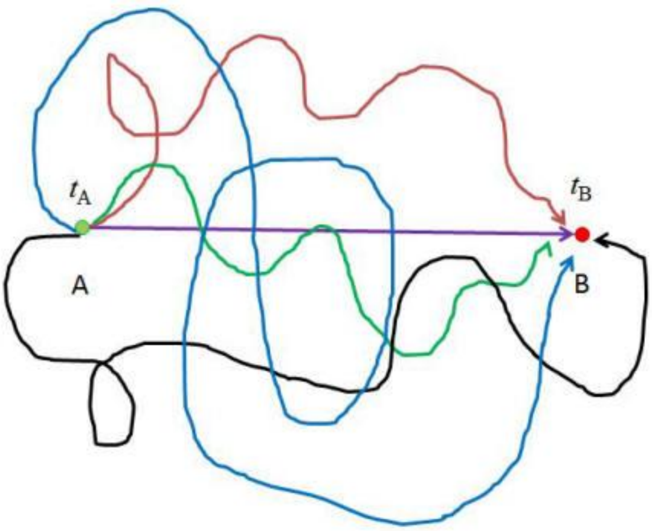


Figure 199 Different ways of getting from A to B

It can travel at any speed as well. The diagram does not show the speed. There are an infinite number of paths that can be travelled. The idea is that these can be added up, and sometimes the sum gives reinforcement, and other times the sum results in cancellation. This reminds us of waves. The sum of the paths gives rise to an **interference wave pattern**. This results in the **probability wave** of detecting the particle. This is called the **path integral** or the **sum over histories**. The mathematical models that underpin this are complex, involving the use of the square root of -1.

It is tempting to think about the paths in terms of the random paths taken by molecules in a gas. However, the random paths are caused by collisions with many other molecules. Between collisions, the molecules travel in straight lines. The behaviour of the molecules is governed entirely by classical physics.

There is only one particle in this explanation. Its path is random as a result of the probability.

15.175 The Many Worlds Interpretation

This was worked out in 1957 by the American physicist, Hugh Everett (1930 - 1982). He promoted the idea of many parallel universes. Each universe would have one out of the possible outcomes for a particular event. At each event, the universe split into two different universes, and each resulting universe carries each particular outcome. The idea is shown in the diagram below (*Figure 200*):

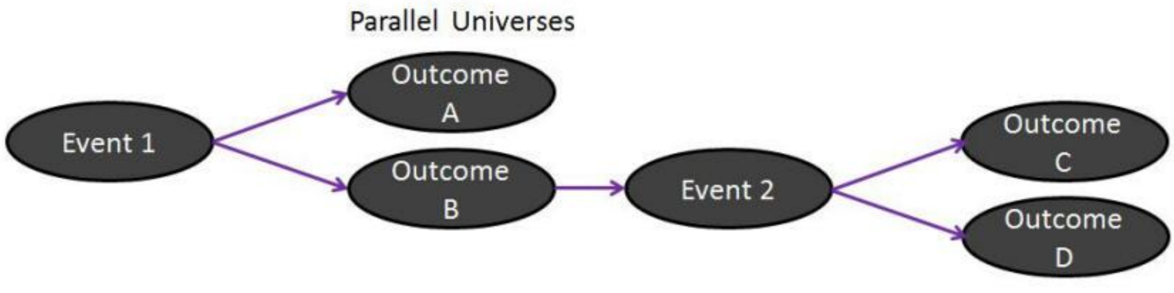


Figure 200 Parallel outcomes for subsequent events

Therefore, a physicist in one universe measures the classical behaviour of a particle. In the parallel universe, a second physicist can analyse the wave probability properties. However, we cannot be aware of ourselves in parallel universes. Suppose I decided not to go out in the car one weekend. My car's absence on the road that afternoon resulted in there not being an accident in which I was seriously injured. Instead, I ended up doing some woodturning in my workshop. In the parallel universe, I took the car out and had a bad accident. I could be spending that afternoon in hospital with serious injuries, wishing that I was in my workshop. The consequences could be that I could never use my workshop again.

Another event could be that I went for an interview for a job I really wanted. In one parallel universe, I might have got the job, but found they didn't like me that much, and I would find myself being fired. In the second parallel universe, I wouldn't have had a successful interview. I would be very disappointed. My conception would be that it was going to be great and I would thrive. I wouldn't know that I would not have lived up to their expectations and end up being "let go".

15.176 Schrödinger's Cat

The Austrian physicist, Erwin Schrödinger (1887 - 1961) found the Copenhagen Interpretation problematic. At what point does a quantum system cease to be a superposition of many states, and the wave function collapses to become one state or another in order to be observed? He illustrated his concern in 1935 using a famous **thought experiment**, called **Schrödinger's Cat**. The idea is shown below (*Figure 201*):

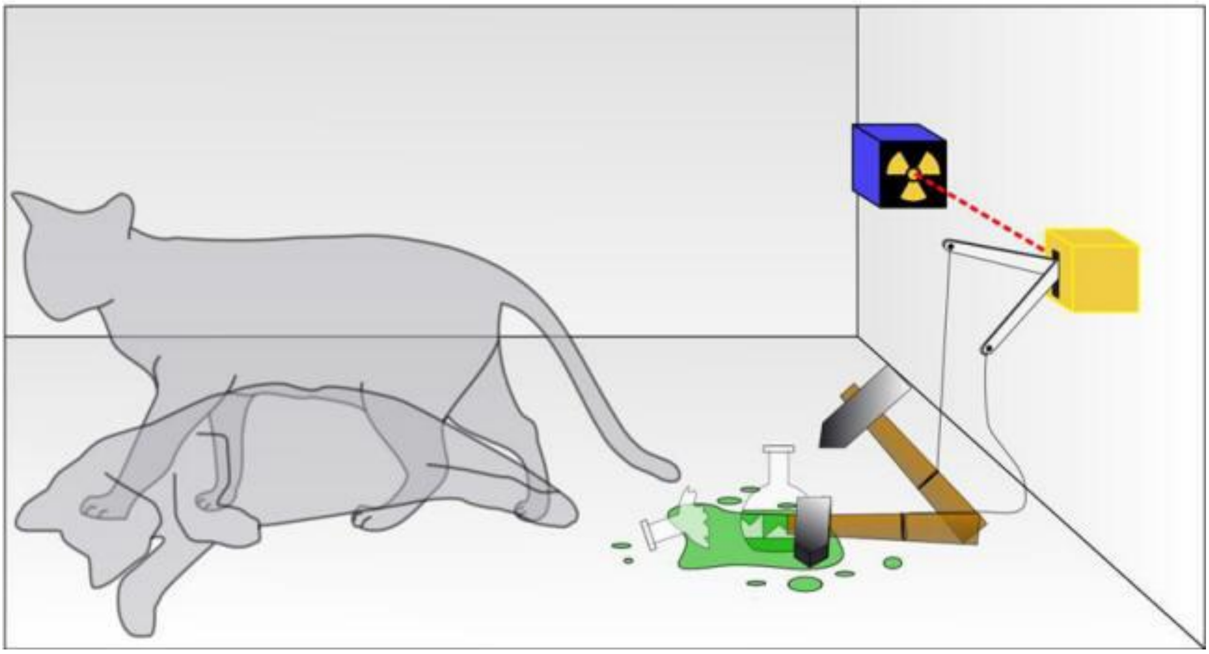


Figure 201 Schrödinger's cat (Image by Dhatfield, Wikimedia Commons)

A cat is placed in a sealed box with a radioactive source. When a nucleus decays, the radioactive emission is detected by a Geiger counter that trips a mechanism that drops a hammer on a flask of hydrogen cyanide (HCN) solution. The flask is smashed to release hydrogen cyanide which will kill the cat. The box is kept shut until an observer opens it. The cat can have two states, **dead** or **alive**. It cannot be both dead and alive. When the box is opened, either the nucleus has not decayed, and the cat is alive, or the nucleus has decayed and the cat is dead. The cat is alive or dead long before the box is opened. (No cats were harmed in this experiment.)

This thought experiment can be interpreted in the Everett Many Worlds Interpretation. In one parallel universe, it is alive. In a second, it's dead.

More thought experiments have been carried out to try to explain the difficulty of making observations in quantum systems. On such is **Quantum Suicide** devised by Max Tegmark (1967 -) in 1997. Thought experiments are useful to discuss and debate fundamental concepts. Factors can be changed easily and "what if?" scenarios can be used to probe different aspects of the debate.

15.177 Heisenberg's Uncertainty Principle

The principle of uncertainty says that there are two properties of a subatomic particle. Firstly, there is the position (x) of the particle, and secondly the momentum (p). The more we know one, the less we know of the other. In other words, if we are chasing an electron, the closer we get to catching the little brute, the less likely we are to get it. There is uncertainty in the position, if we know the momentum. Similarly, if we know the position, we find there is uncertainty in the momentum. This is called **Heisenberg's Uncertainty Principle**. In the Bohr Model of the hydrogen atom (see Topic 15, Tutorial 2), we could predict the position and the momentum of the electron. However, it only gave predictable results in very specific conditions.

The minimum uncertainty is the product of the position and the momentum. This product is **greater than** the **Planck Constant divided by** 2π. We write this as:

$$\Delta p \Delta x \geq \frac{h}{2\pi}$$

.... Equation 301

- Δx - the uncertainty in the position (m);
- Δp - the uncertainty in the momentum (kg m s^{-1});
- h - Planck's constant (= 6.63×10^{-34} J s).

In the Pre-U syllabus, we will use the Planck Constant divided by 2π, which gives us a value of **1.06 × 10^{-34} J s**. In some texts you will see the equation written as:

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

.... *Equation 302*

The reason for this is because due to large numbers of particles in the same state, there is **uncertainty** in their positions and momenta given by the relationship:

$$\Delta x \Delta p > \frac{\hbar}{2}$$

.... *Equation 303*

The term $\hbar$ (with the slash through it ("h-bar")) is sometimes called the **shortened Planck Constant**, and is related to h by:

$$\hbar = \frac{h}{2\pi}$$

.... *Equation 304*

We can, of course, give a value for $\hbar$:

$\hbar$ = 1.06 × 10^{-34} J s

Which is why we can write (*Equation 302*):

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

There is also a minimum for the products of the uncertainty for energy and uncertainty for time. In other words, if you know the energy of the electron, you will have a lot of uncertainty in predicting the time for the little brute:

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

.... *Equation 305*

Worked example A neutron has a mass of 1.67×10^{-27} kg and a speed of 1.56×10^{6} m s^{-1}. Calculate the minimum uncertainty for the position.
Answer Momentum: $p = 1.67 \times 10^{-27} \text{ kg} \times 1.56 \times 10^{6} \text{ m s}^{-1} = 2.605 \times 10^{-21} \text{ kg m s}^{-1}$ Equation: $$\Delta x = \frac{h}{2\pi \Delta p}$$ Minimum uncertainty in position = Δx = $(6.63 \times 10^{-34} \text{ J s}) \div (2 \times \pi \times 2.605 \times 10^{-21} \text{ kg m s}^{-1})$ = **4.05×10^{-14} m**

Although 4×10^{-14} m does not seem that far, it is about twenty times the diameter of the neutron (*Figure 202*).

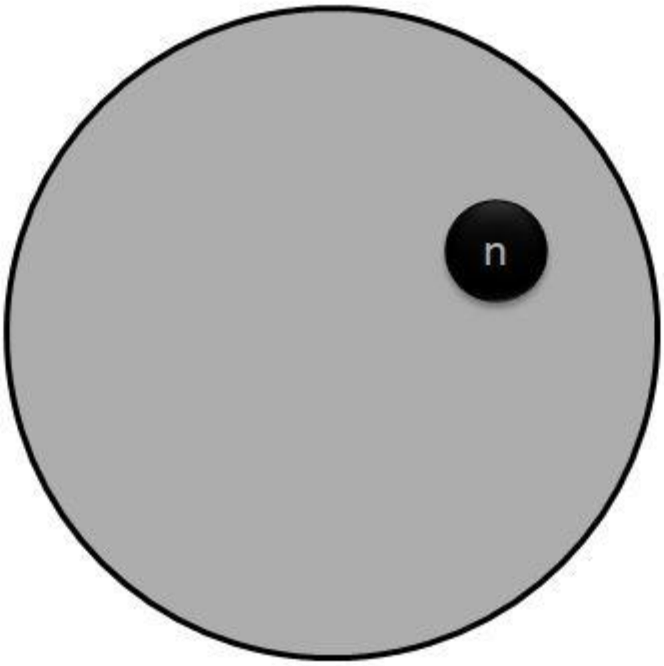


Figure 202 Neutron in its probability cloud

Remember also that the neutron is not a neat black snooker ball with an 'n' written on it. It is fuzzy. And it exists as a **probability**.

The Heisenberg Uncertainty Principle **limits** what we know about the way a quantum system works. Heisenberg himself carried out a **thought experiment** whereby he had a gamma ray microscope. In order to observe with gamma photons, the photons would have to interact with the electrons. Photons have high energy and when they interact with an electron, they will pass on that energy. Since energy in an electron is only kinetic, the increased kinetic energy leads to a higher speed. Therefore, the momentum will increase. If Heisenberg could track an electron down to observe it with the gamma microscope, he could get a lot of information about the **position** of the electron, but there would be a lot of **uncertainty** about the **momentum** of the electron.

No gamma-ray microscope has ever been invented, but that's the beauty of a thought experiment. Werner Heisenberg was also hopeless at practical physics.

15.179 Determinism in Physics

There is a whole branch of philosophy that discusses **determinism**. Philosophers think that things will happen as a series of pre-determined and consequential events, just like a train going down a track will pass a series of stations, level crossings, and tunnels. I am not very philosophical at all, so I am not going to go down that track. There are two things that are certain, death and taxes.

Instead, I will confine things to how determinism is relevant to Physics.

The kind of physics that we experience from day to day is **classical** or **Newtonian** physics. **Determinism** in this context means **cause and effect**, for example:

- A force applied to an object causes it to accelerate in the direction of that force;
- Work is done when a force is moved through a certain distance in the direction of that force;
- Two electrical currents come together at a junction and sum to make a third current.

The results are predictable and can be easily observed.

Quantum Physics is **indeterministic**. There is no direct cause and effect. It works on **probability**. Even if the **starting point** is known, there are many possibilities as to where the process may lead. Some paths are more probable, while other are less so. But they are not impossible.

Consider this argument:

"*Fingers*" is a criminal. He is in prison, but he doesn't think he should be inside. He wants to be outside to commit more crimes. This is our **starting point** (*Figure 203*).

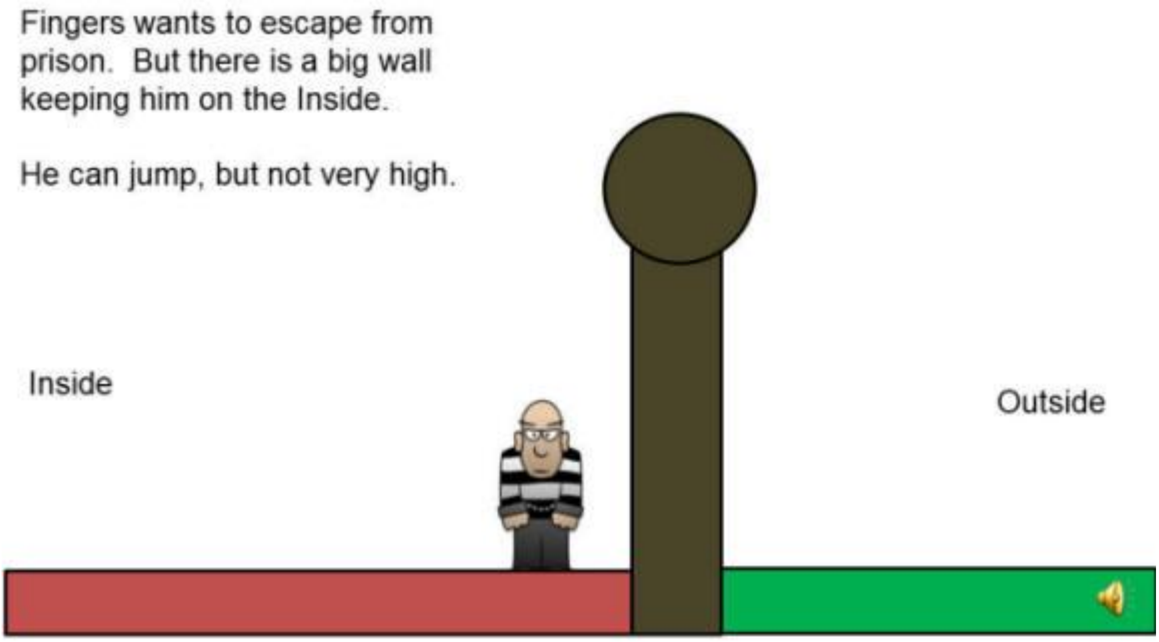


Figure 203 Fingers wants to escape from prison

In the real world, Fingers can jump, but not that high. He could not jump from the Inside to the Outside. So, he remains in the nick. The possible ways for Fingers to get to the Outside include:

- He does his time;
- Some other villains on the outside lift him over the wall with a helicopter (yes, it has been done before);
- He tunnels out;
- He escapes in a laundry basket (that has been done before as well).

All of these are the result of cause and effect.

But in the quantum world, Fingers is in a **probability** cloud.

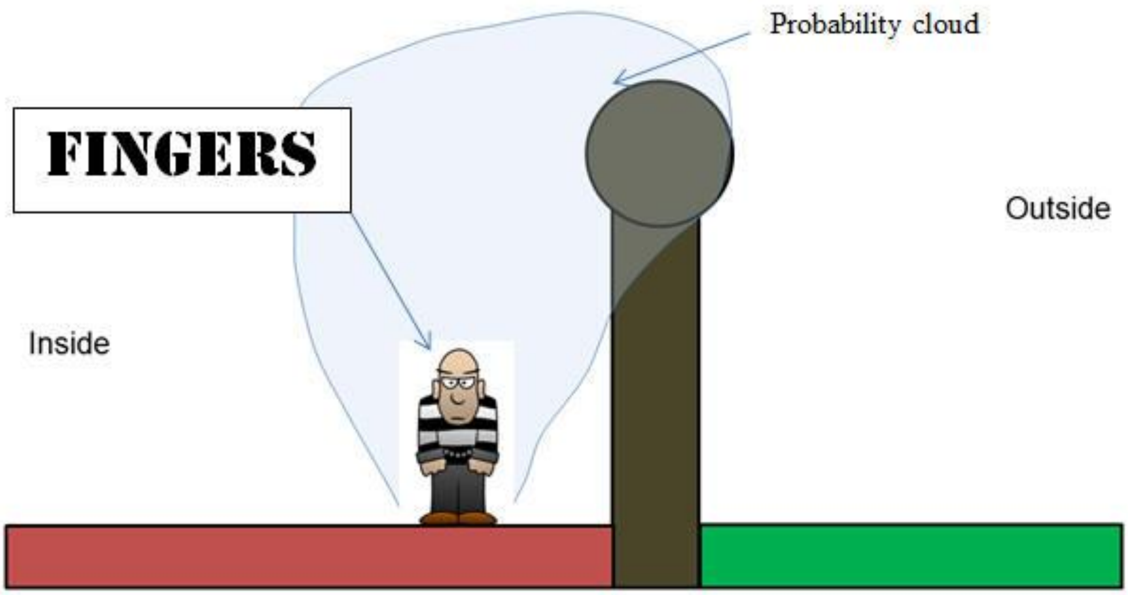


Figure 204 Fingers in a probability cloud

Fingers' probability cloud extends to just over the prison wall. So, there is a tiny, but real, probability that he could happen to be just on top of the prison wall, so he could make good his escape.

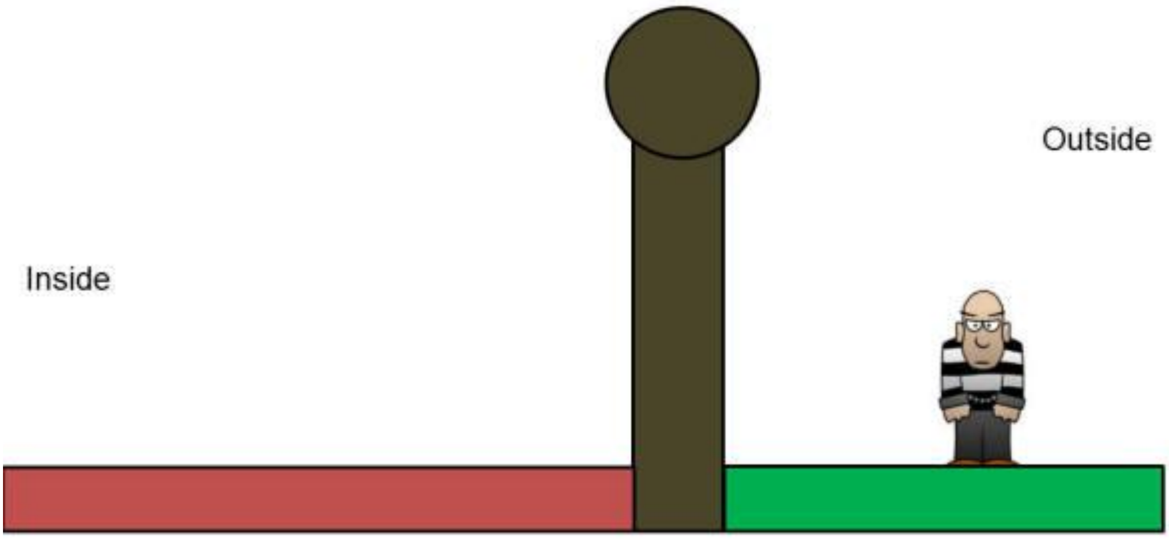


Figure 205 Fingers gets out

Fingers does not need to do anything in the probability world, except to wait for the moment that probability takes him over the wall. We would observe that he has tunnelled through the wall, except that there is no evidence of a tunnel.

Quantum tunnelling is a real world application of probability. For further discussion of this, go to Topic 15 Tutorial 3.

15.179 Einstein and Quantum Theory

Albert Einstein (1879 - 1955) was one of the founding fathers of quantum physics. He proved the photon nature of light as we have seen in the section on Quantum Physics. However, as quantum physics evolved in the early part of the Twentieth Century, he found it harder and harder to accept the indeterministic nature of the quantum world. In 1916 Einstein had predicted the **stimulated emission** of light, which is now used in the laser. In this a photon bumps into an electron and knocks it to a lower level, releasing a photon. But then he discovered that an atom could emit a photon **randomly** without the interaction of a passing photon, using a very similar mechanism to stimulated emission. Being a random process, the photon could go in any direction, and this could not be predicted. This troubled him.

As Einstein produced more papers along with Satyendra Nath Bose (1894–1974) and Erwin Schrödinger, he became more troubled by the probability nature of the quantum world. He particularly disliked the idea that a quantum particle did not move along a **predictable** physical path. And he disliked the idea that one could not measure physical parameters like momentum and energy at any particular moment in time at any point (Heisenberg's Uncertainty Principle).

To sum up Einstein's objections:

1. Quantum Theory was indeterministic, so all outcomes were based on probability. There was no cause and effect. ("God does not play dice.")

2. Quantum Theory was not local. The behaviour of an electron in one atom could affect another atom a long way away (**quantum entanglement**).

3. Quantum Theory could not predict accurately the motion, energy, or other properties of a particle.

The nature of the quantum world still continues to baffle physicists to this day.

Questions

Tutorial 15.17

There are no questions for this tutorial.

7. Further Relativity

Tutorial 15.18 Space Time Diagrams

IB Syllabus only

Contents

(*For IB students*)

Before you attempt this tutorial, you should read Topic 15 Tutorial 1.

The content of this tutorial is challenging. Go through it slowly and carefully.

15.181 Simple Spacetime Diagrams

A simple space diagram reflects **events** that occur within space and time. These events can be anything, for example:

- Answering your mobile;
- A car driving off;
- A molecule of hydrogen reacting with an oxygen atom.

We can show these in a very simple diagram (*Figure 206*). Space in this case is one dimension.

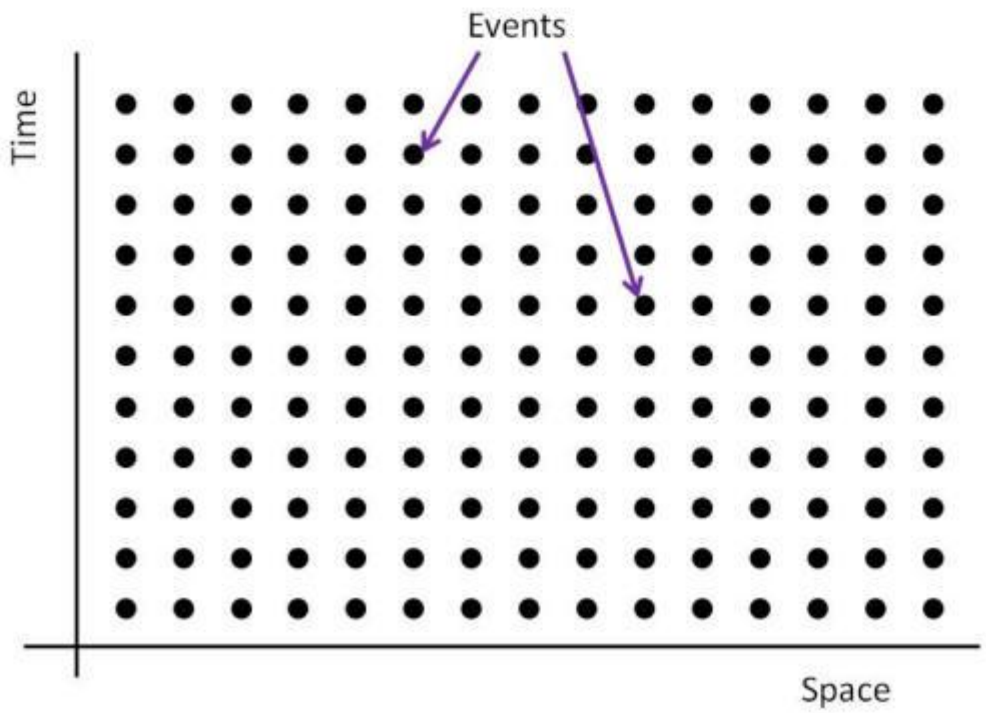


Figure 206 A simple spacetime diagram

Connecting two events gives us a **space-time interval**.

We can represent **length** on a spacetime diagram like this (*Figure 207*):

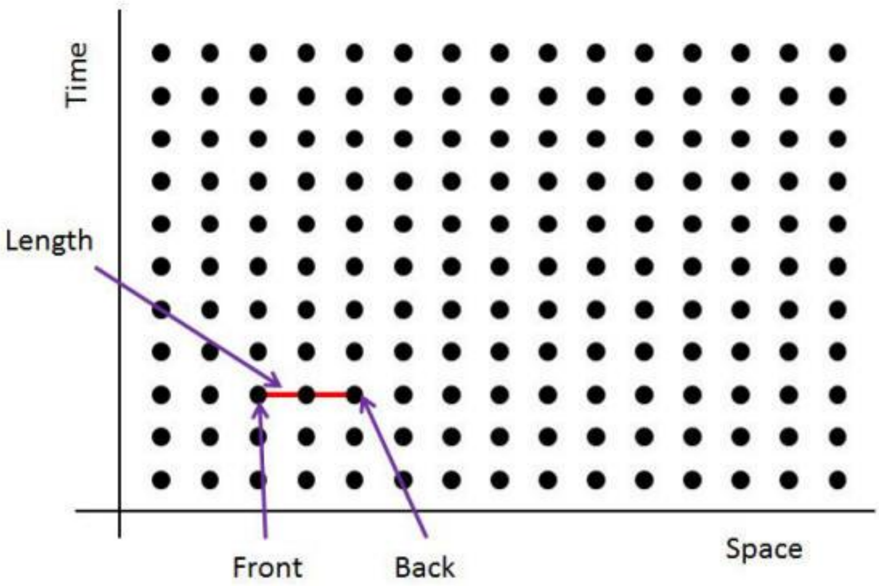


Figure 207 Representing length on a spacetime diagram

Placing a tape measure at the front of a stationary plane is **one event** in space time. Reeling out the tape measure to the back of a plane is a **second event**. The line that links the two represents **length**.

Consider a plane taking off from a point. It travels to another point, where it lands. When a plane takes off, it obviously has to land somewhere (take-offs are optional; landings are not). We can represent the two events linked together by a **spacetime** path (*Figure 208*).

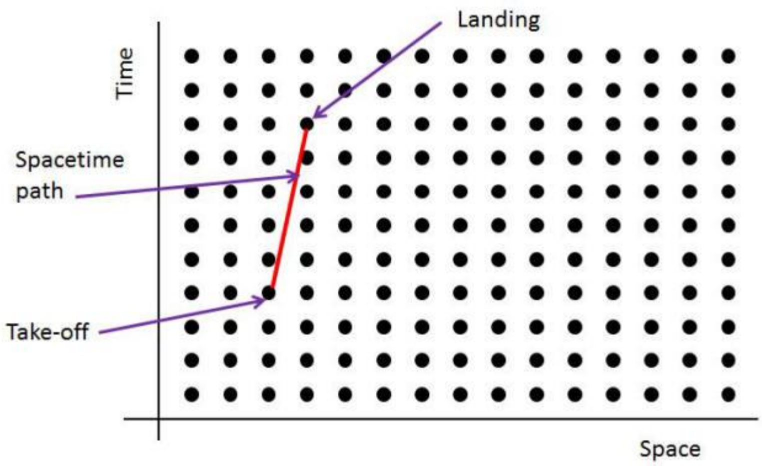


Figure 208 Spacetime diagram for an aeroplane in flight

The slope of the spacetime path gives us the speed, and leads us to an important rule:

All objects with mass must travel more through time than space.

If the object has **zero** mass, for example a photon, it travels through time and space on a path where the space component is the **same** as the time component (*Figure 209*):

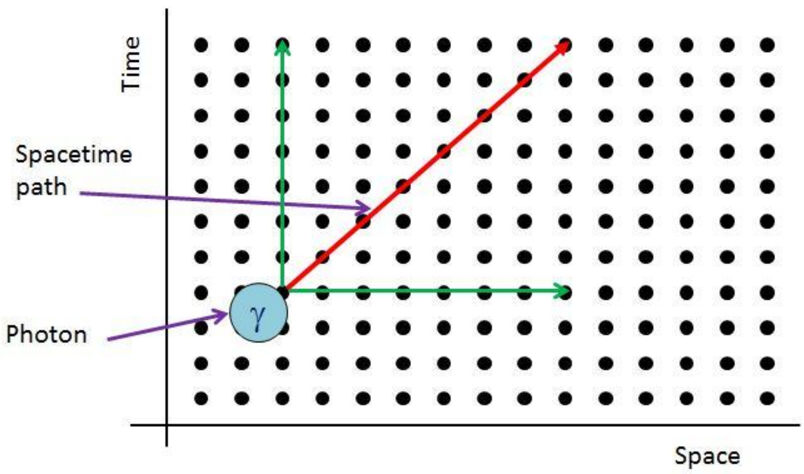


Figure 209 Spacetime diagram for a zero mass object travelling at the speed of light

15.182 Invariance

In the relativity equations we have used, we have rightly assumed that the speed of light is constant, which is consistent with Einstein's postulate. Since it does not change, it is called a **physical invariance**. Another invariant is the **space-time interval**, Δs, which uses the ideas of **four** dimensions.

$$\Delta s^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

.... *Equation 306*

The minus sign shows that time is different from space in space-time. When we use this equation at this level, we assume that $c = 1$. For the speed terms, we can say that they are fractions of c.

Worked example What is the space time interval if the time interval is 1.0 s, and the object is moving at $0.7\ c$ in the x direction, $0.8\ c$ in the y direction, and $0.9\ c$ in the z direction?
Answer $\Delta s^2 = -(1 \times 1.0\ \text{s})^2 + (0.7)^2 + (0.8)^2 + (0.9)^2 = -1.0 + 1.94 = 0.94$ $\Delta s = (0.94)^{0.5} =$ **0.97 s**

15.183 Space in Three Dimensions

We are creatures of three dimensions. We can travel from left to right (x-axis), forwards and backwards (y-axis), and upwards and downwards (z-axis). Any object (*A)* can be placed at any point in space, determined by three coordinates, (x, y, and z), as shown in the diagram (*Figure 210*):

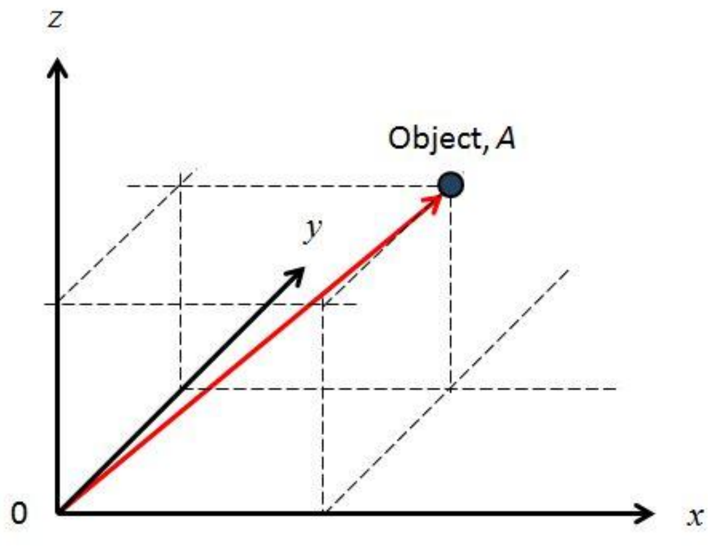


Figure 210 Determining the position of an object in space

The distance 0A is d. We can work out d easily by Pythagoras in 3 dimensions:

$$d^2 = x^2 + y^2 + z^2$$

.... *Equation 307*

We can turn our axes around to any angle we like (*Figure 211*):

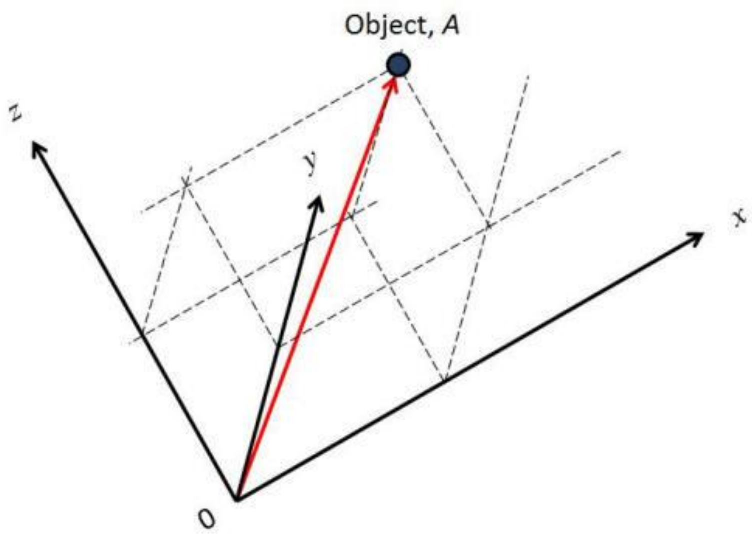


Figure 211 Turning the axes through an angle

We still find that the distance, d, is given by:

$$d^2 = x^2 + y^2 + z^2$$

15.184 Adding a Fourth Dimension

Time is the **fourth** dimension. As three-dimensional creatures, we only experience going forward in time at a set rate of passage. We don't have the luxury of going backwards... The only way to get out of a space determined prison (one with cells and walls) is to complete the sentence. A villain can't move time forward to complete his stretch in double quick time...

Representing a fourth dimension in a three dimensional world is not at all easy. We will show the x and y axis with a time (t) axis (*Figure 212*):

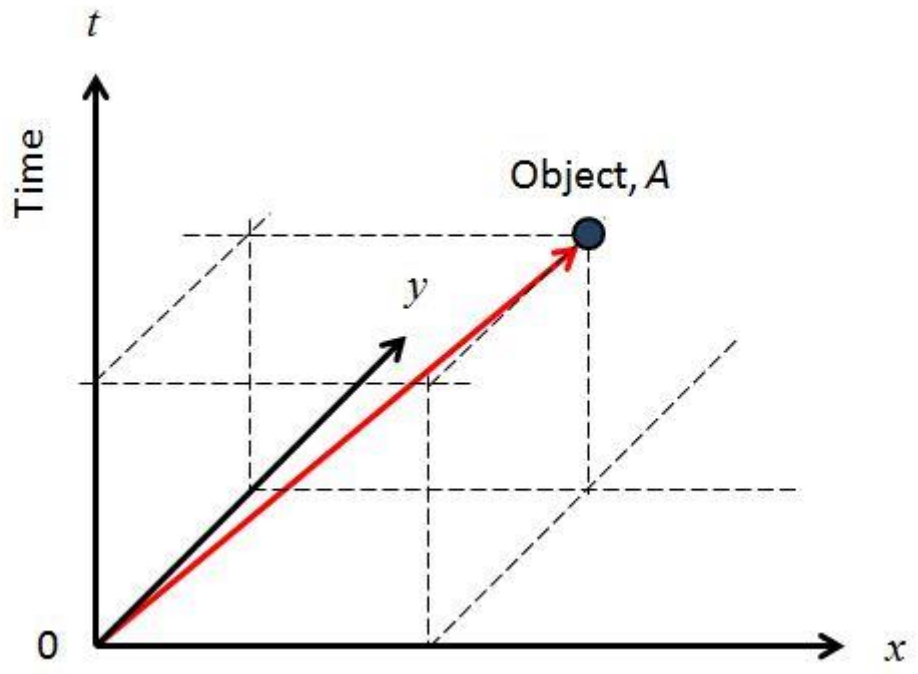


Figure 212 Representing a fourth dimension.

Every point in space can be represented as **three spatial coordinates**, and a **coordinate along the time axis**. When an object is moving along the time axis, it is NOT moving through space, but through time at the speed of light (*Figure 213*).

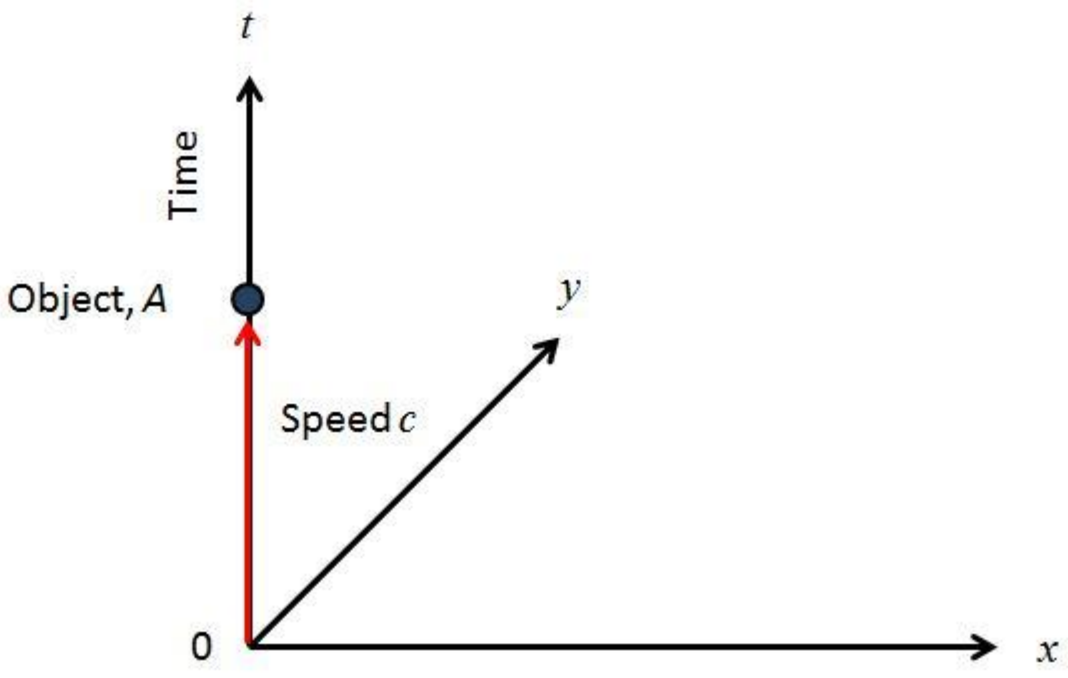


Figure 213 Object A is moving at the speed of light

We show the axes like this for convenience. As before, we can rotate the x and y (and the z) axes to any angle we like, to make a new coordinate system. However, the time axis rotates in a different way to the spatial coordinates, like this (*Figure 214*):

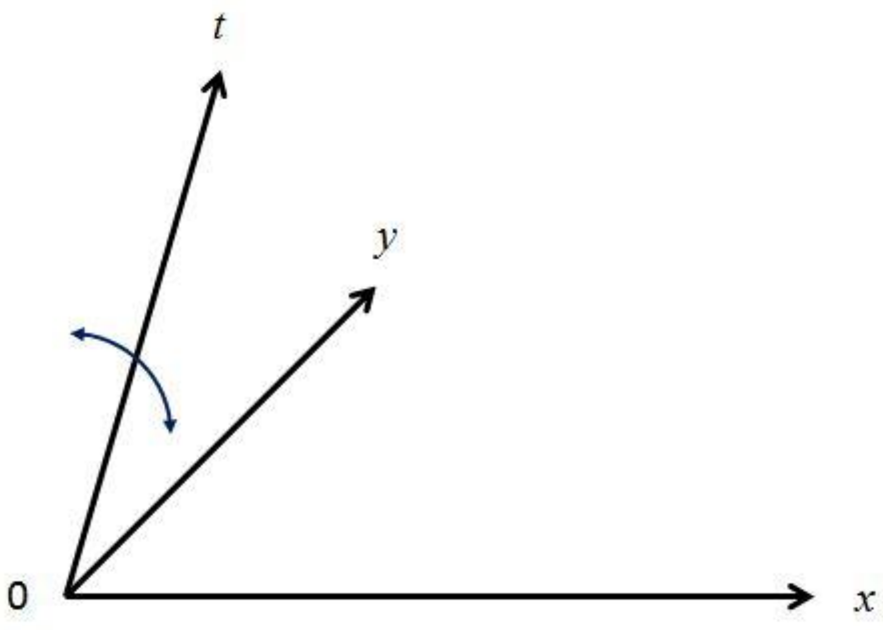


Figure 214 Rotating the t axis

If we link this with Einstein's Theory of Special Relativity, all observers can be considered to be standing still in their own frame of reference, and that the rest of the universe is moving past them. Let's think of one observer travelling through space and time (*Figure 215*).

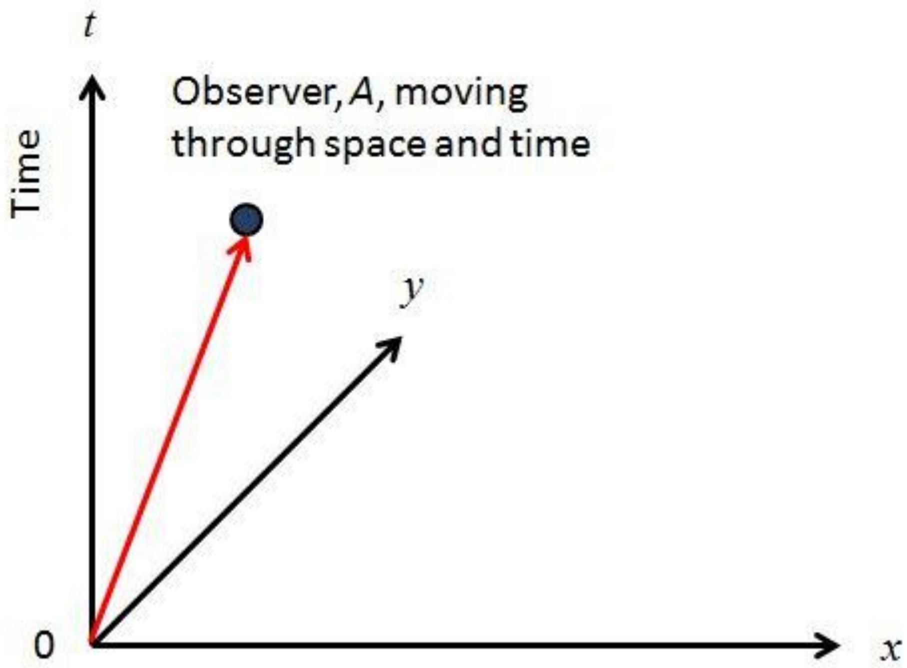


Figure 215 Observer moving through space and time

The observer's views of the other axes for spacetime are rotated so that the observer is moving exclusively along the time axis (*Figure 216*):

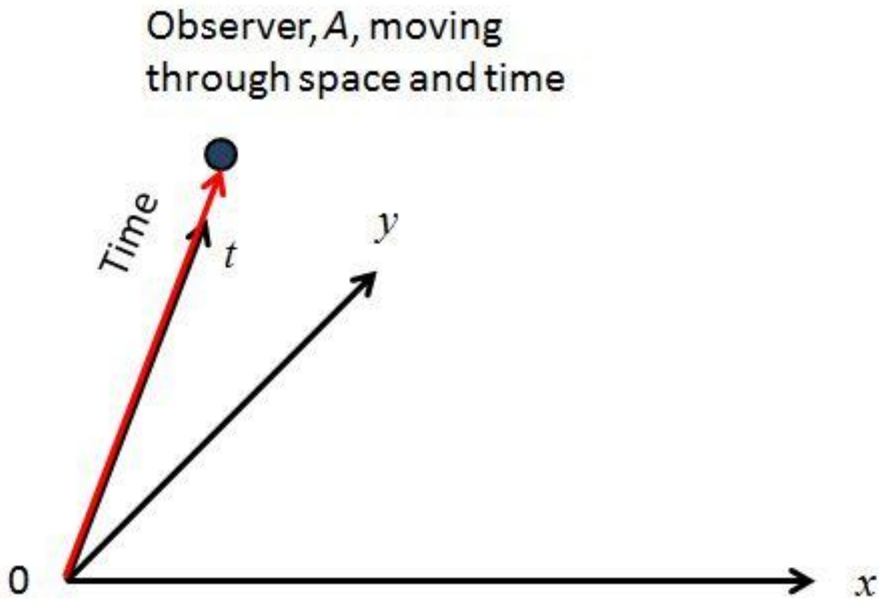


Figure 216 The observer moving along the time axis

However, from Observer A's point of view, the time axis is still perpendicular to the other spatial axes. According to Observer A. every other observer's axes for time and space have been rotated. Therefore, different observers will not agree on the time and space for different events. In traditional three dimensional space, we have seen that we can work out the distance from the Origin 0 to the Object A (*Figure 217*):

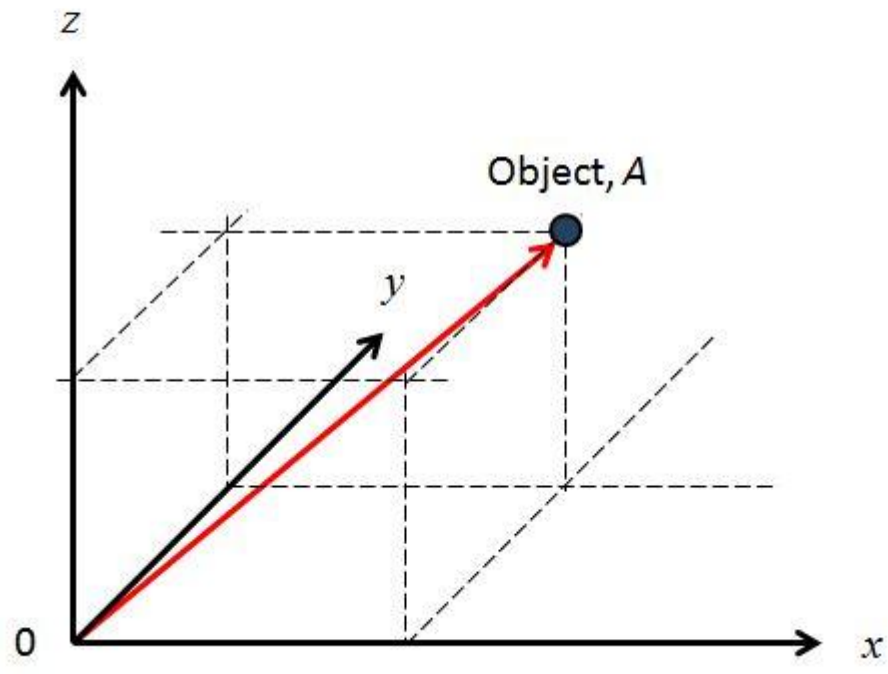


Figure 217 Working out a distance in three dimensions

and we know that the answer is as in *Equation 307*:

$$d^2 = x^2 + y^2 + z^2$$

And that is the case for whatever the way the axes are arranged. All the observers would agree on that.

In the world of Special Relativity, this is no longer the case. Different observers will think their time axis is always perpendicular to the other axes. They cannot agree on the time and space. There is a new quantity that is needed.

15.185 Space-Time Interval

This is determined by the formula:

$$\Delta s^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

.... *Equation 308*

In SI units, the terms are:

- Δs - space time interval (m).
- c - speed of light (m s^{-1}).
- Δt - time interval (s).
- Δx - distance of *x*-coordinate (m).
- Δy - distance of *y*-coordinate (m).
- Δz - distance of *z*-coordinate (m).

The terms may also be measured in:

- Light years and years.
- Units of 30 cm and nanoseconds.
- Astronomical units (distance from Sun to Earth) and 500 s (time for light to travel from the Sun to the Earth).
- Light seconds and seconds.

This quantity remains constant, regardless of how axes for speed and time are rotated. It is the same for all observers.

Light always travels along paths where the space-time interval between two points is zero. The speed of light will always be the same for all observers. This is consistent with what we said about the speed of light in relativistic contexts.

15.186 Minkowski Diagrams

The **Minkowski diagram** was devised by Hermann Minkowski (1864 - 1909) who was a German mathematician. These are diagrams with **one time dimension** and **one space dimension**. On the **horizontal** axis, there is **space**. On the **vertical** axis, the **time** is represented as the **product between the speed of light and the time**. The easiest to think about for the spatial dimension to be the **light-second** (= distance travelled by light in 1 second = 3.0×10^8 m), and the time to be in **seconds**.

A photon travels 3.0×10^8 m in 1 second. So, it travels 1 light second every second. On a Minkowski diagram this looks like (*Figure 218*):

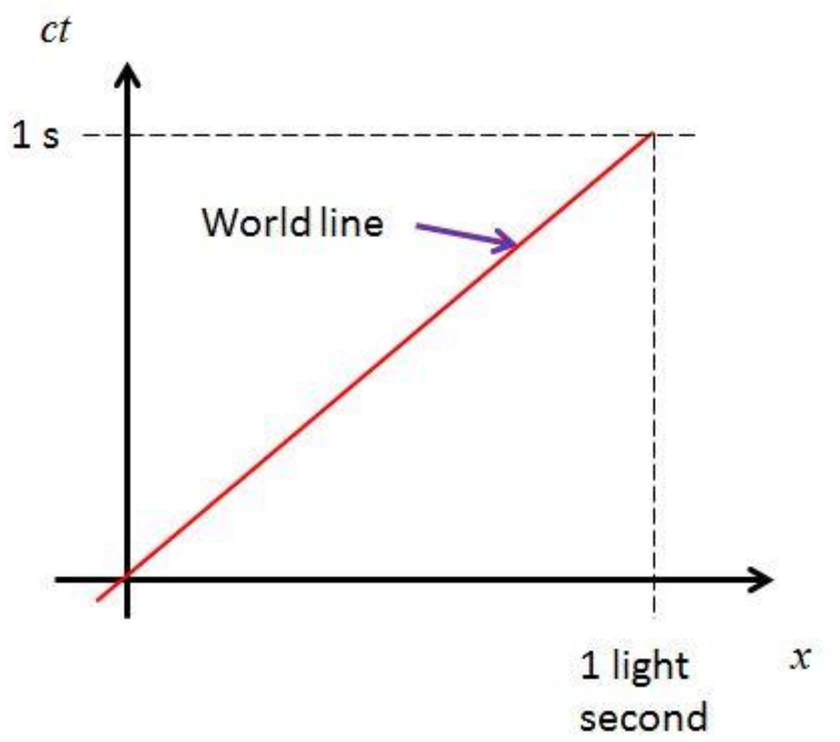


Figure 218 A world line on a Monkowski diagram

The gradient is the speed of light. The line is called a **world line**. On the path, any point can describe a point in space-time, called an **event**, regardless of whether anything relevant happens there. The line is at 45 ° and the angle between the x axis and the line cannot be any less than 45°. This is because nothing can travel faster than the speed of light.

Now let's think of one observer who is stationary. The axes for this observer are in **black**. The second observer is moving relative to the first. The axes are in **blue**. The x-

axis is rotated to form the x' axis, as observed from the stationary observer. The ct axis is also rotated to the ct' axis. We see this here (*Figure 219*):

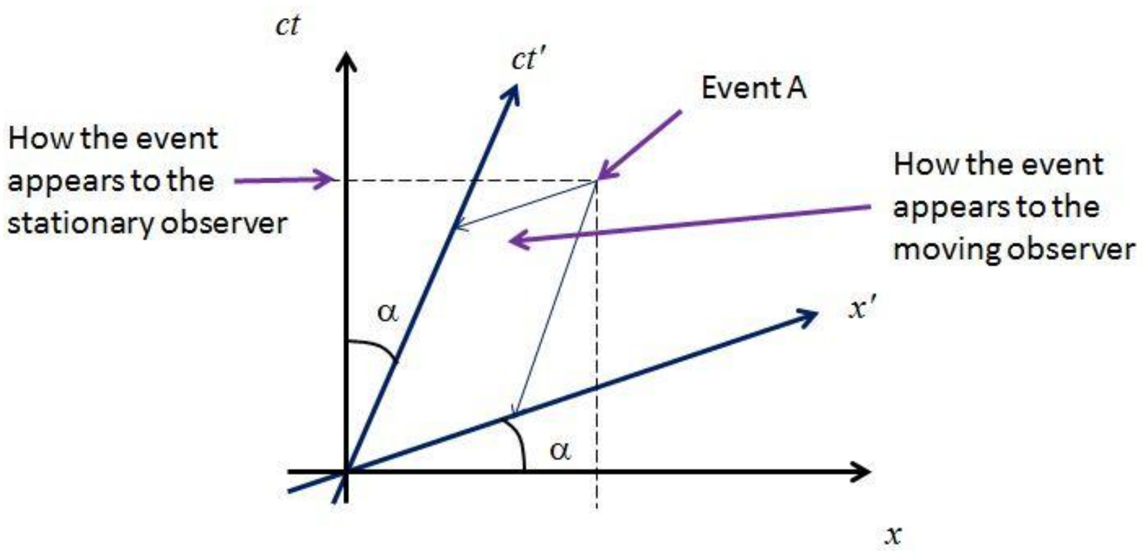


Figure 219 Event appearing to two different observers

This shows that Event A will happen at a different point in space and time depending on whether the observer is stationary or the observer is moving. For the stationary observer, the lines that show the event in time and space are **parallel** to the x-axis and the ct axis. They are shown as the black dotted lines. For the moving observer, the lines that show Event A in time are space are shown by the thin arrows. One of these is parallel to the ct' axis, while the other is parallel to the x' axis.

The difference is shown by the angle α. It is the same angle for the rotation of x to x' and ct to ct'. This is because the speed of light is the same to all observers regardless of the relative motion of their frames of reference.

Let's say that the unit length for the stationary observer is U. This is shown below (*Figure 220*).

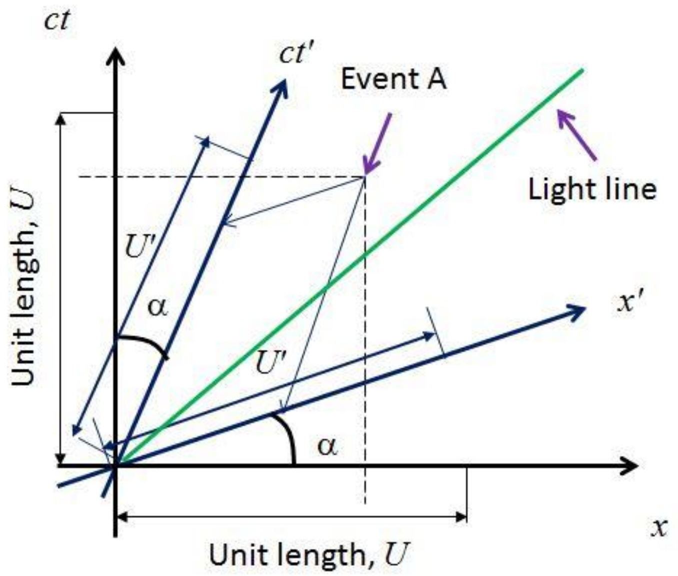


Figure 220 Minkowski diagram showing Unit Lengths

Many Minkowski diagrams show the **light line**, which is shown in green.

The unit length for the moving observer is U'. The angle between the x and x' axes leads us to the Lorentz transformation, which will tell us how the unit length for the stationary observer is related to the unit length for the moving observer.

$$\tan\alpha = \frac{v}{c} = \beta$$

.... *Equation 309*

In the syllabus the angle is written as θ.

The unit length for the x' and ct' axes can be worked out using the equation:

$$U' = U\left(\sqrt{\frac{1+\beta^2}{1-\beta^2}}\right)$$

.... *Equation 310*

15.187 Minkowski Diagrams and Invariance

We saw above the equation (*Equation 308*):

$$\Delta s^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

In a relativity event, the axes of the moving observer are moved through an angle θ (or α, depending on your preference). The equation becomes:

$$\Delta s^2 = -(c\Delta t')^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2$$

.... Equation 311

The term Δs^2 is **invariant**, so we can write:

$$-(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = -(c\Delta t')^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2$$

.... Equation 312

In a Minkowski diagram, we use **only** the ct-axis and the x-axis. So, we can assume that:

- $\Delta y = \Delta y'$.
- $\Delta z = \Delta z'$.

Therefore, we can cancel:

$$-(c\Delta t)^2 + \Delta x^2 + \cancel{\Delta y^2} + \cancel{\Delta z^2} = -(c\Delta t')^2 + \Delta x'^2 + \cancel{\Delta y'^2} + \cancel{\Delta z'^2}$$

.... Equation 313

So, we can write:

$$-(c\Delta t)^2 + \Delta x^2 = -(c\Delta t')^2 + \Delta x'^2$$

.... Equation 314

15.188 Using Minkowski Diagrams

At this level, we take the value of $c = 1$. We will use the time unit as 1 second, and the distance unit of 1 light second (1 ls = 3.0 × 10^8 m). We will express speeds as fractions of the speed of light, for example 1.5 × 10^8 m s^{-1} is $0.5\ c$.

If we have $c = 1$, we can use simplified Lorentz transformations. For the distance, we can write:

$$x' = \frac{x - vt}{(1 - v^2)^{0.5}}$$

.... *Equation 315*

For the time, we write:

$$t' = \frac{t - vx}{(1 - v^2)^{0.5}}$$

.... *Equation 316*

The term x is the distance observed by the stationary observer, while t is the time observed by the stationary observer (the proper distance and proper time). The terms x' and t' are those quantities observed by the moving observer.

Let's put some numbers in.

Worked Example

A spacecraft is travelling at 1.25×10^8 m s^{-1} past a stationary observer. The stationary observer notices a flash of light that lasts 0.80 s. The spacecraft travels 0.80 light seconds in this time as observed by the stationary observer.

(a) What is the angle by which the relative time axis rotated by?

(b) Work out the distance travelled by the moving observer.

(c) How long is the lamp on the spacecraft on for? (The answer is NOT 1.0 s.)

Answer

(a) Use:

$$\tan\alpha = \frac{v}{c} = \beta$$

$\tan \alpha = 1.25 \times 10^8$ m s^{-1} $\div$ 3.0×10^8 m s^{-1} = 0.417

$\alpha = \tan^{-1}(0.417) = 22.6\,°$ = **0.395 rad**

(b) Use:

$$x' = \frac{x - vt}{(1 - v^2)^{0.5}}$$

Put in the numbers:

x' = (0.80 ls - (0.417 c × 0.80 s)) ÷ (1 - (0.417 c)^2)$^{0.5}$

= 0.4664 ls ÷ 0.909 = **0.513 light seconds**

(c) Use:

$$t' = \frac{t - vx}{(1 - v^2)^{0.5}}$$

t' = (0.80 s - (0.417 c × 0.80 ls)) ÷ (1 - (0.417 c)^2)$^{0.5}$

= 0.4664 s ÷ 0.909 = **0.513 seconds**

Now we shall draw a Minkowski diagram using these numbers (*Figure 221*):

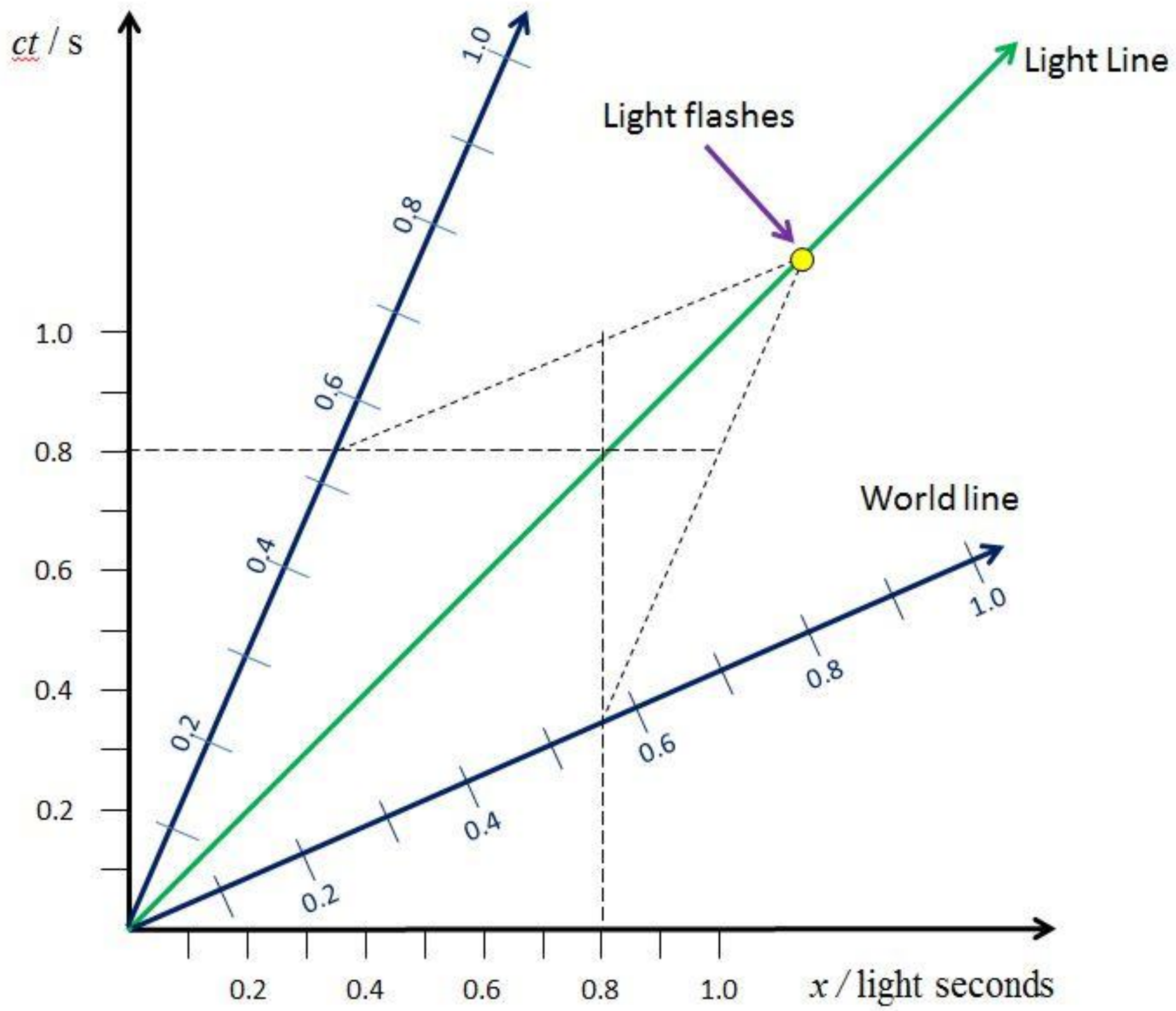


Figure 221 Minkowski diagram to go withe worked example

How do we make up the diagram?

1. We will draw this diagram using $c = 1$.

2. The world lines in dark blue are those of the stationary observer.

3. We calibrate the axes. In this case 1 cm = 1 ls.

4. We draw the world line for the light at an angle of 45 °.

5. Each world line for the moving observer is angled at 23 ° to the axes of the stationary observer (see worked example).

6. Now we need to calibrate the world lines of the moving observer.

7. The horizontal and vertical dotted lines represent the position and time of the event (the light comes on at zero and goes off at 0.8 s). Use the *Equations 315* and *316*:

$$x' = \frac{x - vt}{(1 - v^2)^{0.5}}$$

and

$$t' = \frac{t - vx}{(1 - v^2)^{0.5}}$$

8. Now draw a line from the event parallel to each world line until the lines intersect with the world lines.

The picture below shows the steps as applied to the Minkowski Diagram (*Figure 222*):

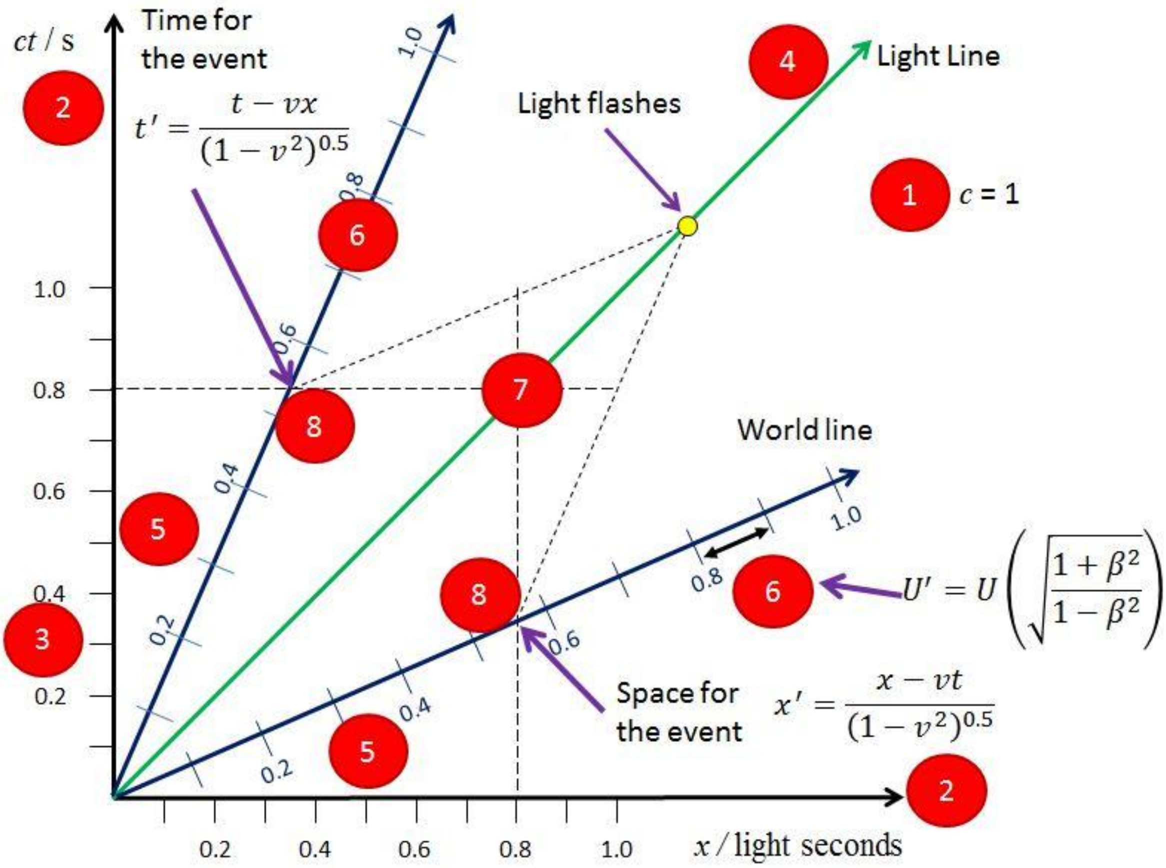


Figure 222 Minkowski diagram showing the steps

15.189 Showing Time Dilation

In the diagram below (*Figure 223*), we see two events, A and B, which happen at the same point in space. Both events are seen by the stationary observer.

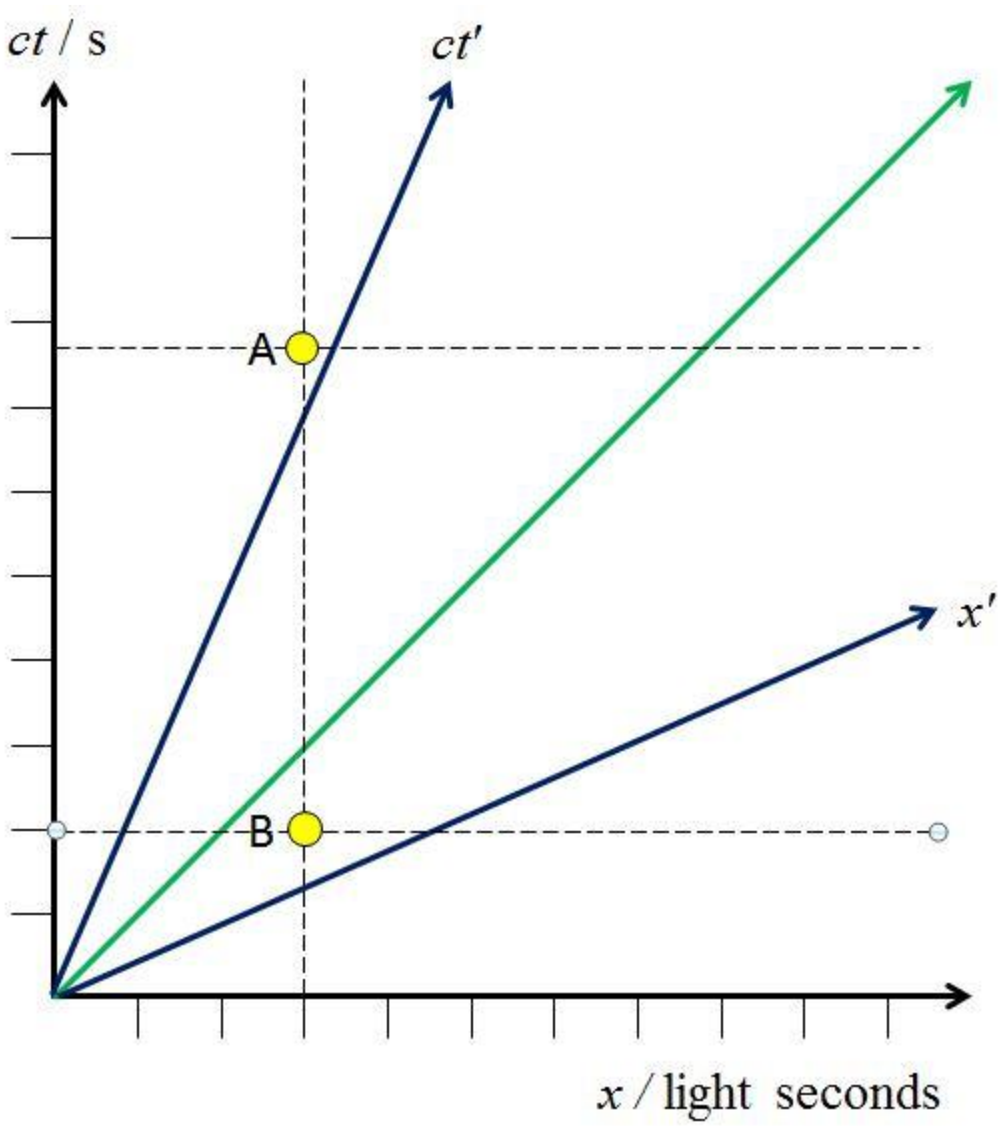


Figure 223 Minkowski diagram for two events happening at the same point before a stationary observer

The horizontal dashed black lines show the times of the events.

Then we put lines parallel with the x' and ct' axes, to intersect at events A and B, like this (*Figure 224*):

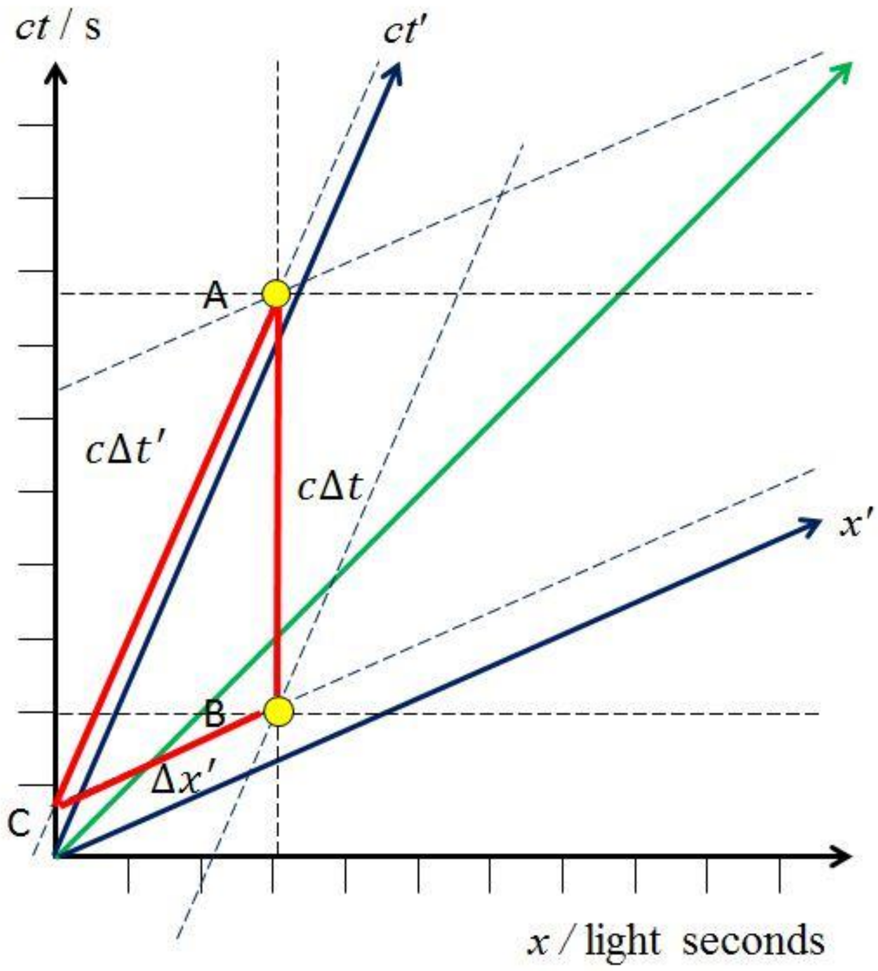


Figure 224 Putting in lines parallel to the x' and ct' axes

We then draw a triangle was shown in red. The third corner of the triangle is at an intersection of lines parallel to the x' and ct' axes at point C. In this diagram the point C is at the ct axis, but that needn't be the case.

15.1810 Showing Length Contraction

Consider two events, A and B, which both happen simultaneously, but a distance D*x* apart as seen by the stationary observer. At the same time a second observer moves past at a constant velocity. The time for the second observer to pass is $c\Delta t$, as measured by the stationary observer. The idea is shown by the diagram (*Figure 225*):

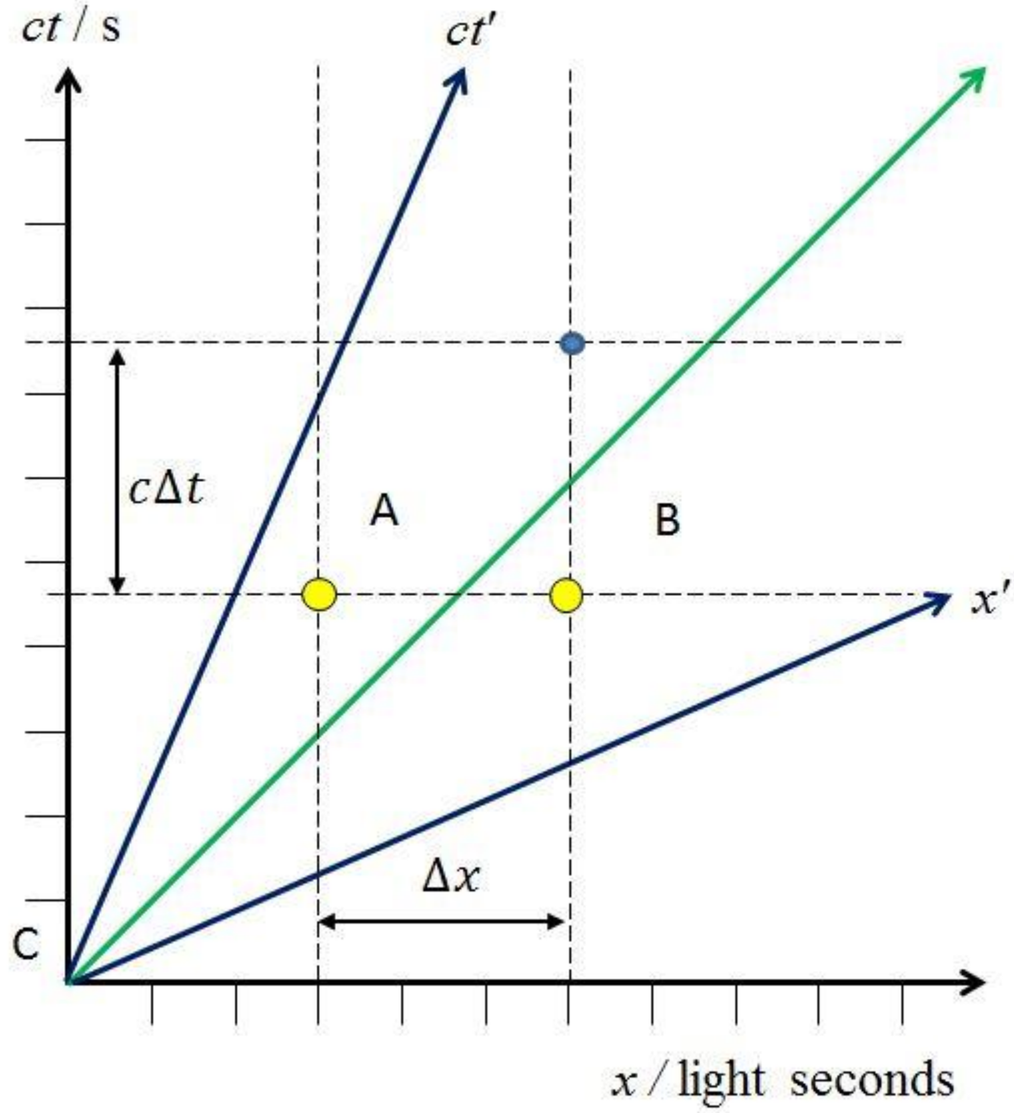


Figure 225 Two events occurring at the same time observed by a stationary and moving observer

Now we draw lines parallel to the ct' and x' axes which intersect at A and B (*Figure 226*).

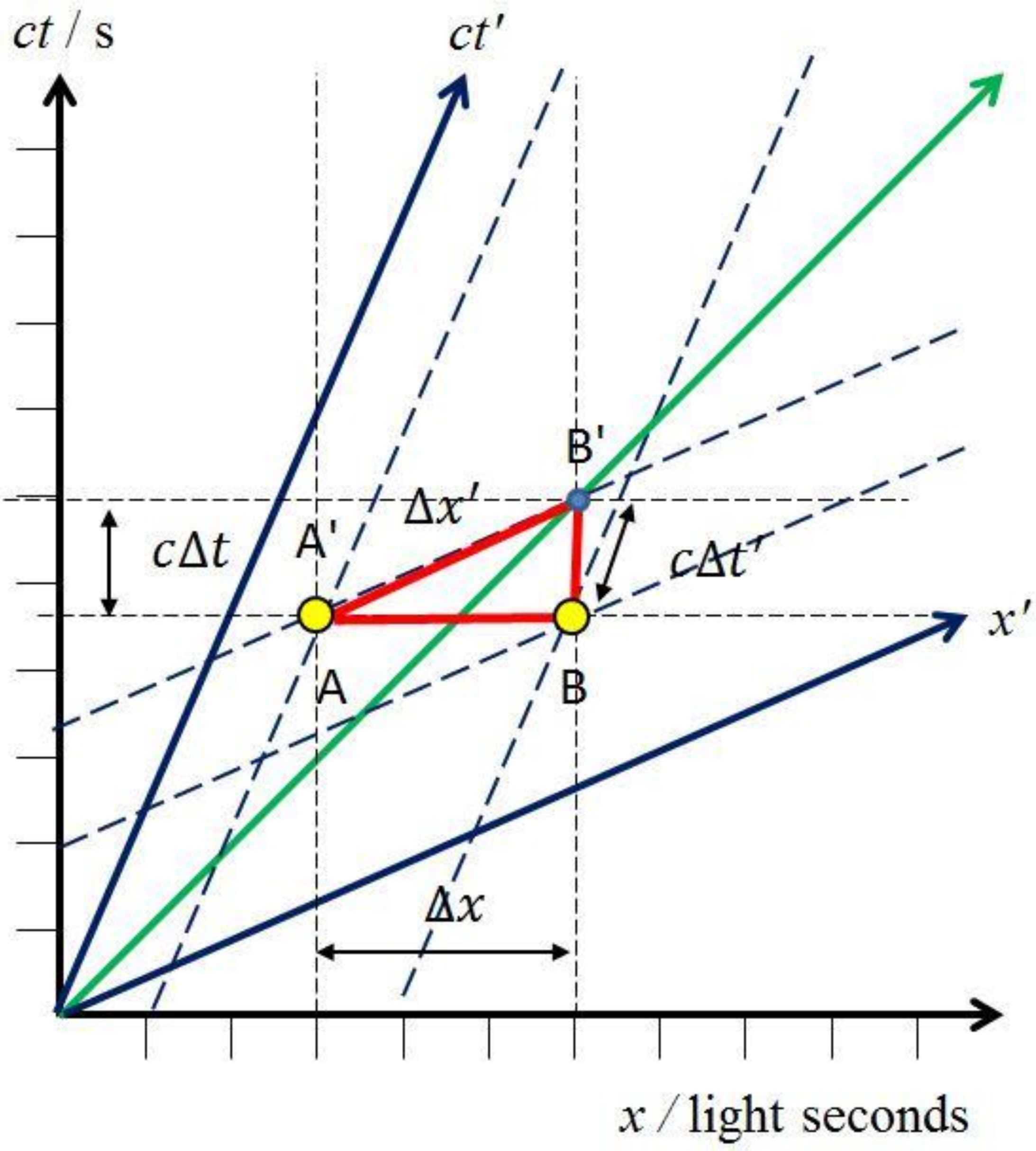


Figure 226 Parallel lines added

This diagram is not so straight forward to interpret. A couple of rules:

- Δx is the proper length. Δt is the proper time.
- We know that time dilates.

1. We are in Frame *S*, looking at a ruler that is length Δx which is 1 m. It is stationary relative to us. Its ends are shown as A and B.

2. The world lines for each end are the vertical dotted lines.

3. The moving observer goes past at a constant velocity in a moving frame of reference, Frame S'.

4. To the moving observer the ends are at A and C. They occur at the same time Dt' to the moving observer.

5. The moving observer measures the length of the ruler as $\Delta x'$.

6. The coordinates for the stationary observer in Frame *S* are different to the moving observer in Frame *S'*. This is because of the Lorentz Transformation.

7. The Lorentz Transformations are:

$$A = \gamma(A' + \beta ct')$$ .... *Equation 317*

and

$$B = \gamma(B' + \beta ct')$$ .... *Equation 318*

8. In the stationary frame S, we can say:

$$\Delta x = B - A$$ .... *Equation 319*

Therefore, we can substitute *Equations 317* and *318*:

$$\Delta x = \gamma(B' + \beta ct') - \gamma(A' + \beta ct')$$ .... *Equation 320*

which gives us:

$$\Delta x = \gamma(B' - A')$$ .... *Equation 321*

9. Since

$$\Delta x' = B' - A'$$ ....*Equation 322*

We can write:

$$\Delta x = \gamma \Delta x'$$ .... *Equation 323*

Therefore:

$$\Delta x' = \frac{\Delta x}{\gamma}$$

.... *Equation 324*

10. The moving observer always sees the object as shorter than claimed. The stationary observer always sees the proper distance. All moving observers see a **contracted** distance.

15.1811 The Twins Paradox

This is a **thought experiment** in which there are two identical twin brothers who were born at **exactly the same time**. (It could be twin sisters of course, and one twin tends to be born before the other, but let's not allow boring things like facts spoil the story.)

One of the twins makes a journey into deep space in a spacecraft that can travel at a speed of about $0.5\ c$. The other twin stays on The Earth (*Figure 227*).

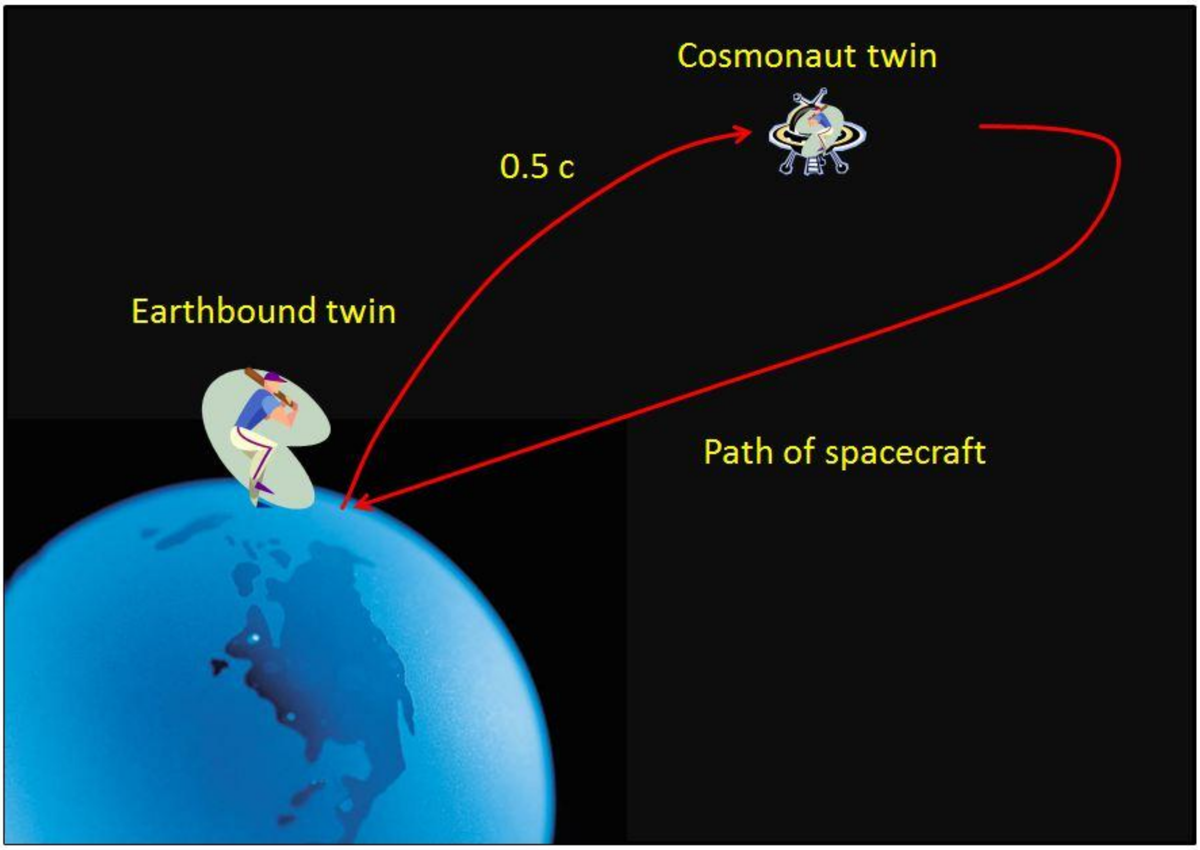


Figure 227 The twins paradox

The first twin gets back and finds that the **earthbound twin is now older than cosmonaut twin**. Why is this? The usual explanation is that for an object travelling at that speed, time will dilate (get longer) compared with the time observed by the stationary earthbound observer.

However, there is a flaw in this argument. The earthbound twin will be moving at about $0.5\ c$ relative to the twin on the spacecraft. (Note that we are ignoring practicalities like the time taken if the spacecraft accelerates at $2\ \mathrm{g}$. It will take about

100 days to get to its maximum speed.) **To the cosmonaut twin, the earthbound twin is younger**.

The resolution of this can be explained using **Minkowski diagrams**. Attempt Question 15.18.6.

Now we can work out the dilated time using the equation:

$$t' = \gamma t_0$$ .... *Equation 325*

Where:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

.... *Equation 326*

and t' is the time for the moving observer, and t_0 is the proper time (as seen by the stationary observer). See Question 15.18.7.

Now we can work out the angle through which the x' and ct' axes are rotated. See Question 15.18.8.

We can now draw the x' axis and draw lines parallel to it. The grid is shown as well (*Figure 228*).

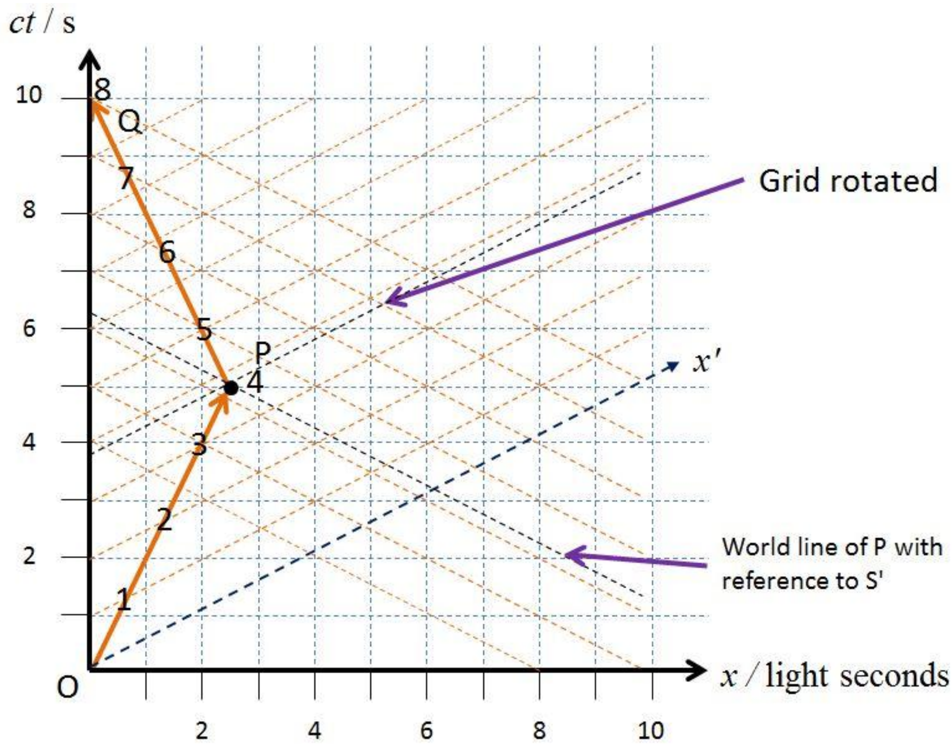


Figure 228 Minkowski diagram for the twins paradox

We can show the time as seen by the observer. The time according to the moving twin is about 8 seconds.

If we include the world lines to P, we see (*Figure 229*):

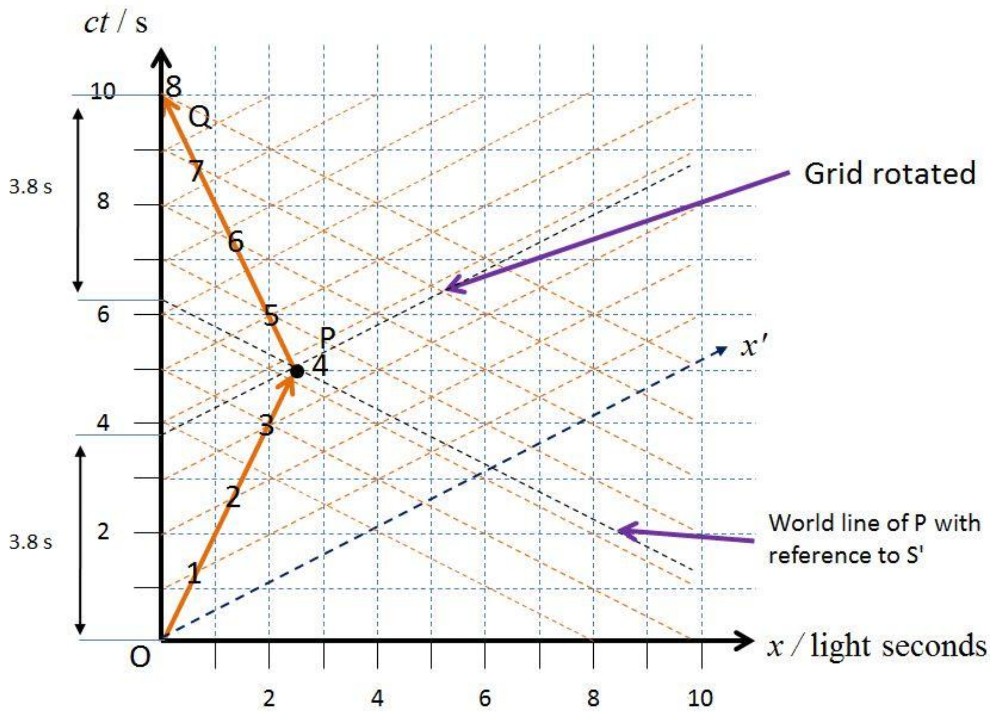


Figure 229 Minkowski diagram for the twins paradox with world line

According to these results, the total taken by the moving twin is 7.6 s. However we need to take into account the space-time that is taken up by the change of velocity, which is shown in the triangle below (*Figure 230*).

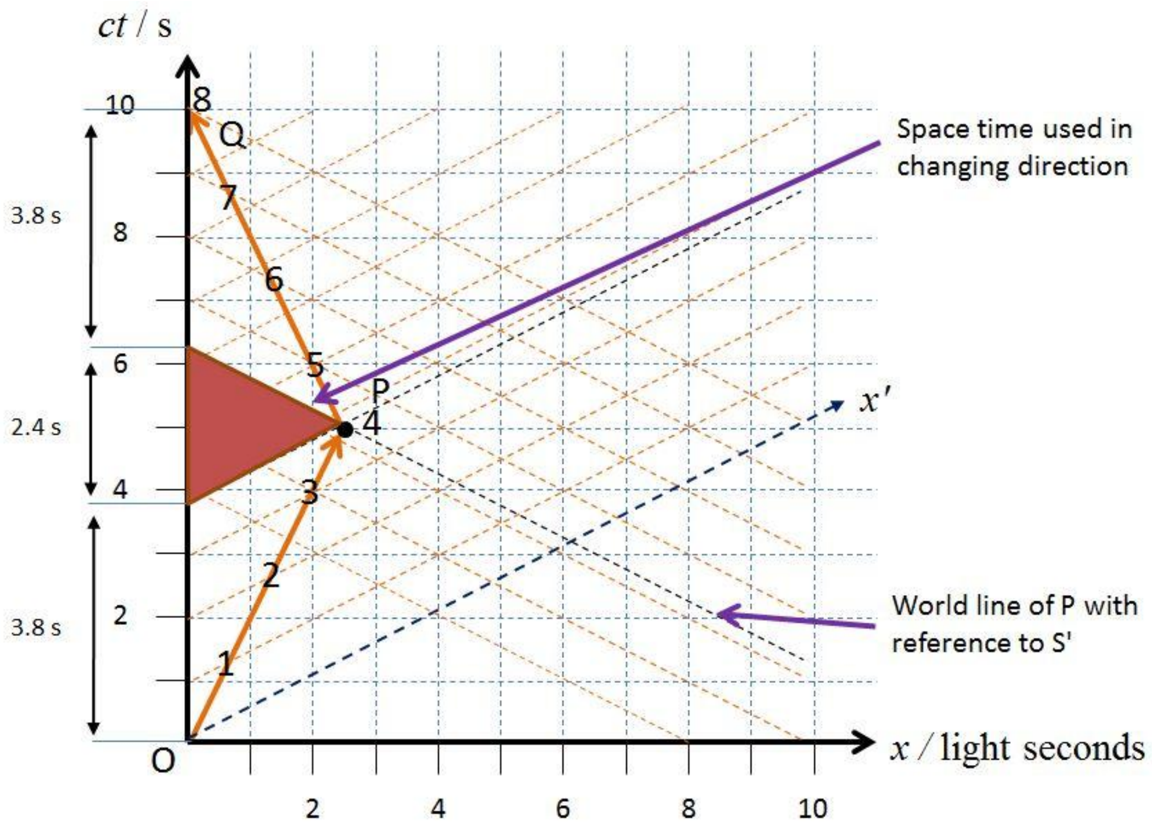


Figure 230 Minkowski diagram for the twins paradox showing the spacetime taken up by the chang in velocity

Therefore, the twins both age by 10 s.

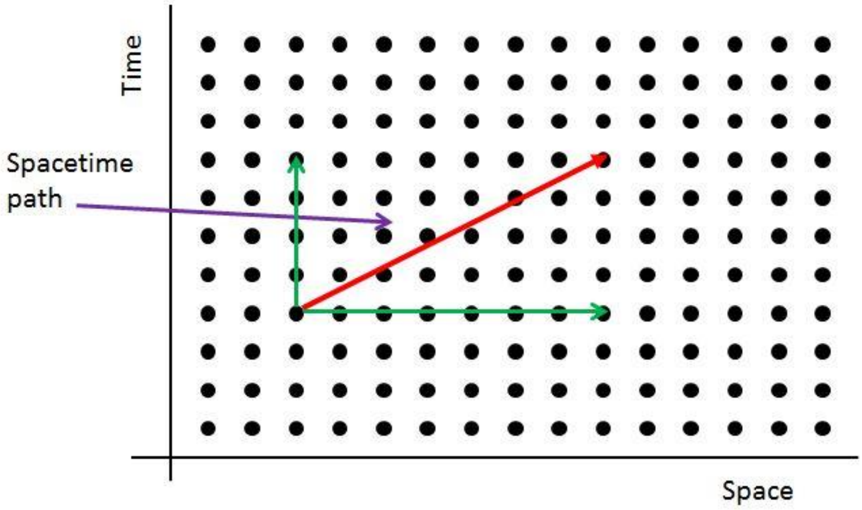
This is a simplistic representation that requires the spacecraft to do a "handbrake turn" at half the speed of light. The world line would be curved in reality, but this is beyond the scope of these notes.

Questions

Tutorial 15.18

15.18.1

A situation like this is impossible:



Explain why.

15.18.2

An object moves from a zero point to x = 4.0 m, y = 5.6 m, and z = 3.8 m. Work out the distance d.

15.18.3

Refer to Page 341. Use this formula to show that the unit length of the calibration x' and ct' axes is about 1.2 cm.

$$U' = U\left(\sqrt{\frac{1+\beta^2}{1-\beta^2}}\right)$$

The spacecraft is moving at a constant speed of 0.417 c.

Assume the spacing for the stationary observer is 1.0 cm

15.18.4

Refer to *Figure 223*. Which event happens first? Explain your answer.

15.18.5

Refer to *Figure 224*. How can we tell that time has dilated compared to that seen by the stationary observer?

15.18.6

The spacecraft travels at 0.5 c for 10 seconds. It goes away from the Earth for 5 seconds. It immediately turns round, and returns, reaching the Earth at a time of 10 seconds as seen by the Earthbound twin.

Draw the Minkowski diagram that shows these events. Mark the events and explain what they are.

15.18.7

If the proper time is 10 s, what is the dilated time?

15.18.8

What angle are the axes for the moving observer rotated by?

Tutorial 15.19 Relativistic Mechanics

IB Syllabus only

Contents

For IB Students.

This is a long tutorial. Go through it slowly and carefully.

15.191 Total Energy and Rest Energy

Particle travelling at low speeds are subject to all the rules of classical mechanics. They have mass, energy, and momentum. The SI units of these quantities, kg, J, kg m s^{-1}, are all far too big to be useful in particle physics. So, particle physicists use much smaller units like the **electron volt**, where 1 eV = 1.6×10^{-19} J.

We know that particles of matter have a mass. For example, the mass of an electron is 9.11×10^{-31} kg. We also know that energy and mass are closely related. Mass can be turned into energy, and energy can be turned into mass. Mass and energy are related by Einstein's simple equation:

$$E = mc^2$$ .... *Equation 327*

In the particle physics notes, the term **rest energy** is used. We say that a proton has a rest energy of 938 MeV when it is **stationary**.

We give the rest energy the physics code E_0. Therefore, the **rest mass** m_0 is related to the rest energy by:

$$E_0 = m_0c^2$$ .... *Equation 328*

The **total energy** of a particle is the **sum** of the **rest energy** and the **kinetic energy**:

$$E_{\text{tot}} = E_0 + E_{\text{k}}$$ .... *Equation 329*

If the speed of the particle is **much less** than the speed of light, we can use the **classical kinetic energy**, i.e.

$$E_{\text{tot}} = m_0c^2 + \frac{1}{2}mv^2$$

.... *Equation 330*

The total energy is given by:

$$E_{tot} = mc^2$$ .... *Equation 331*

So, we can write:

$$mc^2 = m_0c^2 + \frac{1}{2}mv^2$$

.... *Equation 332*

The answer of this last calculation (Question 15.19.3) showed that the contribution of the kinetic energy made very little difference to the total energy of the of the proton. (Also to get any difference to show requires an absurd number of significant figures.)

For high speed particles, the kinetic energy becomes much more significant. However, when the speed of the particles gets towards the speed of light, the classical formula breaks down. The equation can easily deliver speeds of greater than the speed of light, and we know that nothing can travel faster than 3.0×10^8 m s^{-1}. Therefore, we have to think about what happens as the speed of the particle gets close to the speed of light.

The **mass increases** as the particle approaches the speed of light according to the **relativistic equation**:

$$m = \gamma m_0$$

.... *Equation 333*

Or:

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

.... *Equation 334*

The term m_0 is called the proper or **rest mass**. It is the mass as measured by an observer in a frame of reference which is at rest relative to the observer.

The term m is the relativistic mass which is the mass as measured by an observer in a frame that is moving at a constant velocity v.

The equation looks horrendous, but you should be getting used to these equations by now. It is not difficult to work with provided you follow a strategy:

1. Work out the term v^2/c^2.

2. Take the number you work out away from one. You will get a fraction.

3. Find out the **square root** of the answer to step 2.

4. Take the reciprocal to get γ.

5. Multiply the term m_0 by γ to step 4 to get m.

The time m is always bigger than m_0. If it isn't, something has gone wrong.

The **relative kinetic energy** is given by:

$$E_{\text{tot}} = \gamma m_0 c^2$$

.... *Equation 335*

We know how to work out the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

.... *Equation 336*

So, we can write the equation:

$$E_{\text{tot}} = E_0 + E_{\text{k}}$$ .... *Equation 337*

which we can rewrite:

$$E_{\text{k}} = E_{\text{tot}} - E_0$$ .... *Equation 338*

We can also write:

$$E_{\text{k}} = mc^2 - m_0c^2$$

.... *Equation 339*

As in classical mechanics, **energy is always conserved**.

Worked example A proton of mass 1.67×10^{-27} is accelerated to a constant speed of 2.75×10^{8} m s^{-1}. (a) Work out γ. (b) Work out the relative mass of the proton; (c) Work out the kinetic energy in both J and eV.
Answer (a) Use: $$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$ $v^2/c^2 = (2.75 \times 10^8 \text{ m s}^{-1})^2 \div (3.0 \times 10^8 \text{ m s}^{-1})^2 = 0.8403$ $\gamma = (1 - 0.8403)^{-0.5} =$ **2.50**

(b)

$$m = \gamma m_0 = 2.50 \times 1.67 \times 10^{-27} = \underline{\mathbf{4.18 \times 10^{-27}\ kg}}$$

(c)

$$E_k = mc^2 = 3.76 \times 10^{-10}\ \text{J}$$

$$E_k = 3.76 \times 10^{-10}\ \text{J} \div 1.6 \times 10^{-19}\ \text{J eV}^{-1} = \underline{\mathbf{2.35 \times 10^{9}\ eV}} = 2.35\ \text{GeV}$$

The mass of a particle is given as kilograms, as we have used above. As we saw above, the numbers are clumsy. So, we give the units as **eV/c²**, or more commonly **MeV/c²**. This comes from the rearrangement of the equation:

$$m = \frac{E}{c^2}$$ .... *Equation 340*

So, we say that an electron has a rest mass of 0.511 **MeV/c²**. It can be written as 0.511 **MeV c⁻²**, but I have not seen it like this in any of my sources.

15.192 Relativistic Momentum

We know that that momentum is the **product of mass and velocity**. In classical mechanics, we write the equation as:

$$p = mv$$ .... *Equation 341*

Relativistic momentum is simply classical momentum multiplied by γ.

$$p = \gamma m_0 v$$ .... *Equation 342*

The SI units remain kg m s⁻¹. But like Joules, the units are far too big and clumsy. So, we use **MeV/c** or **MeV c⁻¹**.

As in classical mechanics, **momentum is always conserved**.

15.193 Relativistic Momentum and Energy

This is summed up in the **Einstein Relationship**.

We can relate **momentum** to **kinetic energy** in classical mechanics like this:

$$E_k = \frac{mv^2}{2} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

.... *Equation 343*

The energy in relativistic contexts can be worked out by reference to the momentum. There is a quantity, *pc*, the product of the momentum and the speed of light. It is derived from a form of the de Broglie expression. We will look at this later.

In the Einstein Relationship, we combine Einstein's relationship:

$$E = mc^2$$

with that of relativistic momentum:

$$p = \gamma m_0 v$$

The resulting relationship is:

$$E = \sqrt{(p^2c^2 + {m_0}^2c^4)}$$

.... *Equation 344*

We can also write this as:

$$E^2 = p^2c^2 + {m_0}^2c^4$$ .... *Equation 345*

Worked Example

The total energy of an alpha particle is 25 GeV. The mass of an alpha particle is 6.64 × 10^{-27} kg.

Work out the mass of the alpha particle in eV c^{-2}.

Calculate the momentum of the alpha particle. Express your answer in both eV c^{-1} and kg m s^{-1}.

Answer

$$p^2c^2 = E^2 - m_0^2 c^4$$

$$E_0 = 6.64 \times 10^{-27} \text{ kg} \times (3.0 \times 10^8 \text{ m s}^{-1})^2 = 5.98 \times 10^{-10} \text{ J.}$$

$$E_0 = 5.98 \times 10^{-10} \text{ J} \div 1.60 \times 10^{-19} \text{ J eV}^{-1} = 3.74 \times 10^9 \text{ eV}$$

$$m_0 = \underline{\mathbf{3.74 \times 10^9 \text{ eV c}^{-2}}}$$

$$p^2c^2 = (25 \times 10^9 \text{ eV})^2 - (3.74 \times 10^9 \text{ eV c}^{-2})^2 \times 1 \text{ c}^4$$

$$= 6.25 \times 10^{20} \text{ eV}^2 - (1.40 \times 10^{19} \text{ eV}^2 \text{ c}^{-4} \times 1 \text{ c}^4)$$

$$= 6.11 \times 10^{20} \text{ eV}^2$$

$$pc = (6.11 \times 10^{20} \text{ eV}^2)^{0.5} = 2.47 \times 10^{10} \text{ eV}$$

$$p = 2.47 \times 10^{10} \text{ eV c}^{-1}$$

$$p = (2.47 \times 10^{10} \text{ eV} \times 1.6 \times 10^{-19} \text{ J eV}^{-1}) \div 3.0 \times 10^8 \text{ m s}^{-1} = \underline{\mathbf{1.32 \times 10^{-17} \text{ kg m s}^{-1}}}$$

15.194 Particle Acceleration

To carry out particle experiments, we need them to travel at high speed. Charged particles need to be accelerated to high speed using electric fields. Accelerators are covered in Topic 2 Tutorial 4. The tutorial covers various particle accelerators. There is also an example using a classical treatment of a particle being accelerated at speed less than the speed of light.

We are going to look at how charged particles behave when they are accelerated to speeds closer than the speed of light.

The classical model for kinetic energy and charge is given by:

$$E_k = qV$$ .... *Equation 346*

So, we can write:

$$\frac{1}{2}mv^2 = qV$$

.... *Equation 347*

And rearrange:

$$V = \frac{mv^2}{2q}$$

.... *Equation 348*

Your answer to Question 15.19.8 gave you a voltage of 260 kV, which is easily obtainable from a small research accelerator. You would not get such a machine in a school or college lab.

Accelerating voltages of 15 MV are possible from a Van der Graff generator. Using this model, we could have electrons travelling at 60 times the speed of light. Since nothing can travel faster than 3.0×10^8 m s^{-1}, such figures are meaningless. So, we need to turn to **relativity** to say that as the particle gets closer to the speed of light, its **mass** increases.

The kinetic energy in any charged particle is governed by the relationship:

$$E_k = qV$$

This is true regardless of whether it's the energy dissipated by a resistor with a current flowing through it, or whether it's a charged particle accelerated by a voltage. We use this relationship however close to the speed of light the particle is travelling, because:

Charge is an invariant quantity.

This means that the charge on an electron always stays at (-) 1.6×10^{-19} C.

We need to use *Equation 337*, using the **frame of reference** of the **stationary observer**:

$$E_{\text{tot}} = E_0 + E_{\text{k}}$$

where:

- E_{tot} is the total energy.
- E_0 is the rest energy.
- E_{k} is the kinetic energy.

The accelerating voltage gives the particle **kinetic energy**. So, we can write:

$$E_{tot} = E_0 + qV \quad \text{............. Equation 349}$$

The following problem solving strategy to find the **particle speed** may help:

1. Work out the **rest energy** of the particle using:

$$E_0 = m_0c^2$$

The term m_0 is the **rest mass** and is given in a datasheet. For example, the rest mass of the electron is 9.11×10^{-31} kg. The rest energy is in **joules**.

2. Work out the kinetic energy by multiplying the accelerating voltage by the charge. Note that an alpha particle has a charge of $2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$C.

The kinetic energy will be in **joules**.

3. Add the **rest energy** to the **kinetic energy** to give the **total energy**.

4. Use the equation:

$$E_{\text{tot}} = \gamma m_0 c^2$$

to give the **Lorentz transformation**, γ. The Lorentz transformation is simply the ratio of E_{tot} to E_0, in the same way as it is the ratio of the relativistic mass to the rest mass.

5. The **particle speed** can be worked out using:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

This is rearranged to:

$$\gamma^{-2} = 1 - \frac{v^2}{c^2}$$

In this strategy, I have suggested that you should use SI units (i.e. energy in joules). If you are confident in the use of electron-volts, you can do so quite easily. You need to be consistent, though.

Worked example An electron has a total energy of 1.5 MeV. (a) Calculate the kinetic energy (b) Calculate the Lorentz factor, γ (c) Calculate the speed as a fraction of c and in m s^{-1}.
Answer (a) $E_k = E_{tot} - E_0$ $E_{tot} = 1.5 \times 10^6$ eV $E_0 = 9.11 \times 10^{-31}$ kg $\times (3.0 \times 10^8$ m s$^{-1})^2 = 8.199 \times 10^{-14}$ J $E_0 = 8.199 \times 10^{-14}$ J $\div 1.6 \times 10^{-19}$ J eV$^{-1} = 5.12 \times 10^5$ eV $E_k = 1.5 \times 10^6$ eV $- 5.12 \times 10^5$ eV = **0.988×10^6 eV**

(b)

$$E_k = \gamma m_0 c^2$$

$$0.988 \times 10^6 \text{ eV} = \gamma \times 5.12 \times 10^5 \text{ eV}$$

$$\gamma = 0.988 \times 10^6 \text{ eV} \div 5.12 \times 10^5 \text{ eV} = \underline{\mathbf{1.93}}$$

(c) Lorentz Factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

Rearranging:

$$\gamma^{-2} = 1 - \frac{v^2}{c^2}$$

$$1.93^{-2} = 1 - (v^2/c^2)$$

$$(v^2/c^2) = 1 - 0.268 = 0.732$$

$$v^2 = 0.732\, c^2$$

$$v = \underline{\mathbf{0.86\, c}} = \underline{\mathbf{2.6 \times 10^8 \text{ m s}^{-1}}}$$

We can also express the **kinetic energy** in terms of the **Lorentz Transformation** γ, if we do not know the accelerating voltage:

$$E_k = (\gamma - 1) m_0 c^2$$

.... *Equation 350*

The worked example will show us how.

Worked Example

An alpha particle has a mass of 6.64×10^{-27} kg. The total energy of the alpha particle is known to be 10 GeV.

(a) Work out the rest energy.

(b) Work out the Lorentz factor.

(c) Hence work out the kinetic energy.

(d) Show that your answer is consistent.

Electronic charge = 1.6×10^{-19} C

Answer

(a)

$$E_0 = 6.64 \times 10^{-27} \text{ kg} \times (3.0 \times 10^8 \text{ m s}^{-1})^2 = 5.98 \times 10^{-10} \text{ J.}$$

$$E_0 = 5.98 \times 10^{-10} \text{ J} \div 1.6 \times 10^{-19} \text{ J eV}^{-1} = \underline{\mathbf{3.74 \times 10^9 \text{ eV}}} = 3.74 \text{ GeV}$$

(b) Use:

$$E_{\text{tot}} = \gamma m_0 c^2$$

$$E_{\text{tot}} = 10 \text{ GeV} = g \times 3.74 \text{ GeV}$$

$$\gamma = 10 \text{ GeV} \div 3.74 \text{ GeV} = 2.67$$

(c)

$$E_k = (2.67 - 1) \times 3.74 \text{ GeV} = \underline{\mathbf{6.26 \text{ GeV}}}$$

(d)

$$E_{\text{tot}} = E_0 + E_k = 3.74 \text{ GeV} + 6.26 \text{ GeV} = \underline{\mathbf{10 \text{ GeV}}}\text{, which is consistent.}$$

15.195 Momentum of a Photon

We saw above that relativistic **momentum** is given by the relationship:

$$p = \gamma m_0 v$$

This relationship depends on there being a **rest mass**. However, photons are considered to have **zero mass**. Therefore, this relationship will not work.

When an annihilation event happens (see Topic 2 Tutorial 6), the momentum of the resulting gamma photons has to be conserved. Since there is momentum before, resulting from the particle and antiparticle, there has to be momentum after in the gamma photons. But according to the relationship above, the zero mass leads to zero momentum. There is a way around this apparent difficulty.

We can use the **General Relativity Expression**:

$$E^2 = p^2c^2 + m_0^2c^4$$

We can rewrite the expression to:

$$p = \frac{E}{c}$$ ....*Equation 351*

In the relationship above we do not have any mass term, even though the units suggest a mass. That doesn't matter. For zero mass particles, the **momentum is represented by the energy**, since we can regard c = 1.

From Topic 3 Physics Tutorial 1, we know that **energy** and **photon** wavelength are related by:

$$E = \frac{hc}{\lambda}$$

.... Equation 352

We can rearrange this to:

$$\frac{E}{c} = \frac{h}{\lambda}$$

.... Equation 353

And we can write:

$$p = \frac{h}{\lambda}$$

.... Equation 354

And it gives us:

$$\lambda = \frac{h}{p}$$

.... Equation 355

This is familiar to us as the **de Broglie relationship**. The photons are particles, so their wavelength is the de Broglie wavelength.

-

15.196 Particle Interactions

We can use the ideas above to predict the behaviour of particles as they decay. The neutral pion (π^0) decays to two gamma photons in 8.4×10^{-17} s.

$$\pi^0 \rightarrow \gamma + \gamma$$

About 99 % of neutral pions decay like this.

Worked Example

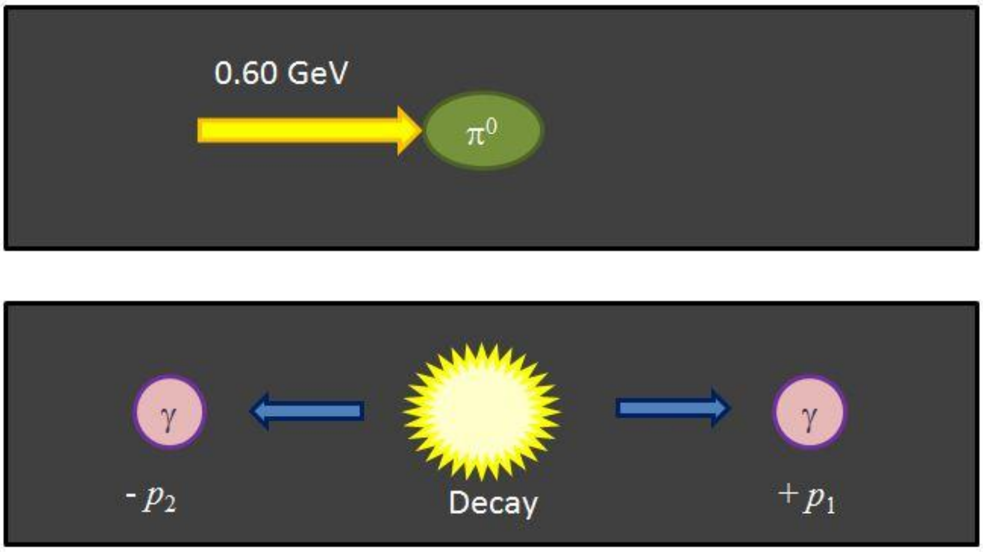
The rest energy of the neutral pion is 135 MeV. It is travelling with a kinetic energy of 0.60 GeV. When it decays the two gamma photons are emitted along the axis of travel, one forward, the other backward. The measurements are taken from the stationary frame of reference.

(a) What is the momentum of each photon when viewed from the moving frame of reference?

(b) What are the measured momenta of the two photons, when viewed from the stationary frame of reference?

Answer

Let's imagine what's happening:



We will use the convention of left to right is positive for analysis of the momentum.

Total energy = rest energy + kinetic energy = 135 MeV + 600 MeV = 735 MeV

(a) In the moving frame of reference, the pion is stationary relative to the frame.

Momentum is conserved, so the values of the momentum p_1 and the value of the momentum p_2 are both the same. The directions are opposite. The value of the momenta are:

p = 135 MeV c^{-1} ÷ 2 = **67.5 MeV c^{-1}**

(b) For the stationary frame of reference:

For momentum:

$$E^2 = p^2c^2 + m_0^2c^4$$

Rearranging:

$$p^2c^2 = E^2 - m_0^2c^4$$

Therefore:

$$pc = (E^2 - m_0^2c^4)^{0.5}$$

Substituting

pc = ((735 MeV)^2 - (135 MeV)^2)$^{0.5}$ = (522000 MeV^2)$^{0.5}$ = 722 MeV

Momentum = **722 MeV c^{-1}**

We can use $c = 1$. And we can write $E = p$. So, we can consider momentum to be energy, as the photons have zero mass.

Momentum is conserved. Momentum before = 722 MeV.

Momentum after = 722 MeV = $E_1 - E_2$

We also know that the energies add up:

735 MeV = $E_1 + E_2$

So, we do simultaneous equations:

$E_1 - E_2$ = 722

$E_1 + E_2$ = 735

E_1 = 722 + E_2

722 + E_2 + E_2 = 735

E_2 = (735 - 722) ÷ 2 = 6.5 MeV

E_1 - 6.5 = 722

E_1 = **728.5 MeV**

If we wanted to know the wavelength of the photons, we would need to use the value of the momentum from the **moving frame**. Otherwise, we would have two different wavelengths. The wavelength is linked to the wave speed through:

$$c = f\lambda$$ ....*Equation 356*

The speed of light **does not change** with the reference frame (**Einstein's Second Postulate**). Therefore, we have to use the frame that gives us the same value for both of the momenta, i.e. the moving frame of reference.

Questions

Tutorial 15.19

15.19.1

Calculate the energy contained within the mass of an electron. Give your answer in J and eV. Give your answer to an appropriate number of significant figures.

15.19.2

Show that the mass of a proton is 1.67×10^{-27} kg.

15.19.3

A proton is travelling at 1.2×10^{5} m s^{-1} in a straight line.

(a) Calculate the kinetic energy and the total energy in both J and eV.

(b) What is the equivalent mass of the moving particle?

15.19.4

Refer to the worked example on Pages 358 – 359. Work out the total energy of the proton in the example.

15.19.5

Show that eV/c^2 is consistent with kilograms.

15.19.6

Explain what happens when a particle reduces speed from $0.8\ c$ to 0 m s^{-1}. No calculation is needed.

15.19.7

A proton of mass 1.67×10^{-27} kg is travelling at a constant speed of 0.5 c. Calculate:

(a) the rest mass of the proton in MeV c^{-2}.

(b) the Lorentz factor;

(c) the momentum in MeV c^{-1}

15.19.8

Use the classical model to show that the accelerating voltage needed to give an electron a speed of 3.0×10^{8} m s^{-1} is about 260 kV.

Mass of electron = 9.11×10^{-31} kg;

Charge on an electron = 1.6×10^{-19} C.

15.19.9

Use the electron energy from Question 15.19.8 to work out the speed of the electron that is accelerated to a kinetic energy of 260 keV.

15.19.10

Rewrite the expression above if m_0 is zero.

15.19.11

Show that the SI units in this relationship are consistent.

15.19.12

A photon has a wavelength of 520 nm. What is its momentum?

Planck's Constant = 6.63×10^{-34} J s.

15.19.13

Explain why the neutral pion decays into two gamma photons.

15.19.14

Work out the wavelength of the gamma photons of energy 67.5 MeV

Tutorial 15.20 General Relativity in the Universe

IB Syllabus only

Contents

IB Students

This is quite a difficult tutorial. Take your time going through it.

15.201 The Pound Rebka Experiment

This experiment was carried out by the physicists R V Pound (1919 - 2010), and G A Rebka (1931 - 2015). It is often called the Pound-Rebka-Snider Experiment. (I can't find anything about Snider.) It was carried out at Harvard University's Jefferson Laboratory in 1958. It involved drilling a hole 30 cm wide through several floors from the attic to the basement, a height of 22.6 m. History does not recall what the Dean had to say when the project was first suggested to him (I would suggest that he would have had a fit), but clearly Pound and Rebka had good persuasive powers. The picture (*Figure 231*) shows the Jefferson Lab where the experiment was carried out.

Figure 231 The Jefferson Laboratory

The experiment was set up like this (*Figure 232*):

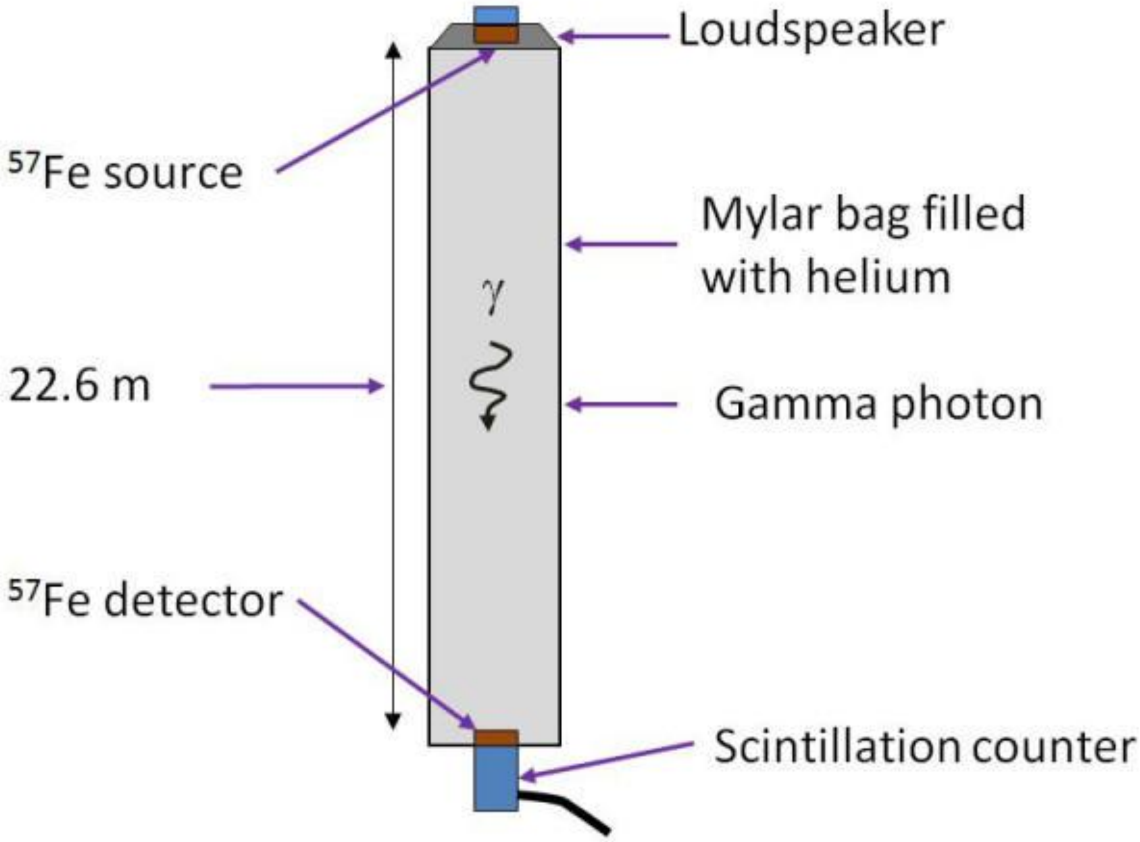


Figure 232 The Pound-Rebka-Snider Experiment

The Mylar bag was filled with helium to minimise the scattering of the gamma rays. The source contained ^{57}Co (cobalt-57) electroplated and annealed to a block of iron. This was mounted on a loud speaker. The cobalt-57 decays by beta minus decay:

$$^{57}_{27}\mathrm{Co} \rightarrow {}^{57}_{26}\mathrm{Fe} + {}^{0}_{-1}e^{-} + \bar{\nu}_{\varepsilon}$$

The iron-57 nucleus is left in an **excited** state once the electron has been emitted. It loses the excess energy in the form of a gamma photon of 14.4 keV.

The detector was a block of material in which there were iron blocks enriched to 32 % iron-57 (the normal level is 2 %).

The results showed a **blue shift**.

Then they swapped the positions of the speaker and the detector (*Figure 233*):

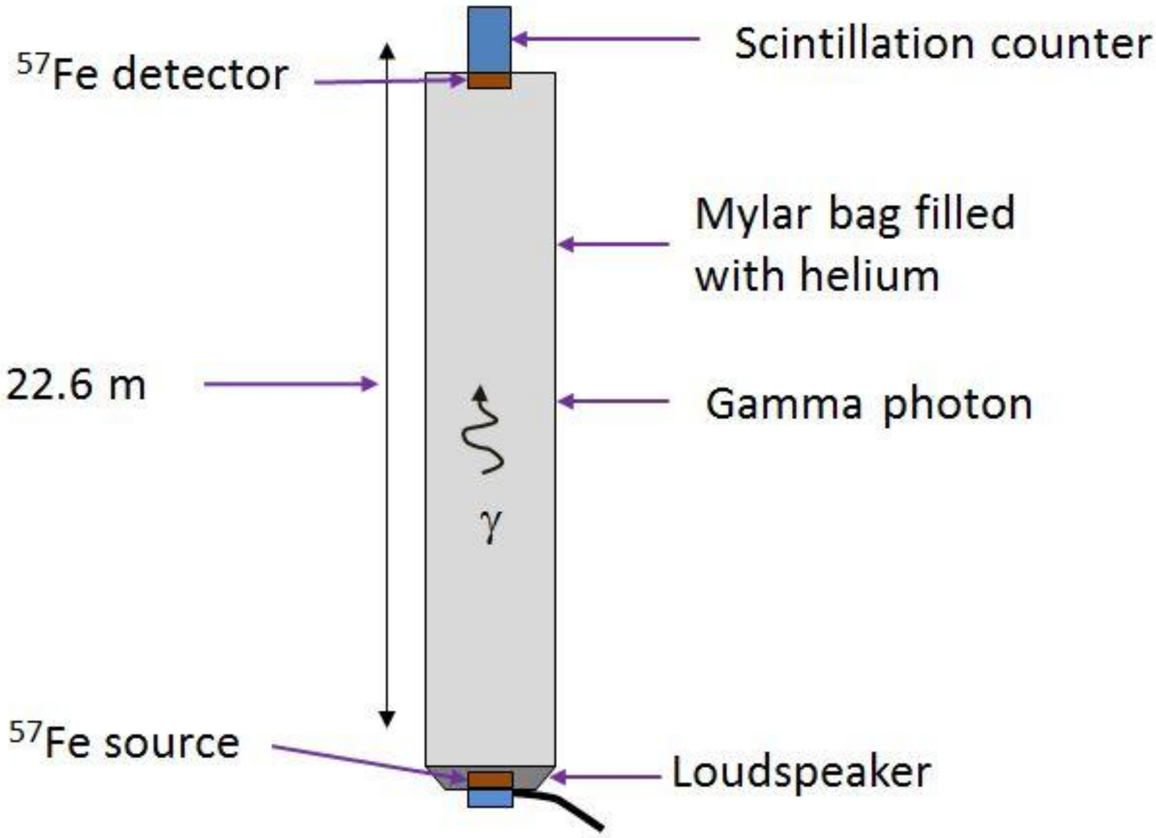


Figure 233 The second part of the experiment

To understand this experiment, we need to look at a couple of physics principles that underpin it.

15.202 Momentum due to Gamma Emission

When Iron-57 nuclei are excited, they give out gamma photons with an energy of 14.4 keV. If the photons strike a second iron-57 nucleus, that nucleus should become excited. It works just like the excitation of electrons in atoms. The energy of the photon has to be **exactly right.** Note that iron-57 is **stable**.

If we have a single iron-57 nucleus firing a single photon at a second iron-57 nucleus, we expect to see (*Figure 234*):

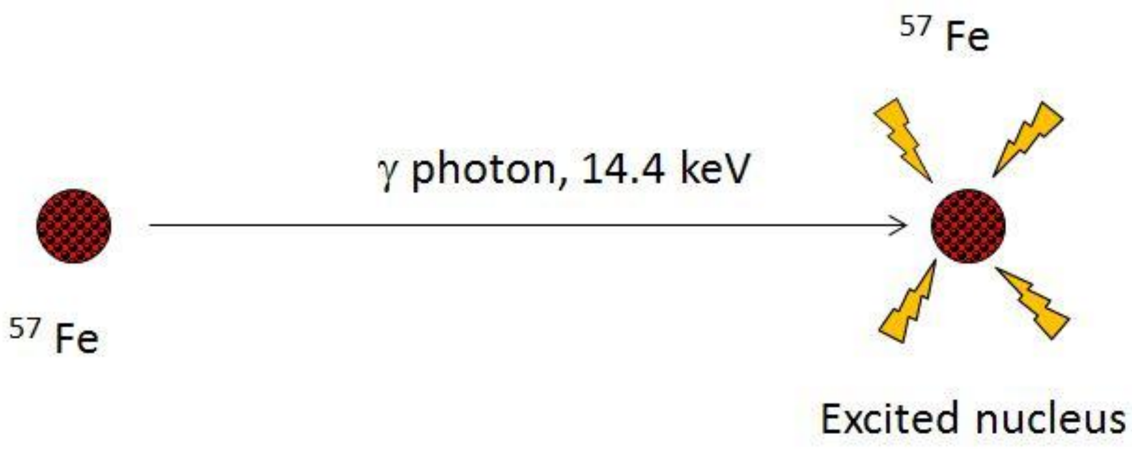


Figure 234 Expected result of an Iron-57 nucleus absorbing a gamma photon of 14.4 keV

However, we don't see this, because the first nucleus **recoils** due to the **conservation of momentum** (*Figure 235*).

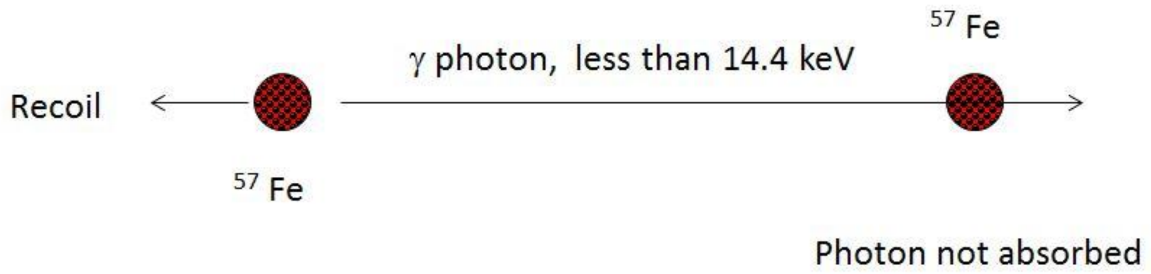


Figure 235 Gamma photon has less energy

The photon has relativistic momentum because it has no mass. The nucleus has classical momentum, because there is mass. The relativistic momentum is given by:

$$E^2 = p^2c^2 + m_0{}^2c^4$$ .... *Equation 357*

Since the photon has **zero** mass, the equation becomes:

$$E^2 = p^2\ c^2$$ .... *Equation 358*

The recoil momentum for the iron-57 nucleus is going to be the same as the momentum of the gamma photon, i.e. 14.4 keV/c. It is easier to express it in terms of **energy**. So, we can write:

$$E_{\mathrm{k}} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

.... *Equation 359*

So, we can write the overall expression as:

$$E_k = \frac{p^2c^2}{2mc^2}$$

.... *Equation 360*

Do not confuse the photon energy with the kinetic energy of the recoiling nucleus.

We can work out the speed of recoil by converting the energy to joules and using the classical kinetic energy equation.

The speed of recoil (see 15.20.5) is about the same as an airgun pellet. As far as particle physics is concerned, this speed is negligible. However, it is sufficient for the energy of the incoming photon to be reduced so that it is NOT absorbed. The photon will pass through the nucleus.

If the recoil speed can be reduced by increasing the mass of the source, the proportion of energy lost is much less, so the photon is absorbed. This is done by mounting the iron 57 nucleus in a much larger crystal to reduce the recoil. This is called the **Mossbauer Effect**. It's like pinning a cannon to the ground.

We can get the same result for the recoil speed using **classical momentum**. In this case, we can say that the momentum of the photon is given by:

$$E^2 = p^2c^2 + m_0{}^2c^4$$

Since the photon has **zero** mass, the equation becomes:

$$E^2 = p^2 c^2$$

Therefore:

$$E = pc$$ .... *Equation 361*

For the nucleus, we know that:

$$p = mv$$ ....*Equation 362*

By using conservation of momentum, we can write:

$$p = \frac{E}{c} - mv = 0$$

.... *Equation 363*

So, we can combine *Equations 362* and *363* to give:

$$\frac{E}{c} = mv$$

.... *Equation 364*

If iron-57 nucleus is embedded in a much bigger structure, for example a block of material, the recoil speed will be much less, as we saw above.

You may be thinking, "So What?" How is this relevant to the frequency-shift in gamma rays?

15.203 Gravitational Red Shift of Gamma Rays

If we were able to move the iron-57 source at a sufficient speed towards the stationary detector, we should observe that the gamma photons are absorbed. Moving a source at 80 m s^{-1} is not that difficult. That is why the loudspeaker was used.

Let us consider the source at the top, and the detector at the bottom (*Figure 236*).

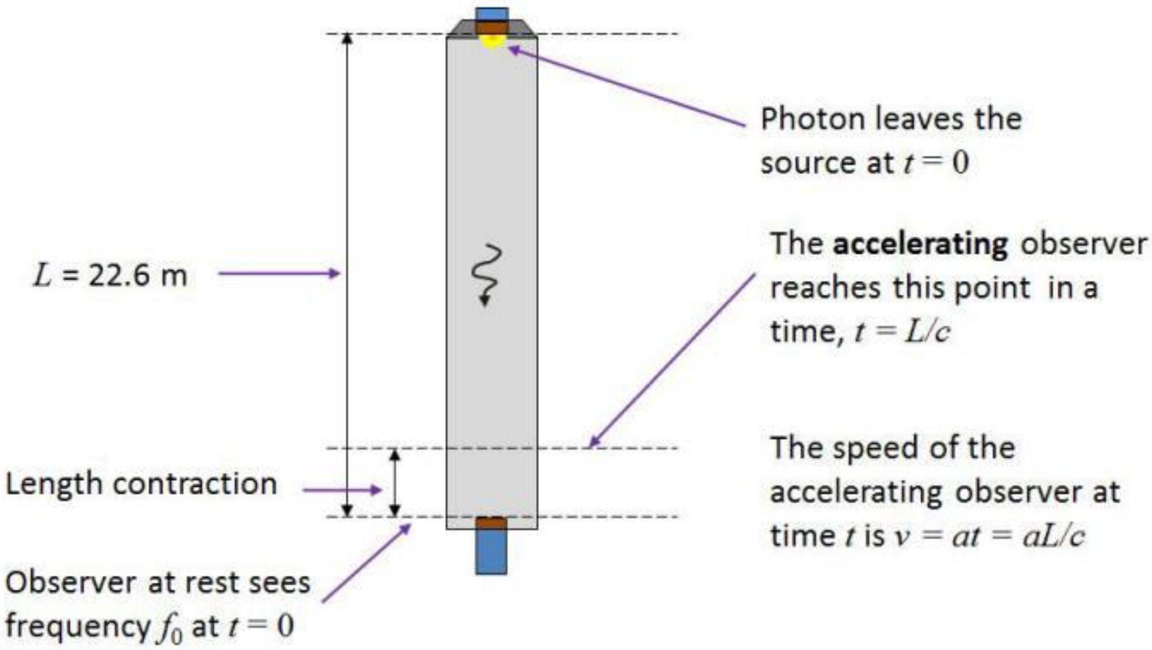


Figure 236 Experiment showing length contraction

Consider the set up with the source at the top. The photon leaves the source with a downward velocity of c. Let us assume that the speaker is stationary at this moment. The key idea in this argument is that a photon **can be accelerated by gravity**. Physicists accept the truth of this by phenomena such as **gravitational lensing**. The speed cannot be affected, as the speed of light is **invariant**. However, the photon **frequency** can be affected by acceleration.

So, let's look at the process, which is summed up in the diagram:

- The photon leaves the source at time 0.
- If we are the **stationary** observer, we see that the time taken for the photon to get to the detector is:

$$t = \frac{L}{c}$$ .... *Equation 365*

Now let's consider the accelerating observer. In previous tutorials, we have considered that a moving observer does so at a constant speed. There is no reason why the observer cannot have a changing speed. In this case, the observer is accelerating at g (9.81 m s^{-2}). You be an accelerating observer by jumping off a diving board into a swimming pool.

For the accelerating observer:

- There is length contraction.
- The acceleration is constant.

The velocity at time $t = L/c$ is given by the simple equation of motion:

$$v = at = \frac{aL}{c}$$ .... *Equation 366*

We can say this because the velocity is **very much less than** the speed of light.

Because c is invariant, we cannot use the Doppler equations to calculate speed. Instead, we look at the change in frequency. The Doppler frequency shift is given by:

$$f = f_0\left(1 + \frac{v}{c}\right)$$ .... *Equation 367*

We can rewrite this to:

$$f = f_0\left(1 + \frac{at}{c}\right) = f_0\left(1 + \frac{aL}{c^2}\right)$$ .... *Equation 368*

Since the acceleration is g, we can now write:

$$f = f_0\left(1 + \frac{gL}{c^2}\right)$$ .... *Equation 369*

The **fractional change in frequency** is the difference between f and f_0.

Since the fractional change in frequency is very small, it would make no sense to try to work out the real values in frequency, as we would need a very large number of significant figures to show any difference in absolute frequency. However, we can say that the change in frequency is sufficient to **prevent the photon being absorbed** by the iron-57 nucleus in the detector. The iron-57 nucleus will remain in the ground state. This is because the nucleus is excited by a photon of **exactly** the right frequency.

If we go back to the first equation:

$$f = f_0\left(1 + \frac{v}{c}\right)$$ .... *Equation 370*

we can say that the fractional change is:

$$\Delta f = \frac{v}{c}$$ .... *Equation 371*

Therefore:

$$\frac{v}{c} = \frac{gL}{c^2}$$ .... *Equation 372*

We can tidy this up to give:

$$v = \frac{gL}{c}$$ .... *Equation 373*

15.204 How was this result used?

Pound and Rebka used a loudspeaker operating at about 10 Hz. In the middle of the cone, there was the gamma source. Let us suppose that the loudspeaker was pointing downwards. To give us the right value of the speed, (7.4×10^{-7} m s^{-1}), for the photon to be absorbed, the cone needs to be moving **downwards**. This will happen when the cone is very close to the maximum amplitude:

- positive accelerating as it leaves the top;
- negatively accelerating as it approaches the bottom.

If we look at the sine wave, the positions where these values of downward velocity are seen are shown here (*Figure 237*):

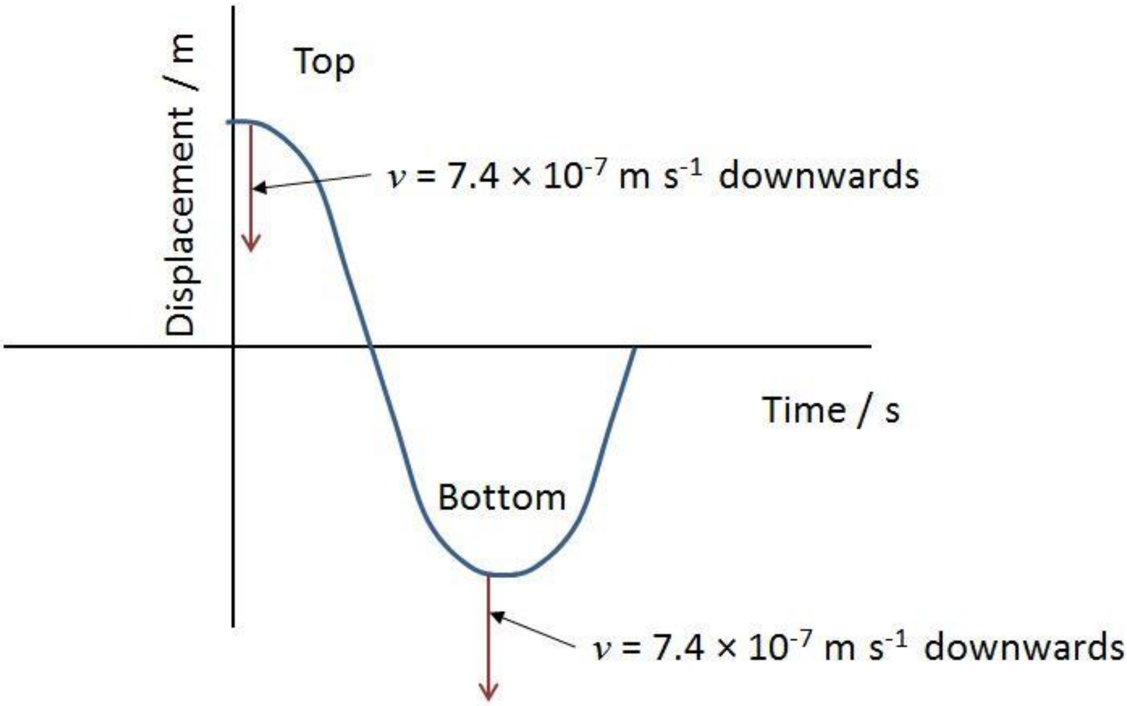


Figure 237 Points on the loudspeaker sine wave where the velocities are just right.

The phase would angle from the amplitude would be very small.

The speaker moves with **simple harmonic motion**. You may want to review SHM in Topic 8 Tutorial 4. The velocity of the oscillator is given by:

$$v = -A\omega \sin(\omega t)$$ .... *Equation 374*

Let us suppose that the amplitude of the speaker is 1.0 mm. We will assume that downwards is negative. We will superimpose the velocity time graph (*Figure 238*):

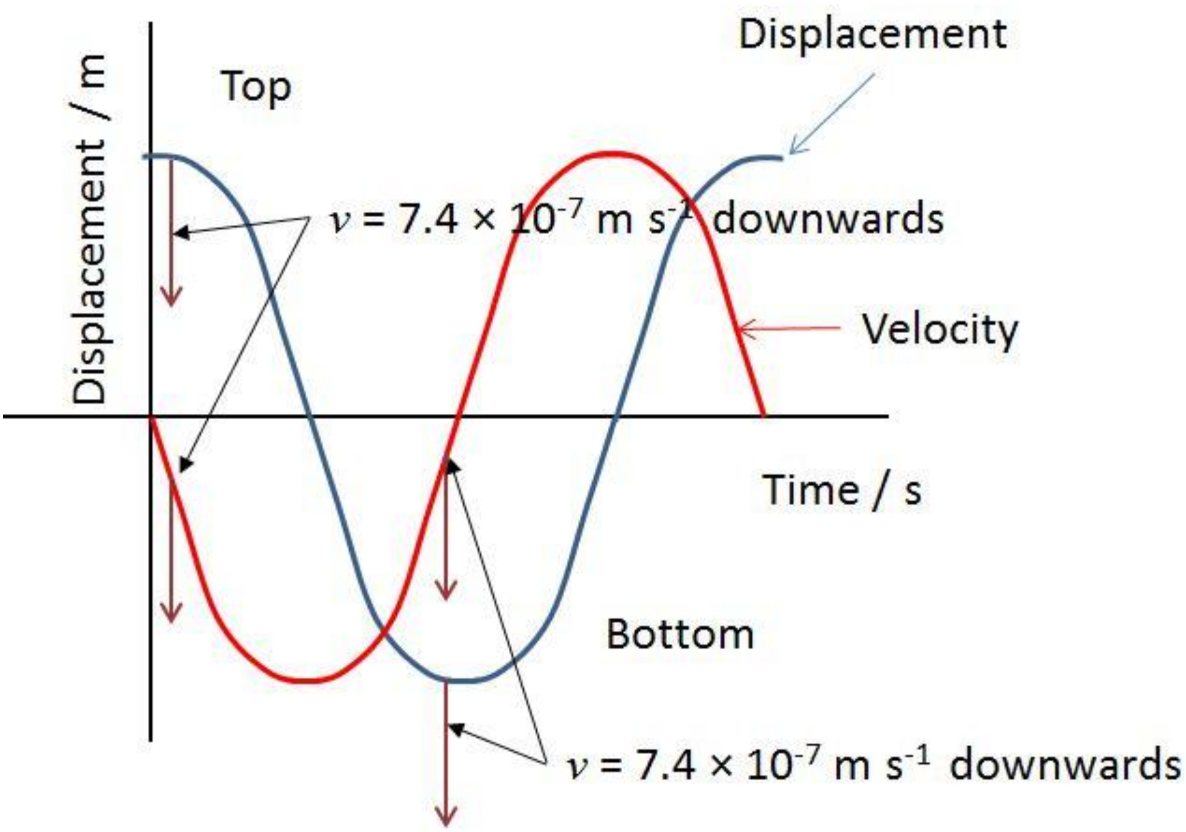


Figure 238 Velocity time graph superimposed on Figure 237

When the speaker is moving down at these points, we should expect to see that the intensity of the transmitted gamma photons falls to a low level. It won't fall to zero, because the photons are not all striking the detector at precisely 90 °, which is the assumption we are making in this model. The picture shows what was seen at the detector (*Figure 239*):

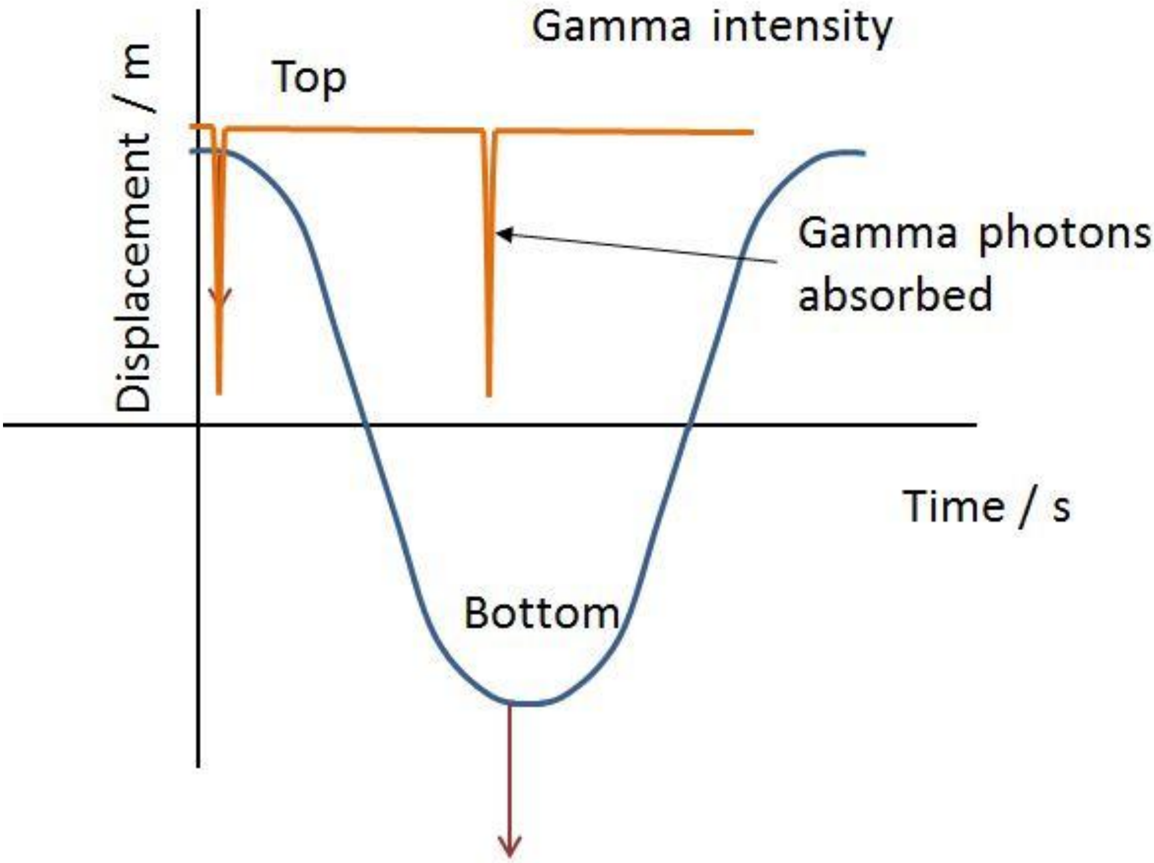


Figure 239 Gamma intensity as seen on a displacement-time graph

In this account, we have assumed that the speaker was at the top of the tower. We also used 10 Hz. Pound and Rebka used frequencies up to 50 Hz. They also put the speaker at the bottom and the detector at the top. In this case, there was red shift as the photons were moving against gravity.

The results were within about 10 % of what they expected. The conclusion is that **photons are subject to gravitational attraction, even though they have zero mass**.

You can also use energy to model this system. This is explained on different websites. Note that some authors uses ν (in italics) for frequency, instead of f, and v (normal) for velocity. Be careful. It caught me out!

15.205 Gravitational Time Dilation

A clock in a gravity field runs a little more slowly than if it were in a place where the gravity field was zero. This means that the time interval required to travel a certain distance is slightly longer in a strong gravity field than it is in a weak gravitational field. This is called **gravitational time dilation**.

Consider two ray of light (*Figure 240*). The ray of light between a and b is in a very weak gravity field, so it travels in (almost) a straight line. The ray of light between c and d is in a stronger field. It is curved, due to the **curving of space-time**. Therefore, the path is **longer**.

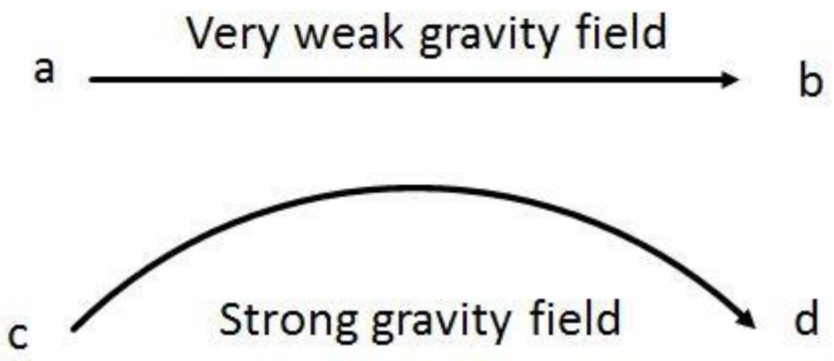


Figure 240 Light travelling in a gravity field

[*Spoiler alert - bad science fiction*] Consider a rocket moving between points a and b at the speed of light. It travels in a time interval of Δt_f. A clock in the rocket ticks slightly faster in this rocket. An identical rocket travels between points c and d, again at the speed of light. The points c and d are within the gravity field of a very large planet of mass M. The time interval now is Δt_0. We call this the **proper time** as observed in the gravity field. The clock ticks at what we consider to be the correct time.

The two time intervals are related by:

$$\Delta t_0 = \Delta t_f \left(1 - \frac{2GM}{rc^2}\right)^{1/2}$$

.... Equation 375

Where:

- Δt_0 - The proper time interval (s).
- Δt_f - The time interval in the very weak field, i.e. an infinite distance from the planet (s).
- G - Universal gravitational constant (= 6.67×10^{-11} N m^2 kg^{-2}).
- M - mass of planet (kg).
- r - the distance of the observer from the centre of the large planet (m).
- c - the speed of light (= 3.00×10^8 m s^{-1}).

You may wish to review the **Schwarzschild Radius** in Topic 14A Tutorial 6. The relationship is:

$$R_S = \frac{2GM}{c^2}$$

.... Equation 376

The term R_s is the Schwarzschild Radius (m).

It does not take a genius to see that the substitution of *Equation 376* into *Equation 375* gives us:

$$\Delta t_0 = \Delta t_f \left(1 - \frac{R_S}{r}\right)^{1/2}$$

.... Equation 377

Gravitational Time Dilation is not just something theoretical that happens. We can work out how the time here on Earth compares with the time observed in a very weak gravity field. We know from Topic 9 Tutorial 1 that:

$$g = -G\frac{M}{r^2}$$

.... *Equation 378*

This rearranges to:

$$GM = -gr^2$$ .... *Equation 379*

Which we can substitute into *Equation 375*:

$$\Delta t_0 = \Delta t_f \left(1 - \frac{2GM}{rc^2}\right)^{1/2}$$

To give us:

$$\Delta t_0 = \Delta t_f \left(1 - \frac{2gr}{c^2}\right)^{1/2}$$

.... *Equation 380*

Note how sign changes.

Worked Example Work out what the time interval of 1.00 s experienced on Earth would be on a space-probe at a point that is far distant from any sources of gravity. g = -9.81 m s^{-2}; Radius of the Earth = 6.37 × 10^{6} m
Answer Rearrange to give Δt_f: $$\Delta t_f = \Delta t_0 \left(1 - \frac{2gr}{c^2}\right)^{-1/2}$$ Note the substitution of the negative value for g. Δt_f = 1.00 s × (1 - ((2 × -9.81 m s^{-2} × 6.37 × 10^{6} m) ÷ (3.00 × 10^{8} m s^{-2})^{2}))$^{-0.5}$ = 1.00 s × (1 - ((-1.25 × 10^{8} m^{2} s^{-2}) ÷ (9.00 × 10^{16} m^{2} s^{-2})))$^{-0.5}$ = 1.00 s × (1 - -1.39 × 10^{-9})$^{-0.5}$ = **0.9999999993 s** This is one part in about 10^{-9} s Note that we have done something that would be frowned on, using excessive significant figures when the data are to just a few significant figures. Issues arising from this bad practice can be dealt with using a binomial expression.

1 second in 1 × 10^{9} represents 1 second in about 30 years. Although that may seem very small, the effect can be significant.

For example, the GPS satellites that enable us to navigate about the Earth so easily are in an orbit that is 26560 km above the Earth's surface. The clocks in GPS satellites have to be adjusted to take the time dilation into account.

Questions

Tutorial 15.20

15.20.1

Refer to Pages 375 to 376. What did the results of this second experiment show?

15.20.2

What happens if the energy of the gamma photon striking the iron-57 nucleus is slightly different to the value quoted on Page 376?

15.20.3

Show that the rest energy of an iron-57 nucleus is about 53 GeV.

Nuclear mass of the nucleus = 56.935 u

1 u = 931.5 MeV

15.20.4

Calculate the kinetic energy of the iron-57 nucleus.

15.20.5

Calculate the speed of recoil of the nucleus.

Nuclear mass of the nucleus = 56.935 u

1 u = 1.661×10^{-27} kg

15.20.6

By considering the classical momentum of the iron-57 nucleus, show that the speed of recoil of the nucleus about 80 m s^{-1}, consistent to your answer to Question 15.20. 5.

Discuss whether this approach is valid.

The mass of an iron-57 nucleus = 56.935 u;

1 u = 1.661×10^{-27} kg

15.20.7

The gamma photon emitted by the iron-57 has energy of 14.4 keV. Calculate the frequency of the photon.

Planck's Constant = 6.63×10^{-34} J s.

15.20.8

Work out the fractional change in frequency for the photon as it moves down the tower.

15.20.9

The source has to be moved to give the fractional change needed to ensure that the 14.4 keV photons are absorbed by the iron-57 nuclei in the detector.

Show that the speed of movement is about 7.4×10^{-7} m s^{-1}.

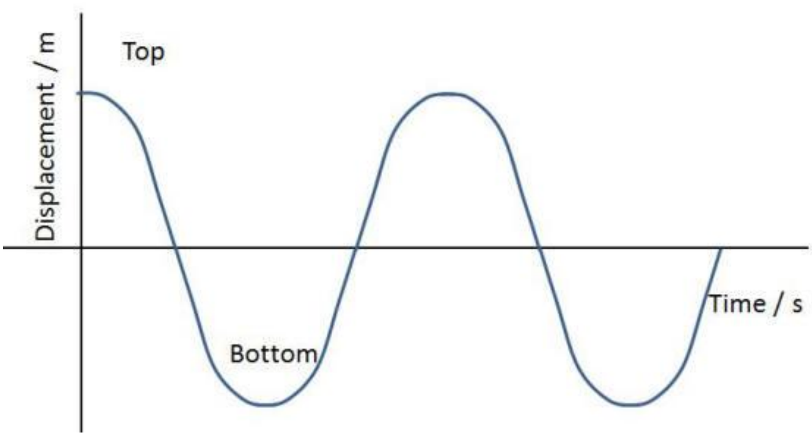
(g = 9.81 m s^{-2})

15.20.10

Refer to Page 394. Assuming that the oscillation of the loudspeaker starts at $t = 0$, calculate the first instance of time t at which the downwards velocity of the speaker is 7.4×10^{-7} m s^{-1}.

15.20.11

The diagram below shows the speaker moving through more than one cycle:



Copy the diagram and sketch where you would expect to see the absorption minima.

15.20.12

Some more very bad science fiction.

Some cosmonauts land on the surface of a neutron star which has a mass of 1.95×10^{30} kg and has a radius of 20.0 km. Their colleagues in the far distant mother ship observe tell them they have 60 minutes on the surface of the neutron star. The 60 minutes is relative to the mother ship.

Calculate how this time interval will appear on to the observers on the mother ship.

G - Universal gravitational constant = 6.67×10^{-11} N m^2 kg^{-2}.

Answers to Questions

Tutorial 15.01

Since there are no questions in this tutorial, there are no answers.

Tutorial 15.02

15.02.1

a) Formula:

$$E_{tot} = \frac{-Ze^2}{8\pi\varepsilon_0 r}$$

$E_{tot} = (-1 \times (1.60 \times 10^{-19}\ C)^2) \div (8 \times \pi \times 8.85 \times 10^{-12}\ F\ m^{-1} \times 5.29 \times 10^{-11}\ m)$

$= -2.56 \times 10^{-38}\ C^2 \div 1.177 \times 10^{-20}\ F$

= **-2.18×10^{-18} J**

(b)

Energy in eV = -2.18×10^{-18} J ÷ 1.60×10^{-19} J eV^{-1} = **-13.6 eV**

(c)

It represents the ionisation energy which is 13.6 eV.

15.02.2

(a) Formula:

$$mvr = \frac{nh}{2\pi}$$

L = (1 × 6.63 × 10^{-34} J s) ÷ 2 π = **1.06 × 10^{-34} kg m^2 s^{-1}**

(b)

v = (1.06 × 10^{-34} kg m^2 s^{-1}) ÷ (9.11 × 10^{-31} kg × 5.29 × 10^{-11} m) = **2.20 × 10^6 m s^{-1}**

(c)

λ = (6.63 × 10^{-34} J s) ÷ (9.11 × 10^{-31} kg × 2.20 × 10^6 m s^{-1}) = **3.31 × 10^{-10} m**

(d)

ω = 2.20 × 10^6 m s^{-1} ÷ 5.29 × 10^{-11} m = **4.16 × 10^{16} rad s^{-1}**

(e)

f = 4.16 × 10^{16} rad s^{-1} ÷ (2 π) = **6.62 × 10^{15} Hz**

15.02.3

(a) Formula:

$$\lambda = \frac{hc}{E}$$

E = 13.6 eV × 1.60 × 10^{-19} J eV^{-1} = 2.176 × 10^{-18} J

λ = (6.63 × 10^{-34} J s × 3.00 × 10^8 m s^{-1}) ÷ 2.176 × 10^{-18} J = **9.14 × 10^{-9} m**

(b)

No. de Broglie wavelength = 3.31 × 10^{-10} m.

The de Broglie wavelength is 276 times smaller than the wavelength of the light.

15.02.4

(a) Formula:

$$mvr = \frac{nh}{2\pi}$$

v = (2 × 6.63 × 10^{-34} J s) ÷ (2 × p × 9.11 × 10^{-31} kg × 2.11 × 10^{-10} m) = **1.10 × 10^{6} m s^{-1}**

(b)

The linear speed worked out in Question 2 (b) is 2.20 × 10^{6} m s^{-1}.

The linear speed worked out here is half of that figure.

Tutorial 15.03

15.03.1

Formula:

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

ΔE = 13.6 eV × 1.60 × 10^{-19} J eV^{-1} = 2.176 × 10^{-18} J

Δt = 6.63 × 10^{-34} J s ÷ (4 × p × 2.176 × 10^{-18} J) = 2.42 × 10^{-17} s

Distance = 3.50 × 10^{6} m s^{-1} × 2.42 × 10^{-17} s = **8.49 × 10^{-11} m**.

15.03.2

The units are not consistent.

The units of h are joule seconds (J s). In base units, this is kg m^2 s^{-2} × s = kg m^2 s

If we put energy and momentum together, we get these base units:

kg m^2 s^{-2} × kg m s^{-1} = kg^2 m^3 s^{-3}

If we put position and time together, we get:

m s

Tutorial 15.04

15.04.1

The photon gets its energy entirely through the frequency of the electromagnetic radiation ($E = hf$).

(The maximum frequency is about 2.4 × 10^{33} Hz.

This corresponds to a wavelength of 1.2 × 10^{-25} m.

Energy is 1.0 × 10^{19} eV.)

15.04.2

Energy = 1.0 × 10^{19} eV × 1.6 × 10^{-19} J = **1.6 J**

15.04.3

It would be beta minus decay,

as the nucleon number is greater than the stable nucleon number of 56.

This indicates that there are too many neutrons.

The neutrons decay by beta minus decay.

15.04.4

(a) Lorentz factor:

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Lorentz factor, $\gamma = 1 \div [1 - ((0.98\ c)^2 \div c^2)]^{0.5} = 1 \div [1 - 0.98^2]^{0.5} = 1 \div 0.199$

γ = **5.025** (about 5, QED).

(b)

Mass = 6.64 × 10^{-27} kg × 5.025 = 3.3366 × 10^{-26} kg = **3.34 × 10^{-26} kg**

(c) Rearrange equation for radius of a charged particle:

$$r = \frac{mv}{BQ}$$

r = (3.3366 × 10^{-26} kg × 0.98 × 3.00 × 10^{8} m s^{-1}) ÷ (4.35 × 10^{-5} T × 2 × 1.60 × 10^{-19} C)

= **7.05 × 10^{5} m** = 705 km

15.04.5

Rearrange equation for radius of a charged particle:

$$r = \frac{mv \sin\theta}{BQ}$$

r = (3.3366 × 10^{-26} kg × 0.98 × 3.00 × 10^{8} m s^{-1} × sin 60) ÷ (4.35 × 10^{-5} T × 2 × 1.60 × 10^{-19} C)

r = 7.05 × 10^{5} m × 0.866 = **6.11 × 10^{5} m** = 611 km

15.04.6

(a)

We know that the radius is 6.11×10^5 m.

Therefore, the circumference is $2 \times \pi \times 6.11 \times 10^5$ m $= 3.839 \times 10^6$ m

The magnitude of the vertical velocity $= 0.98 \times 3.00 \times 10^8$ m s^{-1} $\times$ sin 60

$= 2.546 \times 10^6$ m s^{-1}

$T = 3.839 \times 10^6$ m $\div$ 2.546×10^6 m s^{-1} = **0.0151 s** = 15 ms

(b)

Pitch, $d = 0.98 \times 3.00 \times 10^8$ m s^{-1} $\times$ cos 60 $\times$ 0.0151 s = **2.22×10^6 m** = 2200 km

Tutorial 15.05

15.05.1

E = 3200 eV × 1.60 × 10^{-19} J eV^{-1} = **5.12 × 10^{-16} J**

15.05.2

500 km s^{-1} = 5.0 × 10^{5} m s^{-1}.

This represents a fraction of 0.0016 *c*. The Lorentz factor will be very close to 1.0, so relativistic effects can be ignored.

15.03.3

(a)

Ek = 1/2 × 6.64 × 10^{-27} kg × (5.0 × 10^{5} m $s^{-1})^{2}$ = 8.30 × 10^{-16} J

Ek = 8.30 × 10^{-16} J ÷ 1.60 × 10^{-19} J eV^{-1} = 5188 eV = **5.2 keV**

(b) Use:

$$r = \frac{mv}{BQ}$$

r = (6.64 × 10^{-27} kg × 5.0 × 10^{5} m s^{-1}) ÷ (2.3 × 10^{-5} T × 2 × 1.6 × 10^{-19} C)

r = 451 m = **450 m** (2 s.f.)

15.05.4

(a) Use:

$$r = \frac{mv \sin \theta}{BQ}$$

$r = (6.64 \times 10^{-27}\text{ kg} \times 5.0 \times 10^{5}\text{ m s}^{-1} \times \sin 70) \div (2.3 \times 10^{-5}\text{ T} \times 2 \times 1.6 \times 10^{-19}\text{ C})$

$\mathbf{r = 424\text{ m}}$

(b)

$t = (2 \times \pi \times 424\text{ m}) \div (5.0 \times 10^{5}\text{ m s}^{-1} \times \sin 70) = \mathbf{1.804 \times 10^{-3}\text{ s}}$

(c)

$d = (5.0 \times 10^{5}\text{ m s}^{-1} \times \cos 70) \times 1.804 \times 10^{-3}\text{ s} = \mathbf{309\text{ m}}$

15.05.5

$p = 5\text{ N} \times (800\text{ m})^{2} = \mathbf{8 \times 10^{-6}\text{ Pa}}$

15.05.6

(a)

$a = F/m = 10\text{ N} \div 1500\text{ kg} = \mathbf{6.67 \times 10^{-3}\text{ m s}^{-2}}$

(b)

$v = u + at = 5000\text{ m s}^{-1} + (6.67 \times 10^{-3}\text{ m s}^{-2} \times 86400\text{ s})$

$= 5000\text{ m s}^{-1} + 576\text{ m s}^{-1} = \mathbf{5576\text{ m s}^{-1}}$

Tutorial 15.06

15.06.1

a.

$\omega = 2\pi f$ = 2 × π × 6.0 Hz = **37.7 rad s^{-1}**

b.

$$x = A\sin(\omega t)$$

x = 0.20 m × sin (37.7 rad s^{-1} × 1.1 s)

= -**0.118 m** (i.e. 0.12 m below the zero displacement level)

c.

$$v = \frac{dx}{dt} = A\omega\cos(\omega t)$$

v = 0.20 m × 37.7 rad s^{-1} × cos (37.7 rad s^{-1} × 1.1 s) = **-6.10 m s^{-1}** (i.e. downwards)

d.

$$a = \frac{dv}{dt} = \frac{d_2x}{dt^2} = \frac{d}{dt}(A\omega\cos(\omega t)) = -A\omega^2\sin(\omega t)$$

a = -0.20 m × (37.7 rad s^{-1})^2 × sin (37.7 rad s^{-1} × 1.1 s) = **167 m s^{-2}**

15.06.2

ϕ = (2 × π × 0.20 m) ÷ 3.5 m = **0.36 rad**

15.06.3

(a) Use the equation:

$$c = f\lambda$$

λ = 3.5 m s^{-1} ÷ 50 m = 0.07 Hz

T = 1/f = 1 ÷ 0.07 Hz = **14.3 s** (QED)

(b)

x = 2.0 m × sin (2 × π × 0.07 Hz × 2.5 s) = **1.78 m**

(c)

y = 2.0 m × sin (2 × π × ((0.07 Hz × 2.5 s) - (6.5 m ÷ 50 m)))

= 2.0 m × sin (2 × π × ((0.175 - 0.13)) = 2.0 m × sin (2 × π × 0.045)

y = **0.58 m**

You did remember to set your calculator to **radians**, didn't you?

15.06.4

The period is 20 s.

The frequency is 0.050 Hz

15.06.5

f_1 = 0.50 Hz.

f_2 = 0.55 Hz.

f_{beats} = 0.55 Hz - 0.50 Hz = 0.050 Hz

This result is consistent with the data.

15.06.6

We can see that the peaks are in whole-number multiples of 55 Hz.

The higher peaks are in the odd-order harmonics.

Tutorial 15.07

15.07.1

The amplitude of the resultant would be small, but not zero.

15.07.2

Equation:

$$d(n_{glass} - n_{air}) = (m + ½)\,\lambda$$

$$\Delta x = 0.20\text{ m} \times (1.51 - 1.00) = 0.20\text{ m} \times 0.51 = 0.102\text{ m}$$

$$(m + ½) = 0.102\text{ m} \div (6.00 \times 10^{-3}\text{ m})$$

$m = 17 - ½ =$ **16 ½** wavelengths

15.07.3

(a) Equation:

$$\Delta x = d(n_{glass} - n_{air}) = m\,\lambda$$

$$\Delta x = 1.0 \times 10^{-4}\text{ m} \times (1.51 - 1.00) = 1.0 \times 10^{-4}\text{ m} \times 0.51 = 5.1 \times 10^{-5}\text{ m}$$

$$m = 5.1 \times 10^{-5}\text{ m} \div (6.00 \times 10^{-7}\text{ m})$$

$m = 85$ wavelengths

(b)

The bright spot would be unaffected, because there is a whole number of wavelengths.

The waves would still arrive in phase, so there would be constructive interference.

15.07.4

Use:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

(a)

$$1.00 \sin 31 = 1.51 \sin \theta_2$$

$$\sin \theta_2 = \sin 31 \div 1.51 = 0.341$$

$$\theta_2 = \sin^{-1} (0.341) = \underline{\mathbf{19.94^o}} = 20^o \text{ (QED)}$$

(b)

$$1.33 \times \sin \theta_3 = 1.51 \times \sin (19.94)$$

$$\sin \theta_3 = (1.51 \times \sin (19.94)) \div 1.33 = 0.387$$

$$\theta_3 = \sin^{-1} (0.387) = \underline{\mathbf{22.77^o}} = \mathbf{23^o} \text{ (2 s.f.)}$$

15.07.5

At the air-oil boundary, 5 % is reflected = 0.05 × 4.75 = 0.2375 units.

At the oil-water boundary 95 % is transmitted, so 5 % is reflected.

This is 0.05 × 0.2375 units = 0.0119 units.

At the oil-air boundary, 95 % is transmitted.

This is 0.95 × 0.011875 units = 0.0113 units.

The energy of the Ray 3 is much smaller than that of the Ray 2.

Therefore, it can be ignored.

15.07.6

The dark surface is because there is destructive interference.

Equation:

$$\frac{t}{\cos\theta_2} = \frac{m\lambda}{2n_{\text{oil}}}$$

Normally incident on the surface is a pompous way of saying that the angle of incidence is 0.

Therefore, the angle of refraction is zero, and cos 0 = 1.

$t = (10 \times 600 \times 10^{-9}\text{ m}) \div (2 \times 1.51) = \underline{\mathbf{2.0 \times 10^{-6}\ m}}$

15.07.7

This is because the refractive index of the glass is greater than the refractive index of the magnesium fluoride.

15.07.8

Equation:

$$d = \frac{\lambda}{4n_{\text{coating}}}$$

$d = 530 \times 10^{-9}\text{ m} \div (4 \times 1.38) = \underline{\mathbf{9.60 \times 10^{-8}\ m}}$

15.07.9

Air has a refractive index of 1.00.

15.07.10

There will be a dark fringe.

This is because the thickness is zero.

The path difference is zero, but there has been a phase change of π radians

when the ray reflected from boundary of the (extremely thin) air gap and the glass.

15.07.11

It shouldn't affect the fringe spacing as:

$$\Delta x = \frac{\lambda l}{2y}$$

The ratio x/y is constant as is l/t.

However, the fringes near the end where the slides touch can be unclear.

15.07.12

Use:

$$\Delta x = \frac{\lambda l}{2y}$$

Δx = (633 × 10^{-9} m × 0.080 m) ÷ (2 × 3.0 × 10^{-5} m) = **<u>8.44 × 10^{-4} m</u>** = 0.84 mm

15.07.13

Use:

$$\lambda = \frac{\Delta x d}{D}$$

Δx = 1.40 × 10^{-2} m ÷ 10 = 1.40 × 10^{-3} m

λ = (1.40 × 10^{-3} m × 0.400 × 10^{-3} m) ÷ (1.00 m) = **<u>5.60 × 10^{-7} m = 560 nm (to 3 s.f.)</u>**

Tutorial 15.08

15.08.1

(a)

$$I_v = 3.0 \text{ W m}^{-2} \times \cos 50 = \mathbf{\underline{1.9 \text{ W m}^{-2}}}$$

(b)

Work out the intensity absorbed:

$$I_h = 3.0 \text{ W m}^{-2} \times \sin 50 = 2.3 \text{ W m}^{-2}$$

$$\text{Area} = 0.10 \text{ m} \times 0.10 \text{ m} = 0.010 \text{ m}^2$$

$$\text{Power} = \text{intensity} \times \text{area} = 2.3 \text{ W m}^{-2} \times 0.010 \text{ m}^2 = \mathbf{\underline{2.3 \times 10^{-2} \text{ W}}} = 23 \text{ mW}.$$

15.08.2

$$50 = 100 \cos^2 \theta$$

$$\cos^2 \theta = 1/2$$

$$\cos \theta = (1/2)^{0.5} = 0.7071$$

$$\theta = \cos^{-1} (0.7071) = 45^{\circ} \text{ (QED)}$$

15.08.3

$$20 \text{ mV} = 60 \text{ mV} \cos^2 \theta$$

$$\cos^2 \theta = 20 \text{ mV} \div 60 \text{ mV}$$

$$\cos \theta = (1/3)^{0.5} = 0.5773$$

$$\theta = \cos^{-1} (0.5773) = \mathbf{\underline{54.7^{\circ}}} = 55^{\circ}$$

15.08.4

The LCD displays are off.

This is because the figures are dark.

The liquid crystals are arranged so that they don't change the orientation of the vertically polarised light.

15.08.5

The background is dark.

The numbers light up.

This is because the vertically polarised light is still twisted and is absorbed by the vertically polarised analyser.

When the display is on, the vertically polarised light passes through without an orientation change. Therefore, it passes through the vertically polarised analyser.

You can see this on LCDs on some petrol pumps.

15.08.6

Formula:

$$\tan\theta_1 = \frac{n_2}{n_1}$$

$\tan\theta_1 = (1.51 \div 1.33) = 1.135$

$\theta_1 = \tan^{-1}(1.135) =$ **48.6º**

15.08.7

Formula:

$$n_1 \sin \theta_1 = n_2 \sin \theta_1$$

For the ordinary ray:

$$1.000 \sin 35 = 1.6854 \sin \theta_2$$

$$\sin \theta_2 = \sin 35 \div 1.6854 = 0.34032$$

$$\theta_2 = \sin^{-1} (0.34032) = \mathbf{\underline{19.9^o}}$$

Similarly for the extraordinary ray:

$$\sin \theta_2 = \sin 35 \div 1.4864 = 0.38588$$

$$\theta_2 = \sin^{-1} (0.38588) = \mathbf{\underline{22.7^o}}$$

15.08.8

Randomly oriented light is polarised vertically by the first polaroid.

The sample acts as a second polarising filter,

changing the orientation of the light,

so that some of the light gets through the analyser.

Tutorial 15.09

15.09.1

If the ball were closer to the floor, it would not detect whether the ball was going downwards or upwards.

We need to know the speed going down and the speed going up.

15.09.2

$v = 0.675 \times 5.50 \text{ m s}^{-1} = \mathbf{3.71 \text{ m s}^{-1}}$

15.09.3

We consider the magnitudes of the velocities; they will be the same.

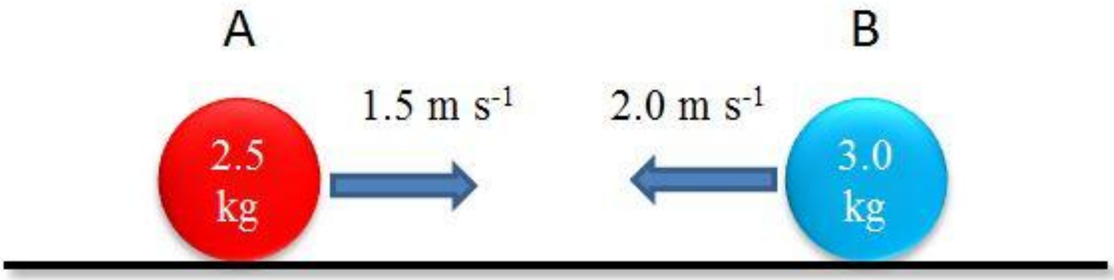
$$e = \frac{v_A - v_B}{u_A - u_B}$$

So, we get $e = (v - v) \div (u_A - u_B) = 0 \div (u_A - u_B) = 0$

It is a perfectly inelastic collision.

15.09.4

Work out the kinetic energies before



For A, E_k = 1/2 × 2.5 kg × (1.5 m s^{-1})^2 = 2.8125 J

For B, E_k = 1/2 × 3.0 kg × (2.0 m s^{-1})^2 = 6.000 J

Total = 8.8125 J

Now for after:

For A, E_k = 1/2 × 2.5 kg × (0.61 m s^{-1})^2 = 0.4651 J

For B, E_k = 1/2 × 3.0 kg × (0.24 m s^{-1})^2 = 0.0864 J

Total = 0.5515 J

Fraction of energy left over as kinetic energy = 0.5515 J ÷ 8.8125 J = **0.062** = 6.2 %

15.09.5

(a) Speed of drop:

$$(v_{\text{drop}})^2 = 2 \times 9.81 \text{ m s}^{-2} \times 1.0 \text{ m} = 19.62 \text{ m}^2 \text{ s}^{-2}$$

v_{drop} = **4.43 m s^{-1}** (which is about 4.4 m s^{-1})

(b) Equation:

$$e = \frac{v_{\text{after}}}{v_{\text{before}}}$$

v_{bounce} = 4.43 m s^{-1} × 0.75 = **3.32 m s^{-1}** = **3.3 m s^{-1}** (2 s.f.)

(c) Equation:

$$e^2 = \frac{h}{H}$$

h = $(0.75)^2$ × 1.0 m = **0.56 m**

Tutorial 15.10

15.10.1

Newton II: $F = ma$.

N= kg × m s^{-2}

N m^{-2} = kg m s^{-2} ÷ m^2 = **kg m^{-1} s^{-2}**

Work: $W = Fs$

J = N m = kg × m s^{-2} × m = kg m^2 s^{-2}

$p = W/V$ = kg m^2 s^{-2} ÷ m^3 = **kg m^{-1} s^{-2}**

The two are consistent

15.10.2

Equation:

$$p = \frac{1}{2}\rho v^2$$

p = 1/2 × 1000 kg m^{-3} × (3.5 m s^{-1})^2 = **6125 Pa**

15.10.3

$p = h\rho g$ = 20 m × 1025 kg m^{-3} × 9.81 m s^{-2} = **2.01 × 10^5 Pa**

15.10.4

(a) Equation:

$$\Delta p = p_0 - p = \frac{1}{2}\rho v^2$$

$\Delta p = p - p_0 = -(1/2 \times 1.23\ \text{kg m}^{-3} \times (5.5\ \text{m s}^{-1})^2) =$ **18.60 Pa** = **19 Pa** (2 s.f.)

(b)

Area = $\pi \times (0.30\ \text{m})^2 \div 4 = 0.07069\ \text{m}^2$

Force = 1.315 N = **1.3 N** (2 s.f.)

(c)

It would act upwards as pressure is less above the top surface.

(d)

The atmospheric pressure as we are interested only in the difference.

15.10.5

The path will be circular.

The force acting on the ball is always at right angles to the direction of motion.

The assumption is that the ball travels at a constant speed.

15.10.6

(a)

Rate of rotation = 500 r p m ÷ 60 s min^{-1} = 8.33 s^{-1}

Speed of circumference = π × 0.0675 m × 8.33 s^{-1} = **1.766 m s^{-1}** = 1.8 m s^{-1} (QED)

(b)

Low speed = 20 m s^{-1} - 1.766 m s^{-1} = 18.23 m s^{-1}.

High speed = 20 m s^{-1} + 1.766 m s^{-1} = 21.77 m s^{-1}

On the high pressure side:

Δp = 1/2 × 1.23 kg m^{-3} × (18.23 m s^{-1})^2 = 204.4 Pa

On the low pressure side:

Δp = 1/2 × 1.23 kg m^{-3} × (21.77 m s^{-1})^2 = 291.5 Pa

Since both pressures are in the same direction:

total pressure = 204.4 Pa + 291.5 Pa = **495.9 Pa**

(c)

Assume the pressure acts equally on the cross section area

Area = $\pi D^2/4$ = π × (0.0675 m)^2 ÷ 4 = 3.578 × 10^{-3} m^2.

F = 495.9 Pa × 3.578 × 10^{-3} m^2 = 1.775 N = **1.8 N** (2 s.f.)

(d)

Equation from circular motion:

$$F = \frac{mv^2}{r}$$

r = 0.0580 kg × (20 m s^{-1})^2 ÷ 1.77 N = 41.16 m = **41 m** (2 s.f.)

15.10.7

The term C_D has no units.

$$F_D = qC_DA$$

Units for F_D are N

Units for q are N m^{-2}

Units for A are m^2

Therefore:

N = N m^{-2} × no units × m^2

15.10.8

(a)

Acceleration = (20 m s^{-1} - 0) ÷ 0.10 s = **200 m s^{-2}**

(b)

Force needed = 0.60 kg × 200 m s^{-2} = **120 N**

(c) Equation:

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

Area of javelin = π × (0.030 m)^2 ÷ 4 = 7.07 × 10^{-4} m^2

F_D = 1/2 × 1.23 kg m^{-3} × (20 m s^{-1})^2 × 0.01 × 7.07 × 10^{-4} m^2 = **1.74 × 10^{-3} N**

(d) Equation:

$$s_x = 2v\cos\theta\left(\frac{-v\sin\theta}{g}\right)$$

s_x = 2 × 20 m s^{-1} × 0.7071 × ((-20 m s^{-1} × 0.7071) ÷ -9.81 m s^{-2}) = **40.8 m** = **41 m**

(e)

E_k = 1/2 × 0.600 kg × (20 m s^{-1})^2 = **120 J**

(f)

Lost energy = drag force × path length = 1.74 × 10^{-3} N × 56 m = **0.097 J**

15.10.9

(a)

Kinetic energy = 1/2 × 750 kg × (5.7 m s^{-1})^2 = **12200 J** (which is about 12 kJ).

(b) Use $v^2 = u^2 + 2as$:

0 = (5.7 m s^{-1})^2 + 2 × a × 40 m

a = -32.49 m^2 s^{-2} ÷ 2 × 40 m = **0.406 m s^{-2}**

(c) Using Newton II:

F_D = 750 kg × 0.406 m s^{-2} = **305 N**

Alternatively, using $W = Fs$:

F_D = 12200 J ÷ 40 m = **305 N**

(d) Use:

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

F_D = 1/2 × 1000 kg m^{-3} × (5.7 m s^{-1})^2 × 0.1 × 0.047 m^2 = **76 N**

(e)

Skin friction = 305 N - 76 N = 229 N = **230 N** (2 s.f.)

15.10.10

Use:

$$v = \frac{\eta R_e}{r\rho}$$

v = (1.93 × 10^{-3} Pa s × 1000) ÷ (0.005 m × 832 kg m^{-3}) = **0.46 m s^{-1}**

Tutorial 15.11

15.11.1

Reasons include:

- Not all places receive the same radiation intensity - the tropics get more, the poles get less.
- Some radiation is reflected by clouds and the snow on mountains and the poles (the **albedo**).
- Water reflects radiation much more, while land absorbs radiation more.
- Radiation is reflected back towards the earth by clouds and greenhouse gases.
- The Sun's radiation is much more at the tropics where the Sun is overhead, while at high latitudes, the intensity is much less.

15.11.2

Heat is a flow of energy from a hot object to a cold object. It is not a reflection of temperature.

Temperature reflects the internal energy of molecules, i.e. the extent to which the molecules are vibrating and the speeds at which they are moving.

15.11.3

$$E = hc/\lambda = (6.63 \times 10^{-34}\ \text{J s} \times 3.00 \times 10^{8}\ \text{m s}^{-1}) \div 15000 \times 10^{-9}\ \text{m} = \underline{\mathbf{1.326 \times 10^{-20}}}\ \text{J}$$

$$E = 1.326 \times 10^{-20}\ \text{J} \div 1.602 \times 10^{-19}\ \text{J eV}^{-1} = 0.0828\ \text{eV} = \underline{\mathbf{0.083\ eV}}\ \text{to 2 s.f.}$$

15.11.4

Points include:

- The intensity is much reduced;
- At 250 nm, the intensity at sea-level is about half that outside the atmosphere.
- The short-wavelength UV radiation is filtered out by the atmosphere;
- A small proportion reaches sea-level (which is why you need your sunscreen).

15.11.5

Points include:

- Ozone absorbs the greater part of UV-C and some UV-B.
- Ozone was reacting with the chlorofluorocarbons.
- Therefore, there was less ozone to absorb the UV-C.
- Therefore, more UV-C could penetrate to the ground;
- This could be damaging to living organisms, both plants and animals.
- It can inhibit photosynthesis.

15.11.6

Equation:

$$\lambda_{\max} T = \textbf{constant} = \textbf{0.00289 m K}$$

$$\mathrm{T} = 0.00289 \text{ m K} \div 10.5 \times 10^{-6} \text{ m} = \underline{\mathbf{275\ K}}$$

15.11.7

We use:

$$P = 4\pi r^2 \sigma T^4$$

$$P = 4 \times \pi \times (6.96 \times 10^{8} \text{ m})^2 \times 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \times (6000 \text{ K})^4$$

$$P = 4.47 \times 10^{26} \text{ W}$$

The power per unit area $= 4.47 \times 10^{26} \text{ W} \div 6.09 \times 10^{18} \text{ m}^2 = \underline{\mathbf{7.34 \times 10^{7}\ W\ m^{-2}}}$

Peak wavelength $\lambda_{\max} = 0.00289 \text{ m K} \div 6000 \text{ K} = \underline{\mathbf{4.82 \times 10^{-7}\ m}} = 482 \text{ nm}$

15.11.8

Volume of the cannon ball:

$$V = \frac{4}{3}\pi r^3$$

V = 4/3 × π × (0.050 m)^3 = 5.26 × 10^{-4} m^3

(You remembered to use the radius, not the diameter, didn't you?)

m = 5.26 × 10^{-4} m^3 × 7900 kg m^{-3} = 4.14 kg

(a)

The weight = 4.14 kg × 9.81 m s^{-2} = **40.6 N**

(b)

Volume of water displaced = 5.26 × 10^{-4} m^3

Mass of water = 5.26 × 10^{-4} m^3 × 1030 kg m^{-3} = 0.542 kg

Weight of water = 0.542 kg × 9.81 m s^{-2} = 5.31 N

Force needed = weight - upthrust = 40.6 N - 5.31 N = **35.3 N** = 35 N

The depth does not matter as the density of the water is constant at any depth, as water is incompressible. The weight of water displaced will be the same, so the upthrust would be the same.

15.11.9

Mass = density × volume = 919 kg m^{-3} × 27 × 10^{-6} m^3 = 0.0248 kg

Weight = 0.0248 kg × 9.81 m s^{-1} = **0.243 N**

15.11.10

Mass = 0.0248 kg

Volume = mass ÷ density = 0.0248 ÷ 1000 kg m^{-3} = 2.48×10^{-5} m^3

Volume in cm^3 = **24.8 cm^3**

The water would rise to the 225 cm^3 mark (224.8 if you are fussy).

15.11.11

It would contribute to the rise in sea-levels, since it has come from land, especially if more water was coming into the sea than was being removed by water leaving the sea as a result of evaporation.

Tutorial 15.12

15.12.1

Fusion is about getting small nuclei to join together. It occurs under extreme conditions of temperature, which ensure that the nuclei have sufficient energy to overcome the strong nuclear force. Specific products are made. The binding energy per nucleon increases by a large amount per fusion event.

Fission only happens with specific large nuclei such as uranium-235. The nuclei have to capture a neutron which makes them very unstable. The nuclei wobble and split, releasing energy. The products of fission cannot be predicted. The increase in energy per nucleon is much smaller in each fission event than that of fusion.

15.12.2

Mass on the left hand side = 3.3425×10^{-27} kg + 6.6425×10^{-27} kg = 9.985×10^{-27} kg

Mass on right hand side = 6.6465×10^{-27} kg + 1.675×10^{-27} kg = 8.3215×10^{-27} kg

Mass deficit = 9.985×10^{-27} kg - 8.3215×10^{-27} kg = 1.6635×10^{-27} kg

Energy = 1.6635×10^{-27} kg × $(3.0 \times 10^{8}\text{ m s}^{-1})^2$ = **1.50×10^{-11} J**

Energy in eV = 1.50×10^{-11} J ÷ 1.6×10^{-19} J eV^{-1} = **9.4×10^{8} eV** = 940 MeV

15.12.3

Equation:

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

$I_1 = (1.50 \times 10^{11}\text{ m} \div 6.96 \times 10^{8})^2 \times 1400\text{ W m}^{-2}$ = **6.5×10^{7} W m^{-2}**

15.12.4

480 cells would be needed in the series array.

15.12.5

4 × 2.5 = **10 A**

15.12.6

Current = 4800 W ÷ 240 V = 20 A.

Number of parallel arrays is 20 ÷ 2.5 = 8.

Each array needs 480 cells to give out 240 V.

Therefore 480 × 8 = **3480 cells**.

They should be arranged in 8 parallel arrays of 480 cells in series.

15.12.7

(a)

Power = intensity × area = 500 W m^{-2} × 25 × 10^{-4} m^2 = **1.25 W**

(Did you remember to convert cm^2 to m^2)

(b)

Real power = (10 ÷ 100) × 1.25 W = **0.125 W**

(c)

Current (A) = 0.125 W ÷ 1.2 V = **0.104 A** (= 104 mA)

(d)

Charge time = 600 mAh ÷ 104 mA = **5.8 h**

15.12.8

(a) Work out area:

$A = \pi \times (20.0\ \text{m})^2 =$ **<u>1257 m²</u>**

Work out the power:

$$P = \frac{1}{2}\rho A v^3$$

P = 1/2 × 1.2 kg m^{-3} × 1257 m^2 × (10.0 m s^{-1})^3 = 7.542 × 10^5 W = **<u>7.5 × 10^5 W</u>** (= 750 kW).

2 s.f. is appropriate here as density was given to 2 s.f.

(b)

This assumes that the machine is 100 % efficient, which it won't be.

15.12.9

Work out area:

$A = \pi \times (20.0\ \text{m})^2 = 1257\ \text{m}^2$

Work out the power:

$$v^3 = \frac{2P}{\rho A}$$

$v^3 = (2 \times 1.0 \times 10^6\ \text{W}) \div (1.2\ \text{kg m}^{-3} \times 1257\ \text{m}^2) = 1326\ \text{m}^3\ \text{s}^{-3}$

$v = (1326\ \text{m}^3\ \text{s}^{-3})^{1/3} =$ **<u>11 m s^{-1}</u>**

This is equivalent to a fresh or strong breeze.

15.12.10

(a) Work out area and flow rate:

$$A = \pi \times (0.50\ \text{m})^2 = 0.785\ \text{m}^2$$

$$\text{Flow rate, } r = 0.785\ \text{m}^2 \times 10\ \text{m s}^{-1} = \underline{\mathbf{7.85\ m^3\ s^{-1}}}$$

(b) Work out the power:

$$P = \rho r g \Delta h k$$

$$P = 1000\ \text{kg m}^{-3} \times 7.85\ \text{m}^3\ \text{s}^{-1} \times 9.81\ \text{m s}^{-2} \times 60\ \text{m} \times 0.60$$

$$= 2.77 \times 10^6\ \text{W} = \underline{\mathbf{2.8 \times 10^6\ W}} = 2.8\ \text{MW}$$

15.12.11

Advantages:

- The energy is renewable and produces no carbon dioxide;
- The tides are predictable;
- The tidal lagoon can be used for recreational purposes, e.g. sailing;
- The power stations are not as visually intrusive as conventional power stations, nuclear power stations, or wind farms;
- The power stations are long-lived (the power station at La Rance was commissioned in 1966 and is still generating 50 years later).

Disadvantages:

- The power stations are very expensive;
- Storage lagoons will flood mudflats which are habitats for many wading birds;
- The power station is useless when the level of the water in the storage pool is at the same level as the sea;
- Marine animals can be killed or seriously injured if they pass through the turbines;
- They can cut off rivers from shipping (although locks can be included in the design.

15.12.12

(a) Use the formula:

$$P = \rho r g \Delta h k$$

P = 1000 kg m^{-3} × 21 m^{3} s^{-1} × 9.81 m s^{-2} × 300 m × 1

P = **6.2 × 10^{7} W** = 62 MW

(b)

Efficiency = (62 MW ÷ 75 MW) × 100 % = 83 %

15.12.13

Relative mass of U-238 hexafluoride = 238 + (6 × 19) = **352**

Relative mass of U-235 hexafluoride = 235 + (6 × 19) = **349**

The difference in the relative masses is **3**, or **0.85 %**

15.12.14

Equation

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + {}^{0}_{-1}\mathrm{e}^{-} + \bar{\nu}_e$$

15.12.15

The helium is inert and can be vented to the atmosphere.

It will diffuse to the edge of the atmosphere.

The isolated neutron is not a stable particle.

It decays into a proton by beta minus decay with a lifetime of about 8 minutes.

Tutorial 15.13

15.13.1

Diesel and petrol are both liquids, so they tend to be sold by the litre, rather than mass. Since the density of diesel is greater than that of petrol, there is a greater mass in each litre than petrol. In other words, the tank of diesel has a greater mass (hence more energy) than an identical tank of petrol.

15.13.2

Diesel and petrol are both liquids, so can be carried about in a tank at atmospheric pressure. Gases need to be compressed to high pressure to accommodate the amounts required for a useful range in a car. High pressure gas tanks are heavy bulky items that take up a lot of space in the boot.

If a fuel tank ruptures in a bad accident, fuel will leak out (dangerous enough), while a ruptured high pressure gas tank will explode, which would be catastrophic.

15.13.3

(a) 50 litres = 0.050 m^3

Mass = density × volume = 750 kg m^{-3} × 0.050 m^3 = **37.5 kg**

(b)

Number of moles in each kg = 1.0 kg ÷ 2.0 × 10^{-3} kg = **500 mol**

(c)

Mass = (49 MJ kg^{-1} ÷ 130 MJ kg^{-1}) × 37.5 kg = **14.1 kg**

(d)

Number of moles = 14.1 kg × 500 mol kg^{-1} = **7067 mol**

(e)

Volume = 7067 mol × 0.024 m^3 mol^{-1} = **170 m^3**

(f) Pressure:

$$p_1V_1 = p_2V_2$$

1 atm × 170 m^3 = p_2 × 0.050 m^3

p_2 = 3390 atm = **3400 atm** (2 s.f.)

15.13.4

The number of series fuel cells needed = 350 V ÷ 0.7 V per cell = 500 cells

15.13.5

(a)

$I = P/V$ = 75000 W ÷ 350 V = **214 A**

(b)

There need to be 214 ÷ 0.8 = 268 parallel arrays.

(c)

Total number = 268 × 500 = 134000 cells.

15.13.6

(a) Equation:

$$H_{2(g)} \rightarrow 2H^+_{(aq)} + 2e^-$$

(b)

Number of electrons per second = 200 A × 1.60 × 10^{-19} C = **1.25×10^{21} s^{-1}**

(c) 2.0 g is 1 mole of H_2 which gives off 2 moles of electrons = 2 × 6.02 × 10^{23} electrons

Time = 12.04 × 10^{23} ÷ 1.25 × 10^{21} s^{-1} = **963 s** (= 16 minutes)

(d) Distance = (963 s ÷ 3600 s h^{-1}) × 100 km h^{-1} = **26.75 km** = 27 km (2.s.f)

(e) This has assumed that the process is 100 % efficient. In reality the efficiency is not 100 %, so more hydrogen fuel is needed. If the efficiency were 50 % (more likely) the distance travelled would be 13 km.

Tutorial 15.14

15.14.1

Equation:

$$\frac{\Delta Q}{\Delta t} = AK\frac{\Delta\theta}{\Delta x}$$

Left hand side units = energy/time = J s^{-1}

Right had side units = area × thermal conductivity × temperature gradient

= m^2 × (K) × °C m^{-1}

Right hand side units = m × (*K*) × °C

Put the two together:

J s^{-1} = m × (*K*) × °C

Units for *K* = J s^{-1} ÷ (m × °C) = J s^{-1} × m^{-1} × $°C^{-1}$.

Since J s^{-1} = W, and $°C^{-1}$ = K^{-1}, we can rewrite this as W **m^{-1} K^{-1}**

(Note that °C was used as the code *K* was used for the thermal conductivity to avoid confusion.)

15.14.2

Equation:

$$\frac{\Delta Q}{\Delta t} = AK\frac{\Delta\theta}{\Delta x}$$

(a)

Temperature gradient = 25 K ÷ 0.006 m = **4167 K m^{-1}** (≈ 4200 K m^{-1})

(b)

Area = 1.3 m × 2.0 m = 2.6 m^2

Heat flow = 2.6 m^2 × 0.80 W m^{-1} K^{-1} × 4167 K m^{-1} = **8.7 × 10^3 W**

15.14.3

Equation:

$$\frac{\Delta Q}{\Delta t} = UA\Delta\theta$$

(a)

Area = 1.3 m × 2.0 m = 2.6 m^2

Δθ = 300 W ÷ (5.1 W m^{-2} K^{-1} × 2.6 m^2) = 22.6 ºC

As it's -5.0 ºC outside, the room temperature will be **17.6 ºC** or 18 ºC (to 2 s.f.)

(b)

Thickness of the glass is not needed.

15.14.4

Equation:

$$\rho = \frac{AR}{l}$$

(a)

σ = 1/ρ

σ = *l*/*AR*

But *G* = 1/*R*

Therefore:

$$\sigma = \frac{lG}{A}$$

(b) Rearrange:

$$\frac{\Delta Q}{\Delta t} = AK\frac{\Delta \theta}{\Delta x}$$

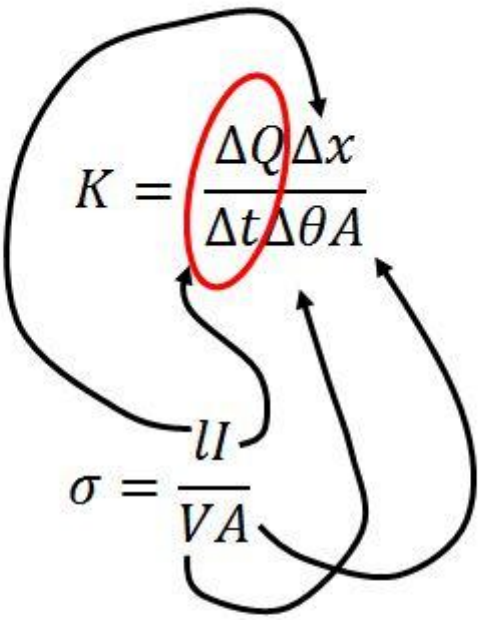
To:

$$K = \frac{\Delta Q \Delta x}{\Delta t \Delta \theta A}$$

Now express the electrical conductivity in terms of voltage and current:

$$\sigma = \frac{lI}{VA}$$

(c) Now compare the two relationships:



15.14.5

The U-values add up:

U value = 0.18 W m^{-2} K^{-1} + 5.1 W m^{-2} K^{-1} + 0.18 W m^{-2} K^{-1}

= 5.46 W m^{-2} K^{-1}

= **5.5 W m^{-2} K^{-1}** (2 s.f.)

15.14.6

Equation:

$$\frac{\Delta Q}{\Delta t} = UA\Delta\theta$$

Temperature difference = 21 °C - 10 °C = 11 °C = 11 K

Heat flow = 2.0 W m^{-2} K^{-1} × 1.35 m^2 × 11 K = **29.7 W** = 30 W (2 s.f.)

15.14.7

Equation:

$$U = \frac{K}{\Delta x}$$

U = 0.80 W m^{-1} K^{-1} ÷ 6.0 × 10^{-3} m = **133 W m^2 K^{-1}**

15.14.8

Equation:

$$U = \frac{K}{\Delta x}$$

Δx = 0.026 W m^{-1} K^{-1} ÷ 10.6 W m^{-2} K^{-1} = **0.00245 m** = 2.5 mm

Tutorial 15.15

Since there are no questions in this tutorial, there are no answers.

Tutorial 15.16

Since there are no questions in this tutorial, there are no answers.

Tutorial 15.17

Since there are no questions in this tutorial, there are no answers.

Tutorial 15.18

15.18.1

The particle is travelling faster than the speed of light.

15.18.2

d^2 = (4.0 m)^2 + (5.6 m)^2 + (3.8 m)^2 = 61.8 m^2

d = (61.8 m^2)$^{0.5}$ = **7.9 m**

15.18.3

Using:

$$U' = U\left(\sqrt{\frac{1+\beta^2}{1-\beta^2}}\right)$$

Work out the terms brackets first:

$1 + \beta^2 = 1 + 0.417^2 = 1.174$

$1 - \beta^2 = 1 - 0.417^2 = 0.826$

$U' \div U$ = (1.174 ÷ 0.826)$^{0.5}$ = 1.19 = **1.2** (2 s.f.)

Distance between graduations = 1.0 cm × 1.2 = **1.2 cm**

15.18.4

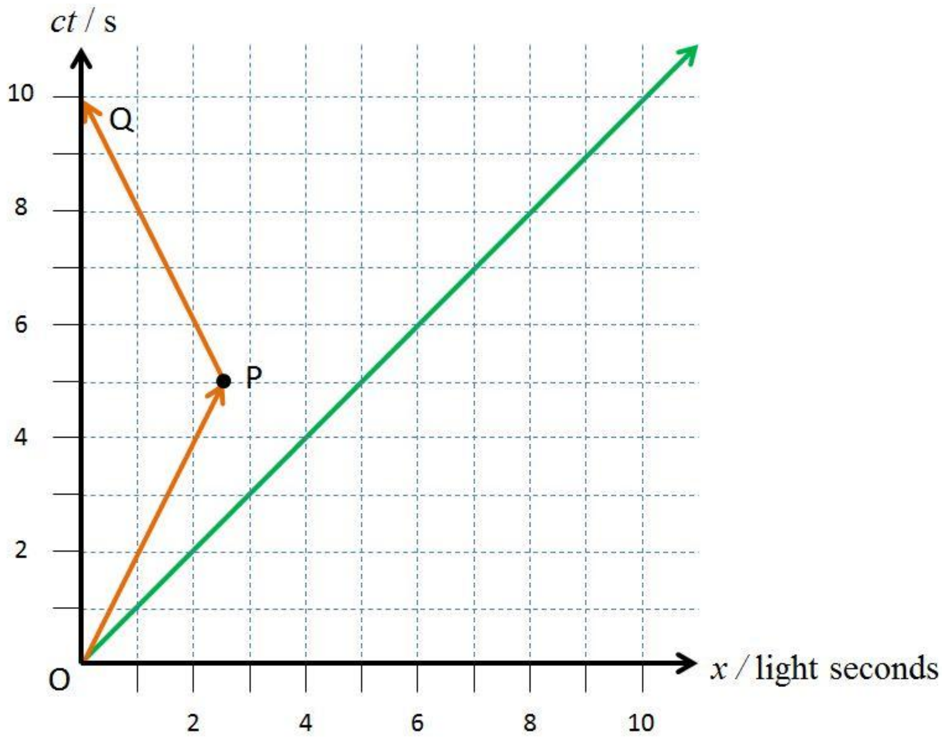
Event B. It happens at a lower value of the ct axis.

15.18.5

The line AC ($c\Delta t'$) is longer than AB ($c\Delta t$). In other words, t' is greater than t.

15.18.6

The Minkowski diagram is like this.



O is when the spacecraft leaves the Earth;

P is where it turns round;

Q is when it gets back to the Earth.

15.18.7

Work out v^2/c^2:

$$v^2/c^2 = (0.5\ c)^2 / c^2 = 0.25$$

Therefore:

$$1 - v^2/c^2 = 1 - 0.25 = 0.75$$

Therefore

$$\gamma = (0.75^{0.5})^{-1} = 0.866^{-1} = 1.15$$

$$t' = \gamma t_0$$

$$t' = 10\text{ s} \times 1.15 = \mathbf{\underline{11.5\ s}}$$

15.18.8

Use:

$$\tan\alpha = \frac{v}{c} = \beta$$

Therefore:

$$\tan\alpha = 0.5$$

Therefore

$\alpha = \tan^{-1} 0.5$ = **26.6°** = 27°

Tutorial 15.19

15.19.1

Use:

$$E = mc^2$$

$E = 9.11 \times 10^{-31}$ kg × $(3.0 \times 10^{8}$ m s$^{-1})^2 = 8.199 \times 10^{-14}$ kg m^2 s^{-2} = **8.2×10^{-14} J**

$E = 8.2 \times 10^{-14}$ J ÷ 1.6×10^{-19} J eV^{-1} = 5.12×10^{5} eV = **0.51 MeV** (2 s.f.)

15.19.2

Use:

$$E = mc^2$$

Convert the energy in eV to J:

$E = 938 \times 10^{6}$ eV × 1.6×10^{-19} J eV^{-1} = **1.5008×10^{-10} J**

Therefore:

$m = 1.5008 \times 10^{-10}$ J ÷ $(3.0 \times 10^{8}$ m s$^{-1})^2$ = **1.67×10^{-27} kg**

15.19.3

(a) Use:

$$E_{\text{tot}} = m_0c^2 + \frac{1}{2}mv^2$$

$E_{\text{tot}} = (1.67 \times 10^{-27}$ kg × $(3.0 \times 10^{8}$ m s$^{-1})^2) + (1/2 \times 1.67 \times 10^{-27}$ kg × $(1.2 \times 10^{5}$ m s$^{-1})^2)$

$= 1.503 \times 10^{-10}$ J + 1.2024×10^{-17} J = $1.50300012 \times 10^{-10}$ J

In eV:

E = $(1.503 \times 10^{-10}$ J ÷ 1.6×10^{-19} J eV$^{-1}) + (1.2024 \times 10^{-17}$ J ÷ 1.6×10^{-19} J eV$^{-1})$

$= 939 \times 10^{6}$ eV + 75.15 eV = 939000075 eV

(b)

$m = 1.50300012 \times 10^{-10}$ J ÷ $(3.0 \times 10^{8}$ m s$^{-1})^2 = 1.670000133 \times 10^{-27}$ kg.

Note the excessive amount of significant figures.

15.19.4

(a) Use:

$$E_{tot} = E_0 + E_k$$

$$E_{tot} = (1.67 \times 10^{-27}\ \text{kg} \times (3.0 \times 10^{8}\ \text{m s}^{-1})^{2}) + (3.76 \times 10^{-10}\ \text{J})$$

$$= 1.503 \times 10^{-10}\ \text{J} + 3.76 \times 10^{-10}\ \text{J} = \mathbf{\underline{5.26 \times 10^{-10}\ J}}$$

In eV:

$$E = (5.26 \times 10^{-10}\ \text{J} \div 1.6 \times 10^{-19}\ \text{J eV}^{-1}) = \mathbf{\underline{3.29 \times 10^{9}\ eV}} = 3.29\ \text{GeV}$$

15.19.5

Use:

$$m = \frac{E}{c^2}$$

E is in J = force × distance moved in direction of force = N m = kg m s^{-2} × m = kg m^2 s^{-2}

c^2 = m^2 s^{-2}

m = kg m^2 s^{-2} ÷ m^2 s^{-2} = kg

15.19.6

The particle has gained mass while travelling at 0.8 c.

As it slows down, that mass is converted to energy.

When it's travelling slowly, the classical equation for kinetic energy applies.

When it reaches zero speed, there remains only the rest mass.

15.19.7

(a)

$$E_0 = 1.67 \times 10^{-27}\text{ kg} \times (3.0 \times 10^{8}\text{ m s}^{-1})^2 = 1.50 \times 10^{-10}\text{ J.}$$

$$E_0 = 1.50 \times 10^{-10}\text{ J} \div 1.60 \times 10^{-19}\text{ J eV}^{-1} = 939 \times 10^{6}\text{ eV} = 939\text{ MeV.}$$

$$m_0 = \underline{\mathbf{939\ MeV\ c^{-2}}}.$$

(b)

$$\gamma = (1 - v^2/c^2)^{-0.5} = (1 - 0.5^2)^{-0.5} = (0.75)^{-0.5} = \underline{\mathbf{1.15}}$$

(c) Use:

$$p = \gamma m_0 v$$

$$p = 1.15 \times 939\text{ MeV c}^{-2} \times 0.5\text{ c} = \underline{\mathbf{542\ MeV\ c^{-1}}}$$

15.19.8

Use:

$$V = v^2\, m/2e$$

$$V = ((3.0 \times 10^{8}\text{ m s}^{-1})^2 \times 9.11 \times 10^{-31}\text{ kg}) \div (2 \times 1.6 \times 10^{-19}\text{ C})$$

$$= 2.56 \times 10^{5}\text{ V} = \underline{\mathbf{2.6 \times 10^{5}\ V}}$$

15.19.9

Use:

$$E_{tot} = E_k + E_0$$

$$m = \gamma m_0$$

$$mc^2 = 2.6 \times 10^5 \text{ eV} + 5.12 \times 10^5 \text{ eV} = 7.72 \times 10^5 \text{ eV}$$

$$\gamma \times 5.12 \times 10^5 \text{ eV} = 7.72 \times 10^5 \text{ eV}$$

$$\gamma = 7.72 \times 10^5 \text{ eV} \div 5.12 \times 10^5 \text{ eV} = \underline{\mathbf{1.508}}$$

To work out the speed, we need:

$$\gamma^{-2} = 1 - \frac{v^2}{c^2}$$

$$0.439 = 1 - v^2/c^2$$

Since we are working in fractions of c^2, we can write:

$$v^2 = (1 - 0.439)\, c^2 = 0.561\, c^2$$

$$v^2 = (0.561)^{0.5}\, c = \underline{\mathbf{0.75\ c}}$$

15.19.10

$$E^2 = p^2c^2 + m_0^2c^4$$

If $m_0 = 0$:

$$E^2 = p^2c^2 + 0$$

Therefore:

$$E = pc$$

15.19.11

$$p = \frac{E}{c}$$

Units for E are Joules = kg m^2 s^{-2}.

Units for c are m s^{-1}.

Units for p are kg m^2 s^{-2} ÷ m s^{-1} = kg m s^{-1}, which are the units for momentum.

15.19.12

$$p = \frac{h}{\lambda}$$

p = 6.63 × 10^{-34} J s ÷ 520 × 10^{-9} m = **1.28 × 10^{-27}** kg m s^{-1}.

15.19.13

The neutral pion consists of a quark-antiquark pair.

In this case it can be up and anti-up OR down and anti-down.

The quark and antiquark pair annihilate to give two gamma photons.

15.19.14

Use:

$$\lambda = \frac{h}{p}$$

We need to convert the energy into momentum.

p = 67.5 MeV c^{-1} = (67.5 × 10^6 eV × 1.6 × 10^{-19} J eV^{-1}) ÷ 3.0 × 10^8 m s^{-1}

= **3.6 × 10^{-20} kg m s^{-1}**

Now we can work out the wavelength:

λ = 6.63 ×10^{-34} J s ÷ 3.6 × 10^{-20} kg m s^{-1} = **1.8 × 10^{-14} m** (2 s.f.)

Tutorial 15.20

15.20.1

The results would show a red shift.

15.20.2

The photon is not absorbed.

It passes straight through the nucleus without interaction.

15.20.3

E = 56.935 u × 931.5 × 10^6 eV u^{-1} = **53.035 × 10^9 eV** = 53.035 GeV

15.20.4

E = (14.4 × 10^3 eV$)^2$ ÷ (2 × 53.035 × 10^9 eV) = **0.00195 eV**

15.20.5

Convert the energy in eV to J:

$E = 0.00195 \text{ eV} \times 1.60 \times 10^{-19} \text{ J eV}^{-1} = 3.128 \times 10^{-22} \text{ J}$

Nuclear mass = $56.935 \text{ u} \times 1.661 \times 10^{-27} \text{ kg u}^{-1} = 9.457 \times 10^{-26} \text{ kg}$

$v^2 = 2E_k/m = (2 \times 3.128 \times 10^{-22} \text{ J}) \div 9.457 \times 10^{-26} \text{ kg} = 6615 \text{ m}^2 \text{ s}^{-2}$

$\mathbf{v = 81.3 \text{ m s}^{-1}}$

15.20.6

Convert the mass in u to mass in kg:

Nuclear mass = 56.935 u × 1.661 × 10^{-27} kg u^{-1} = 9.457 × 10^{-26} kg.

Photon energy = 14.4 × 10^3 eV × 1.60 × 10^{-19} J eV^{-1} = 2.307 × 10^{-15} J

Rearrange:

$$\frac{E}{c} = mv$$

$v = E/mc$ = (2.307 × 10^{-15} J) ÷ (9.457 × 10^{-26} kg × 3.00 × 10^8 m s^{-1})

v = 81.3 m s^{-1}

The answers are consistent.

This is a valid approach, since the recoil speed of the nucleus is very much less than the speed of light.

15.20.7

Convert the energy in eV to energy in J:

E = 14.4 × 10^3 eV × 1.60 × 10^{-19} J eV^{-1} = 2.307 × 10^{-15} J

Use:

$$E = hf$$

f = 2.307 × 10^{-15} J ÷ 6.63 × 10^{-34} J s = 3.48 × 10^{18} Hz

15.20.8

Fractional difference in the frequency:

$$\Delta f = f - f_0 = \frac{gL}{c^2}$$

Fractional difference = (9.81 m s^{-2} × 22.6 m) ÷ (3.0 × 10^8 m s^{-1})^2 = 2.46 × 10^{-15}

15.20.9

Speed:

$$v = \frac{gL}{c}$$

v = (9.81 m s^{-2} × 22.6 m) ÷ (3.0 × 10^8 m s^{-1}) = 7.39 × 10^{-7} m s^{-1} = 7.4 × 10^{-7} m s^{-1} (QED)

You can also use the fractional difference = 2.463 × 10^{-15}

v = 2.463 × 10^{-15} × 3.00 × 10^8 m s^{-1} = **7.39 × 10^{-7} m s^{-1}**

15.20.10

Time after the zero point:

$v = -A\omega \sin(\omega t)$

Therefore:

$$\sin(\omega t) = \frac{-v}{-A\omega}$$

Minus signs cancel.

ω = 2 × π × 10 Hz = 20 π rad s^{-1}

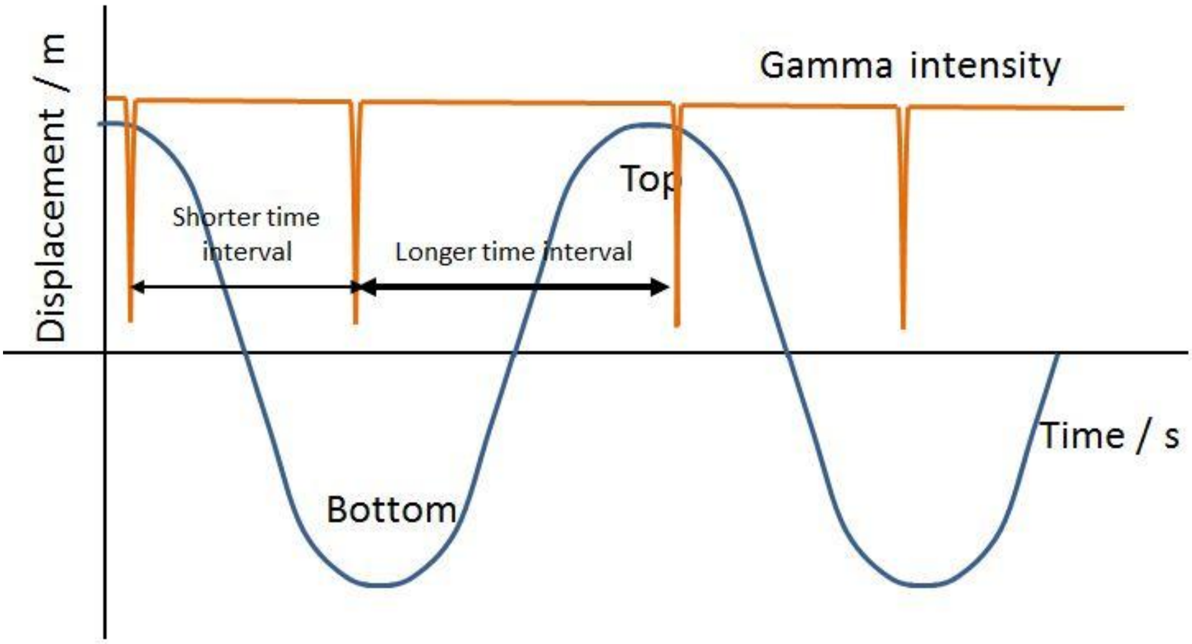
sin (20π t) = (7.4 × 10^{-7} m s^{-1}) ÷ (1.0 × 10^{-3} m × 20 π rad s^{-1}) = 1.178 × 10^{-5}

Now for very small angles, sin θ ≈ θ

20 π t = 1.178 × 10^{-5}

t = (1.178 × 10^{-5}) ÷ 20 π s^{-1} = **1.87 × 10^{-7} s**

15.20.11



Note that there are differences between the time intervals.

These are exaggerated for the purposes of illustration.

15.20.12

Equation:

$$\Delta t_0 = \Delta t_f \left(1 - \frac{2GM}{rc^2}\right)^{1/2}$$

$Dt_0 = 3600\ \text{s} \times (1 - ((2 \times 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 1.95 \times 10^{-30}\ \text{kg}) \div (2.00 \times 10^4\ \text{m} \times (3.0 \times 10^8\ \text{m s}^{-1})^2))^{0.5}$

$= 3600\ \text{s} \times (1 - (2.60 \times 10^{20}\ \text{m}^3\ \text{s}^{-2}) \div (1.80 \times 10^{21}\ \text{m}^3\ \text{s}^{-2}))^{0.5}$

$= 3600\ \text{s} \times (1 - 0.141)^{0.5}$

$= 3600\ \text{s} \times (0.856)^{0.5} = 3600\ \text{s} \times 0.925 = 3330\ \text{s}$

The crew on the surface would see the time period as being 5 minutes less. If they timed from the surface, the mother ship would observe the time interval as being 65 minutes.

Now the surface crew have got to think about how they get back to the mother ship - easier said than done.